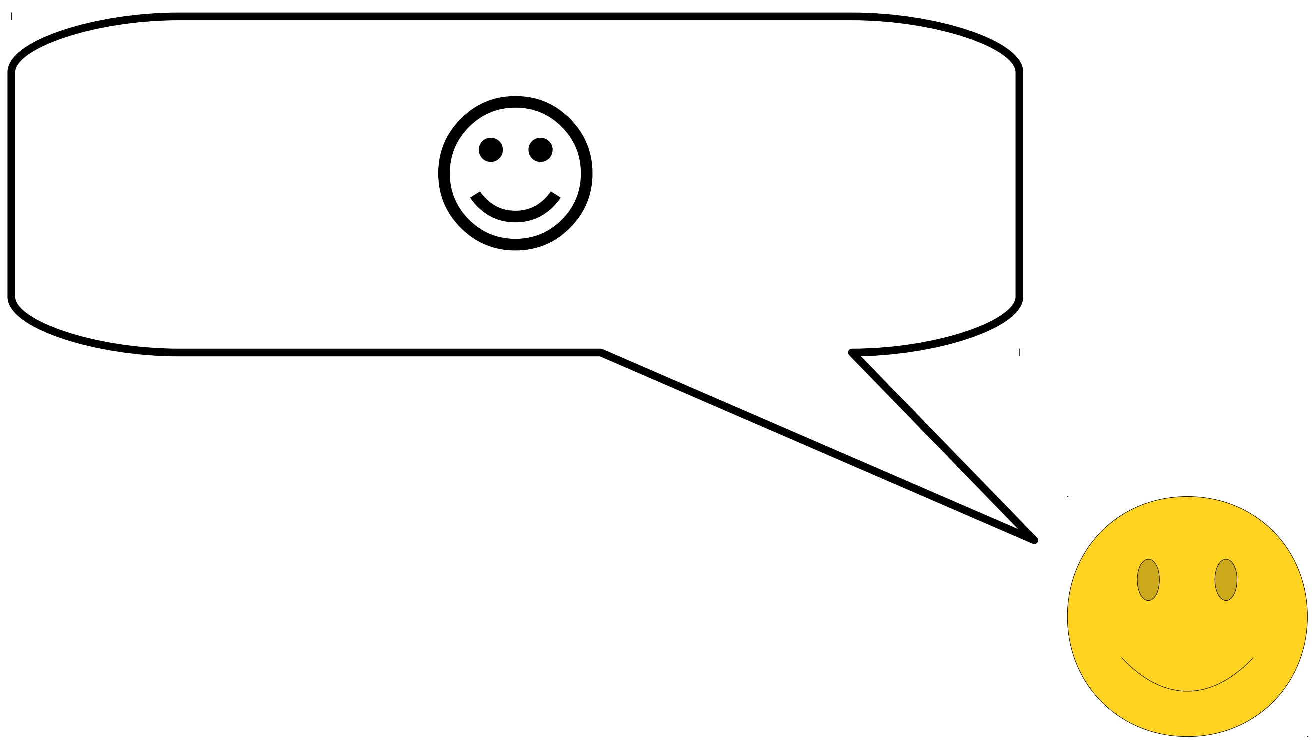
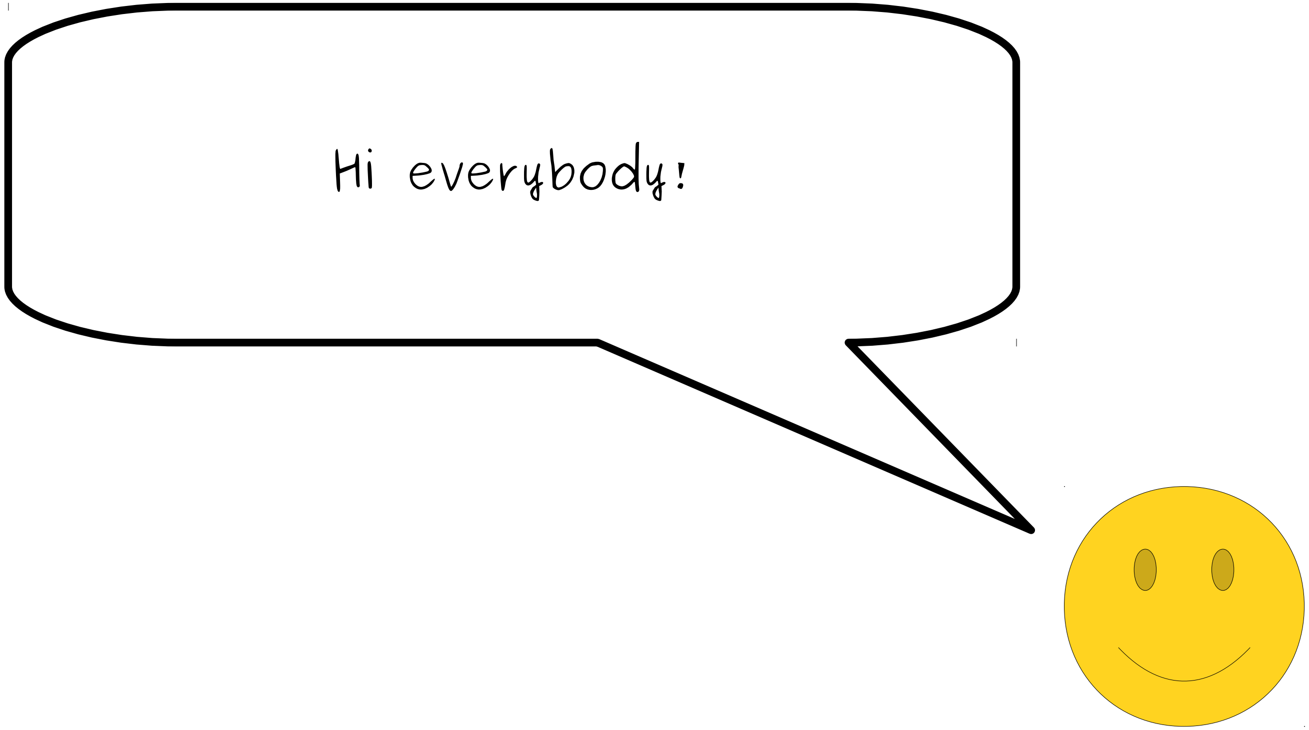
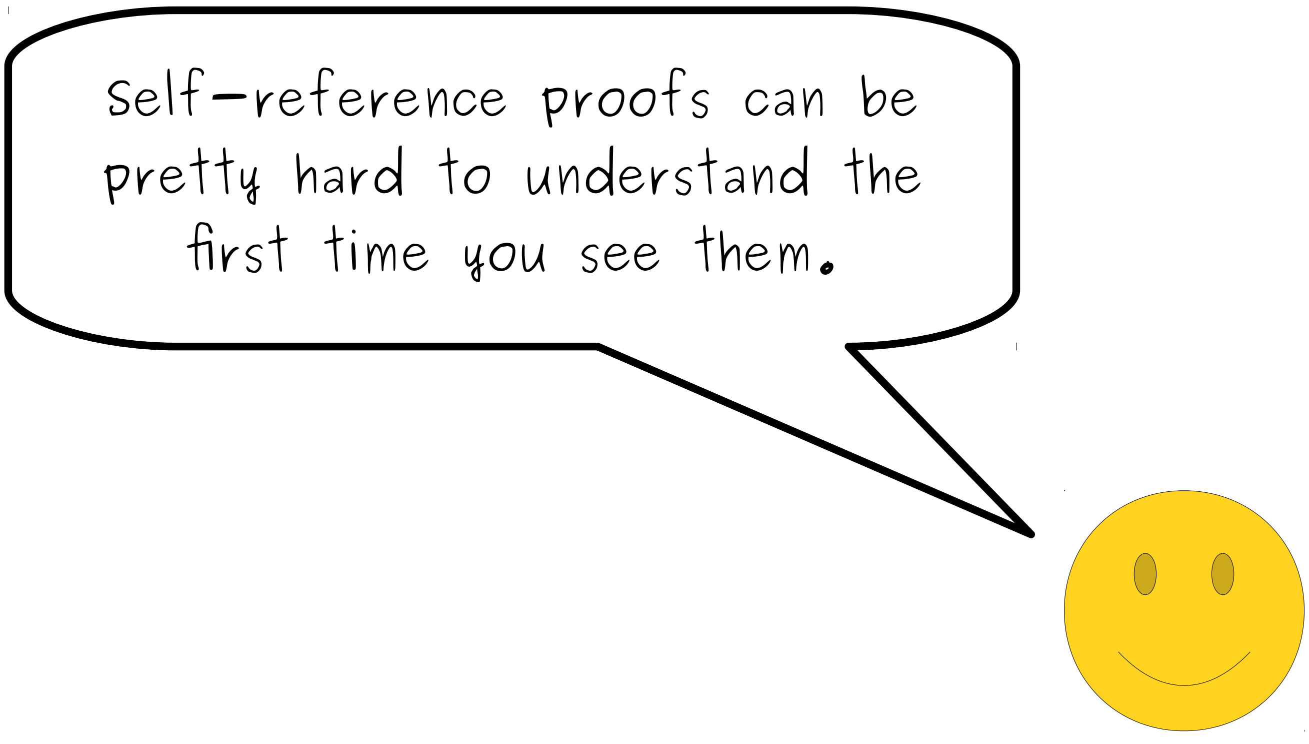
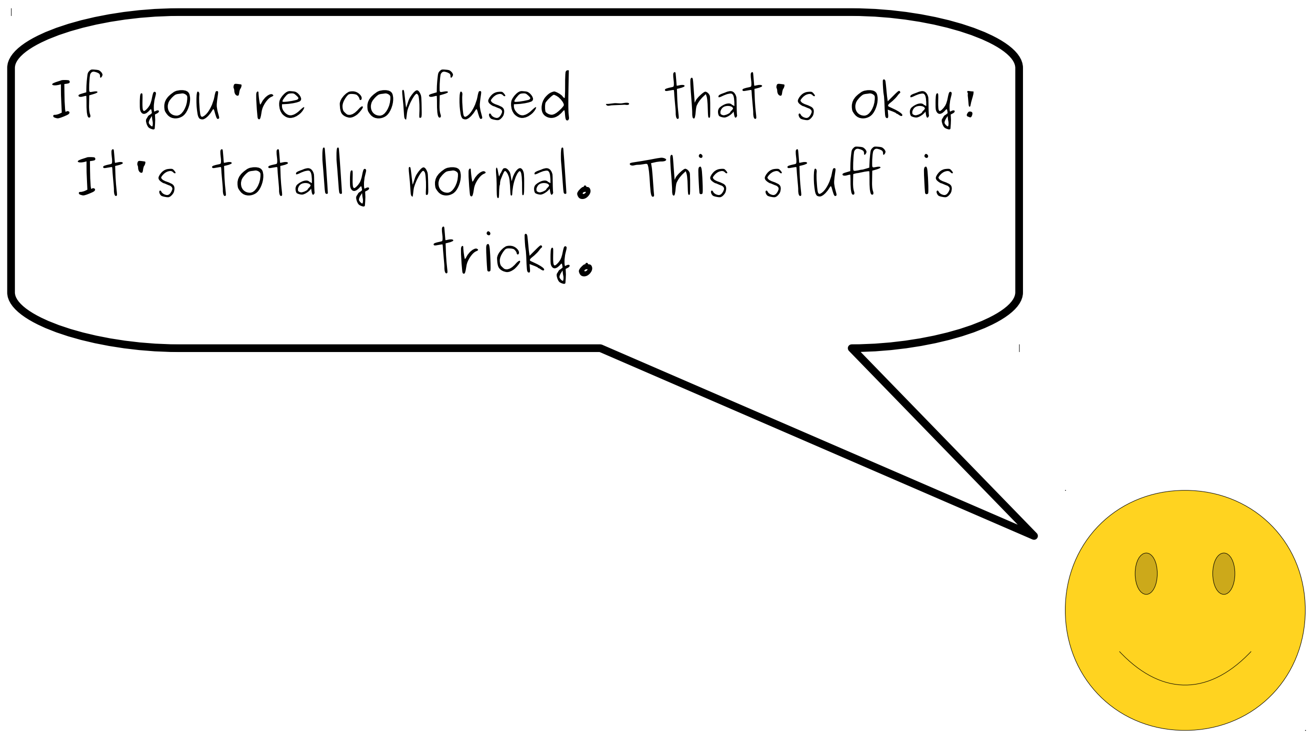
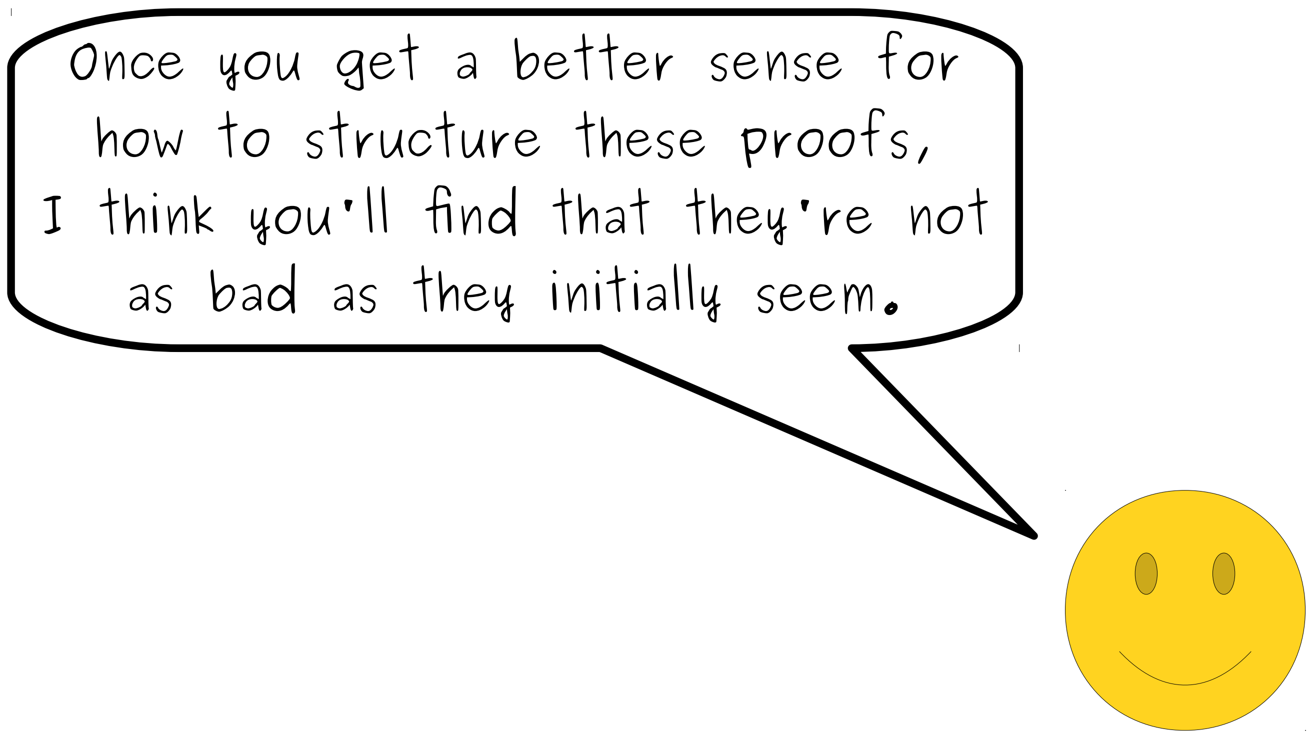
The Guide to Self-Reference




Hi everybody!


Self-reference proofs can be pretty hard to understand the first time you see them.


If you're confused - that's okay! It's totally normal. This stuff is tricky.


Once you get a better sense for how to structure these proofs, I think you'll find that they're not as bad as they initially seem.

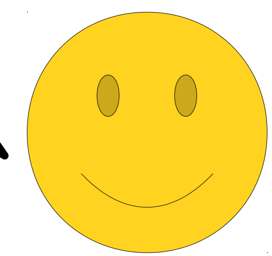
What does this program do?

```
bool willAccept(st
   /* … some im
}

int main() {
   string me = 
   string input

   if (willAccept(me, input)) {
      reject();
   } else {
      accept();
   }
}
```

This lecture slide was the first time that we really saw self-reference, and a lot of you got pretty tripped up by what was going on.



Try running this program on any input.
What happens if

… this program accepts its input?
It rejects the input!

… this program doesn't accept its input?
It accepts the input!

What does this program do?

```
bool willAcc
    /* … some
}

int main() {
    string me
    string inpu

    if (willAccept(me, input)) {
        reject();
    } else {
        accept();
    }
}
```

Part of the reason why this can be tricky is that what you're looking at is a finished product. If you don't have a sense of where it comes from, it's really hard to understand!

Try running this program on any input.
What happens if

… this program accepts its input?
It rejects the input!

… this program doesn't accept its input?
It accepts the input!

What does this program do?

```
bool willAccept(string program, string input) {
    /* … some implementation … */
}

int main() {
    string me = mySource();
    string input = getInput();

    if (willAccept(me, input)) {
        reject();
    } else {
        accept();
    }
}
```

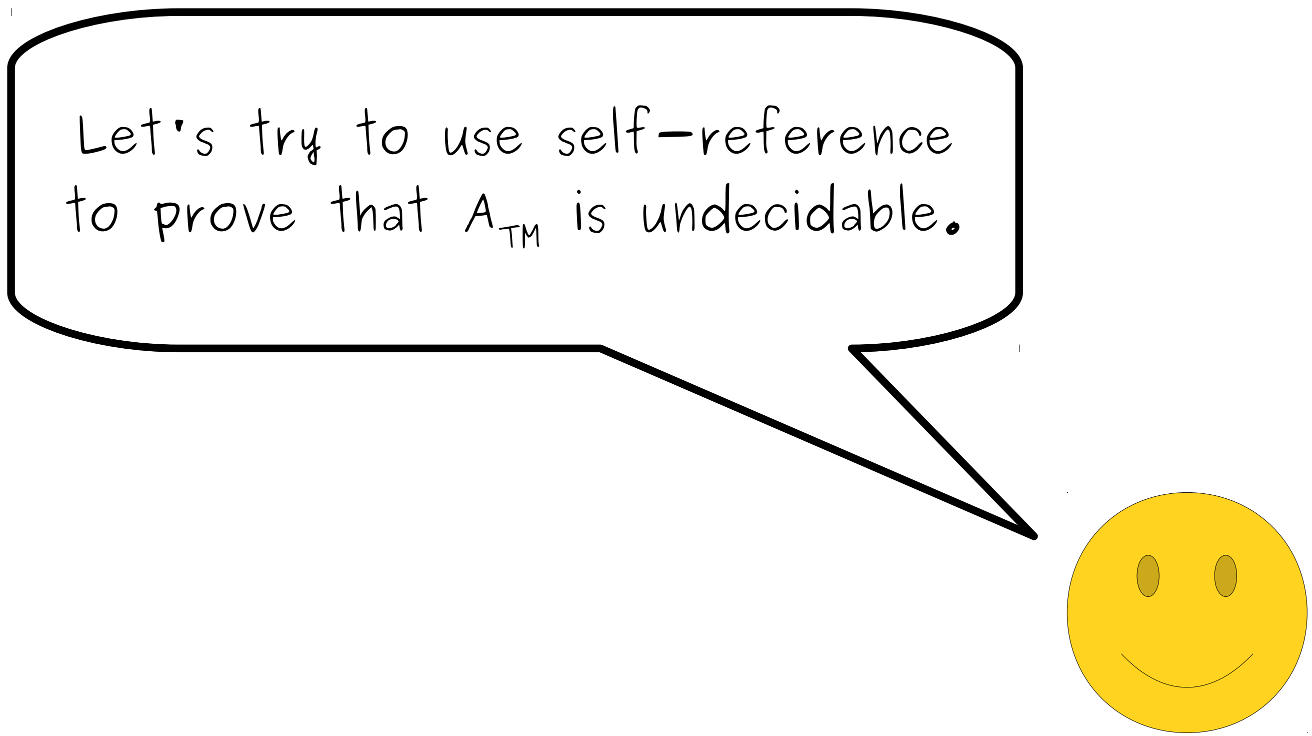
Let's see where it comes from!

We'll take it from the top.

Try running this program on any input.
What happens if

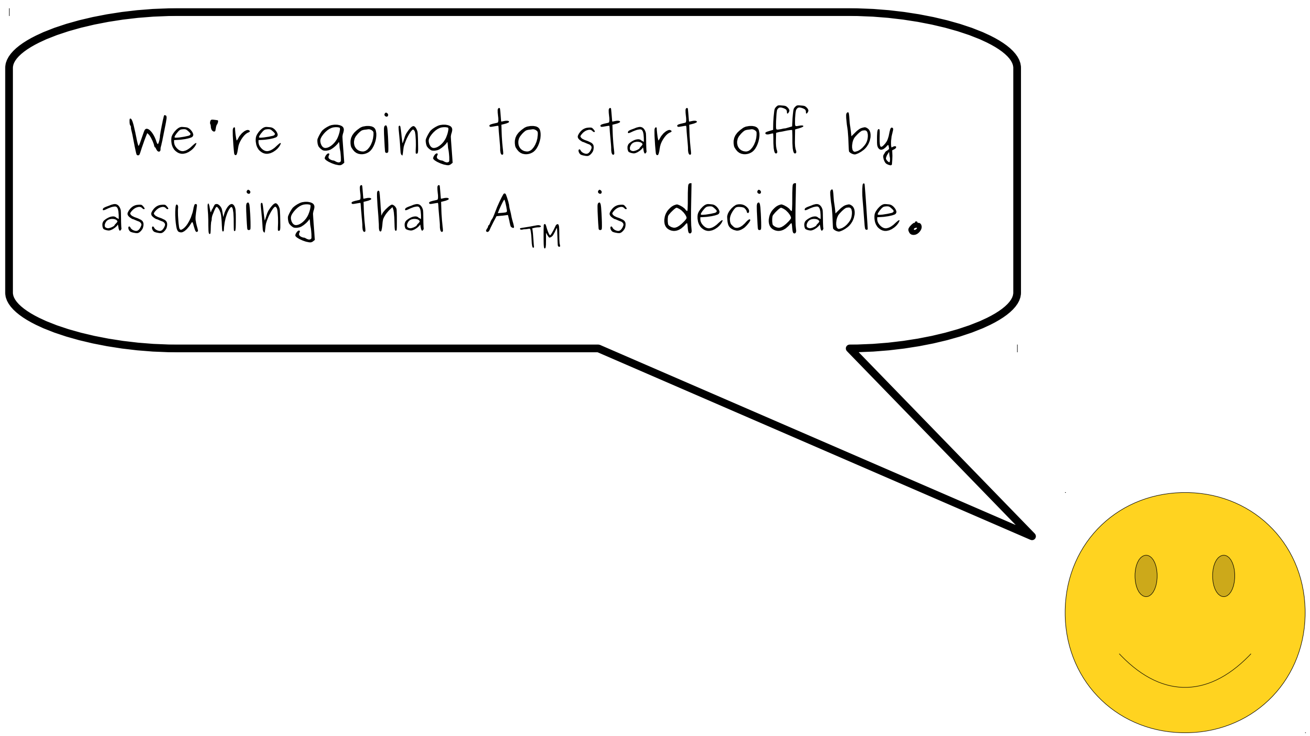
… this program accepts its input?
It rejects the input!

… this program doesn't accept its input?
It accepts the input!


Let's try to use self-reference to prove that A_{TM} is undecidable.


At a high level, we're going to do a proof by contradiction.

$A_{TM} \in \mathbf{R}$



$A_{TM} \in \mathbf{R}$

↓

Contradiction!

$A_{TM} \in \mathbf{R}$

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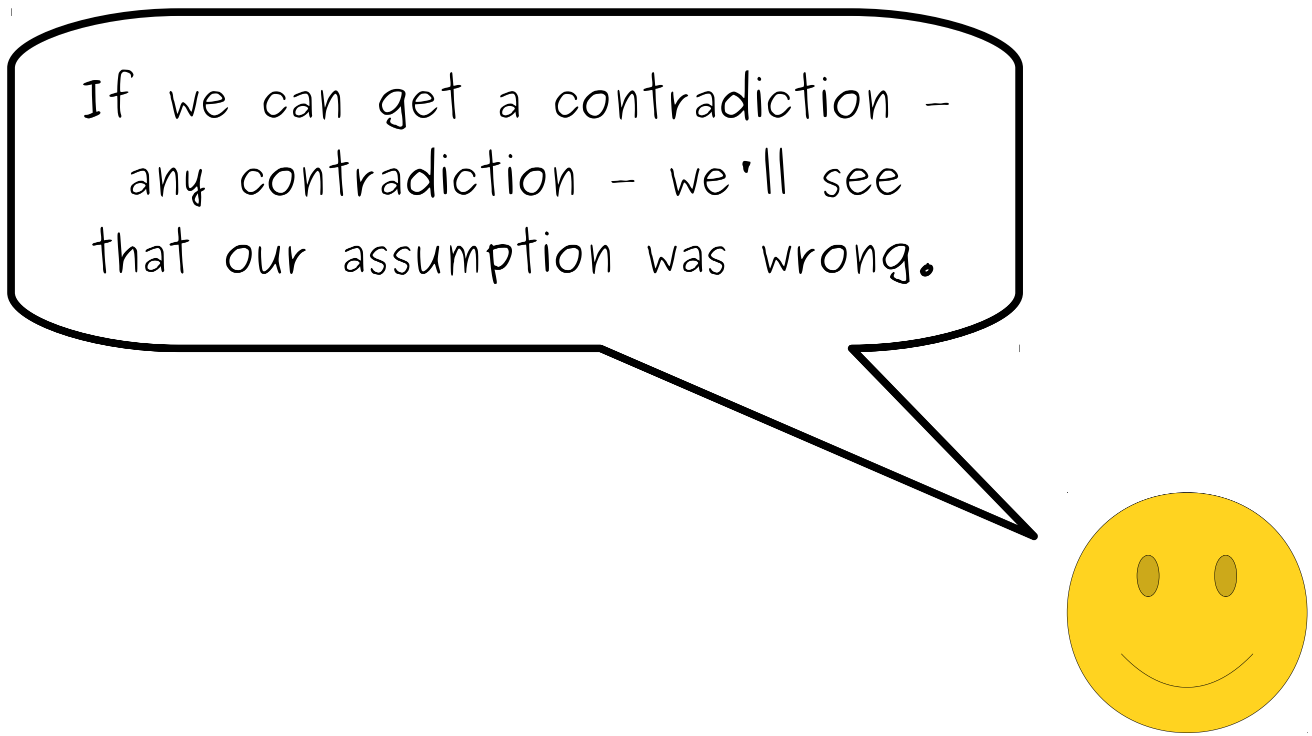

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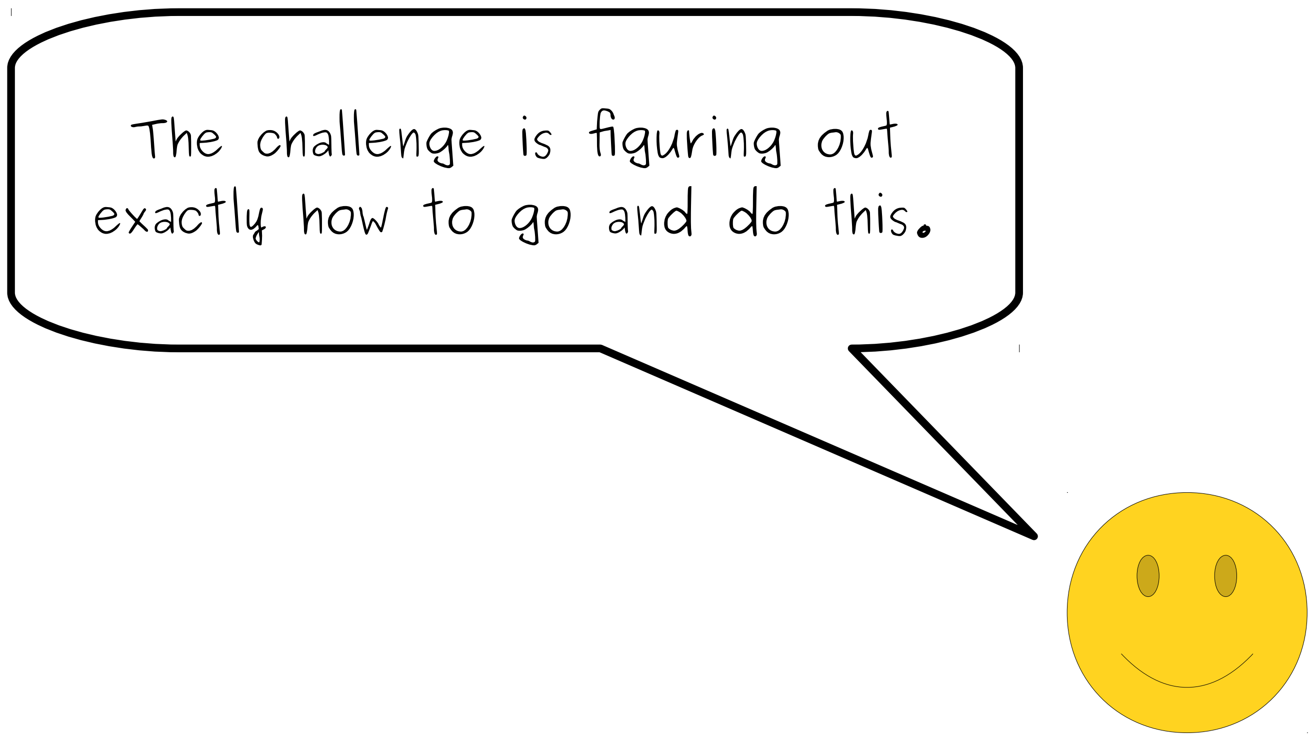

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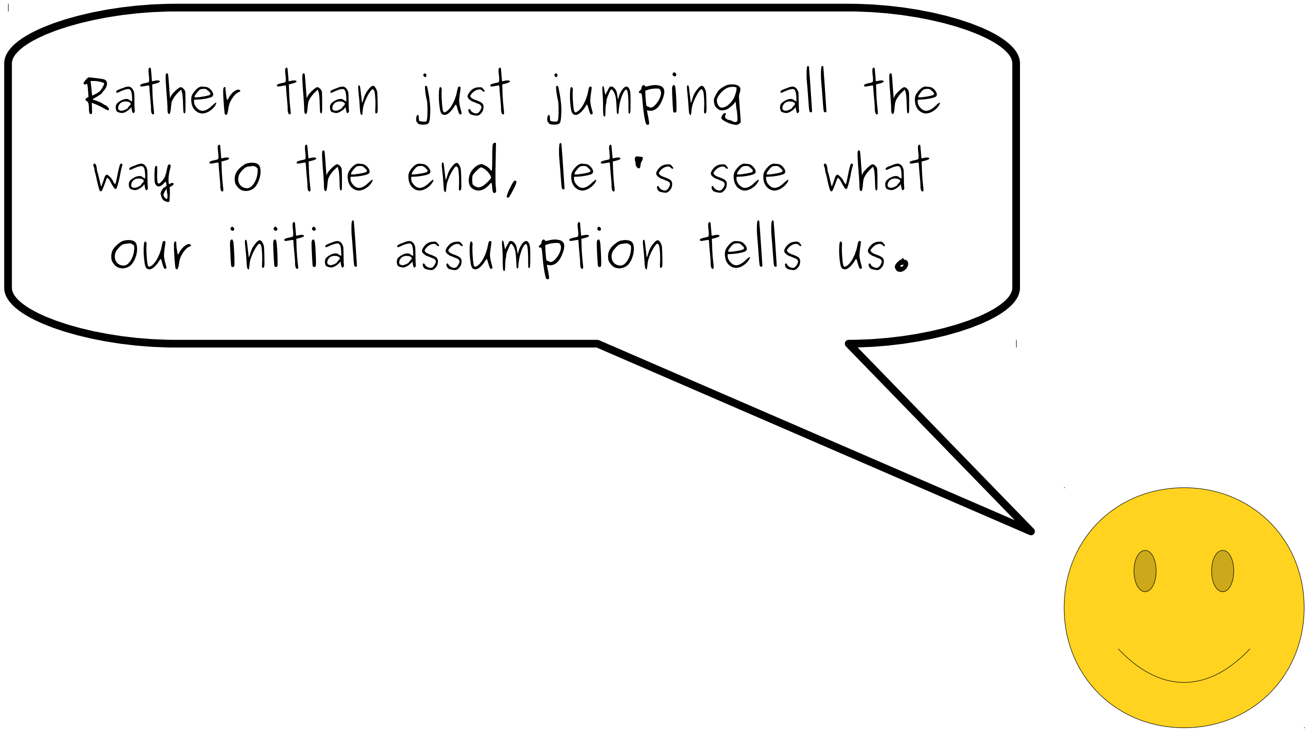

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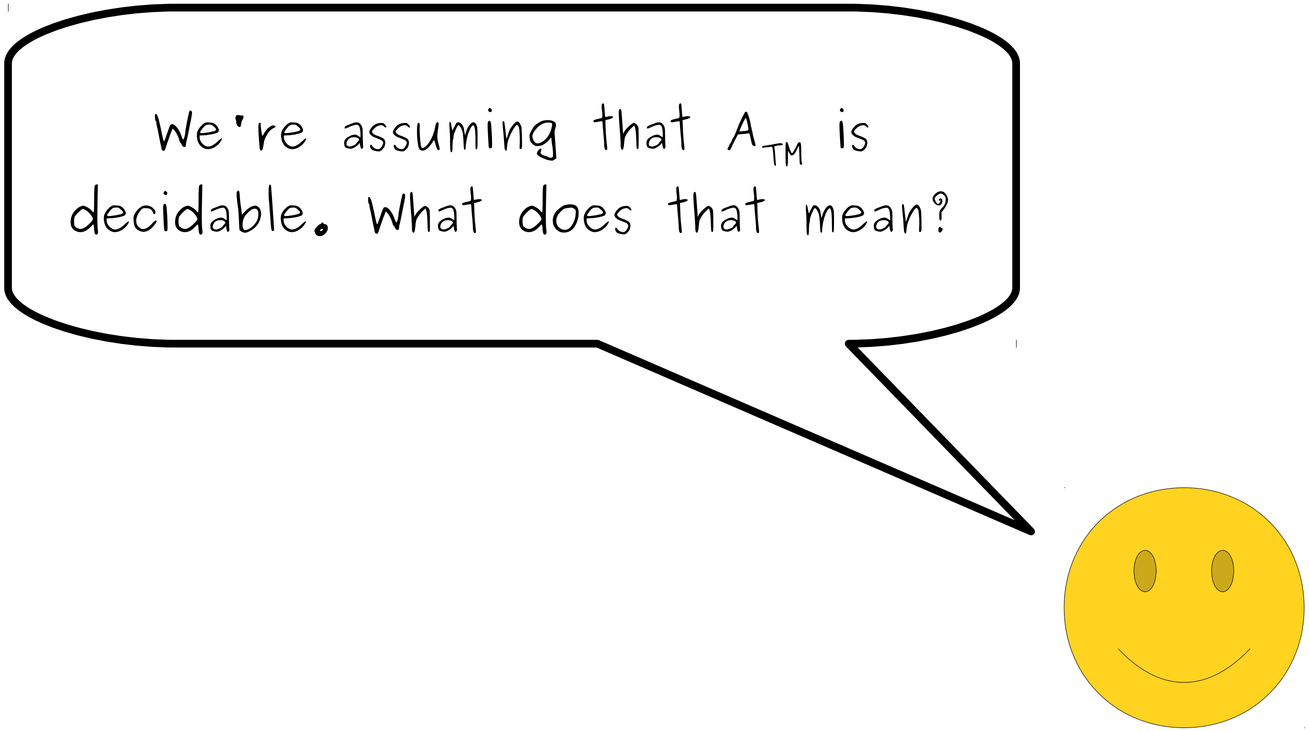

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There is a decider *D* for A_{TM}

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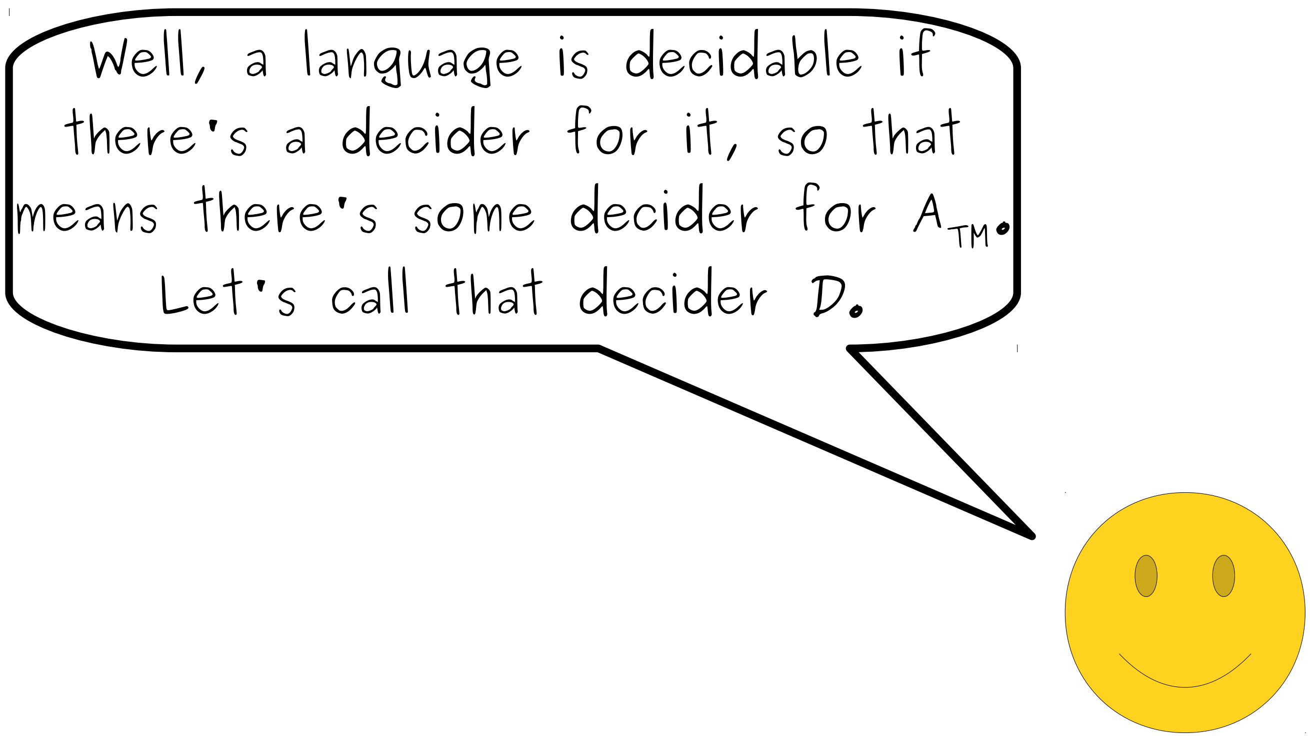

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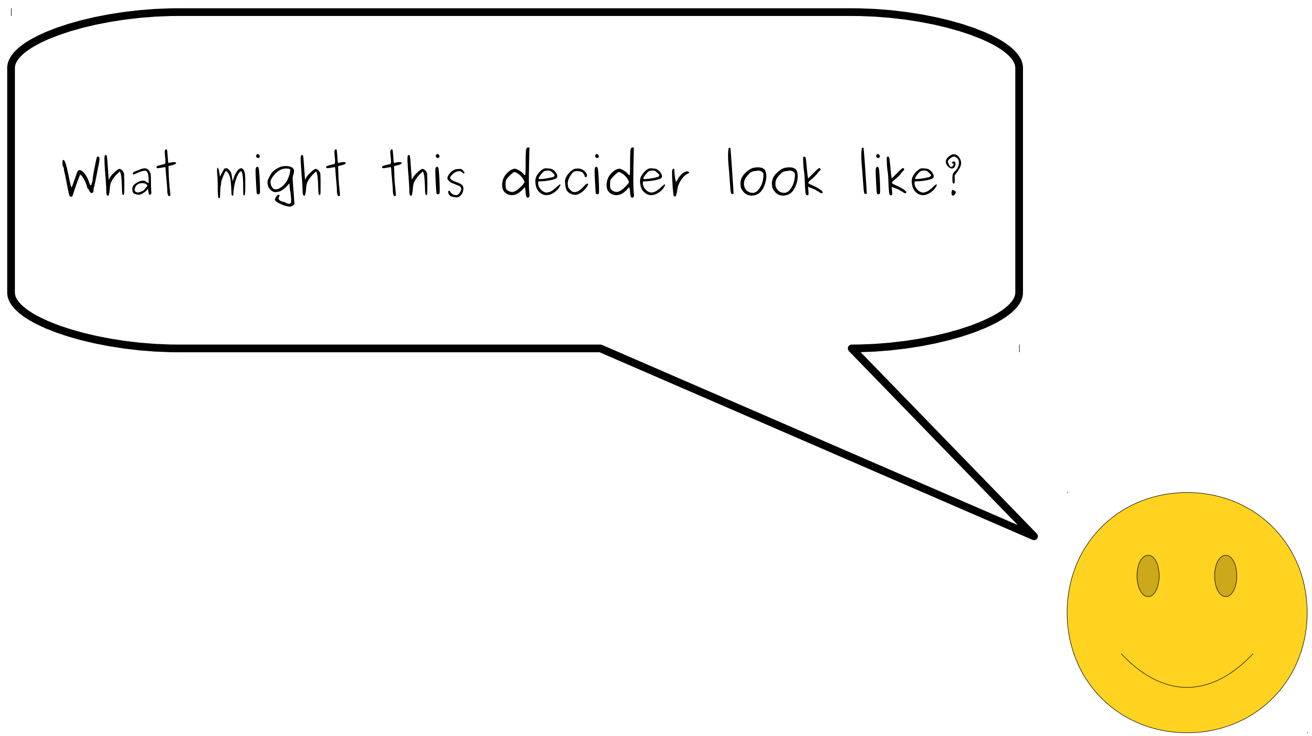

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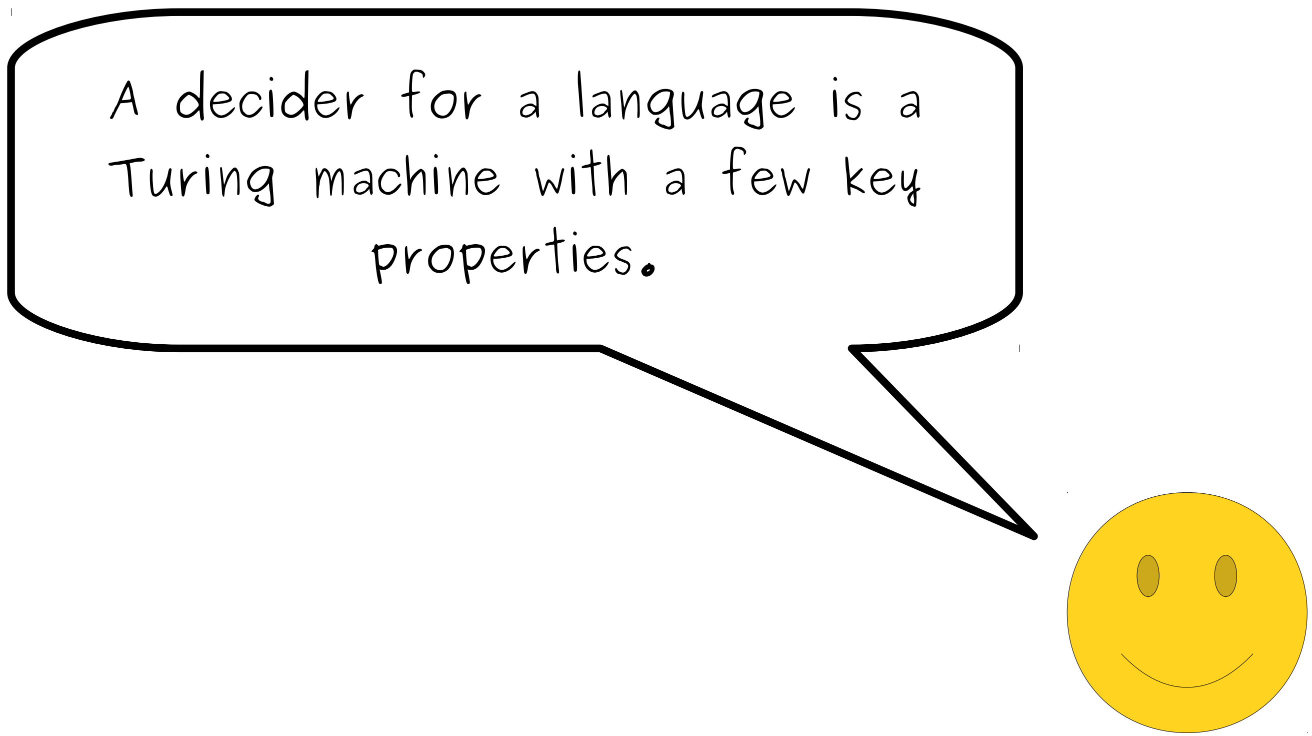

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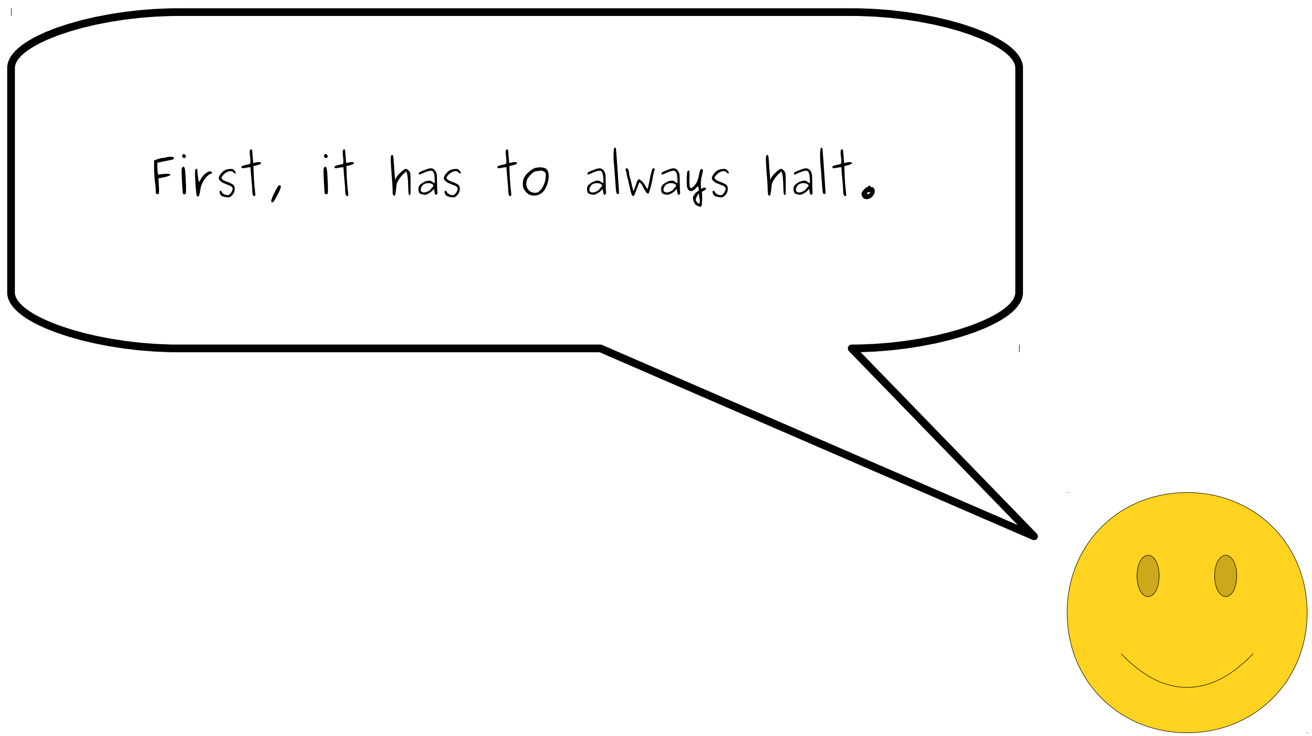

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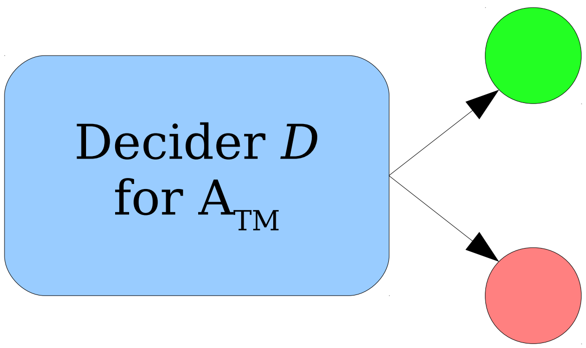

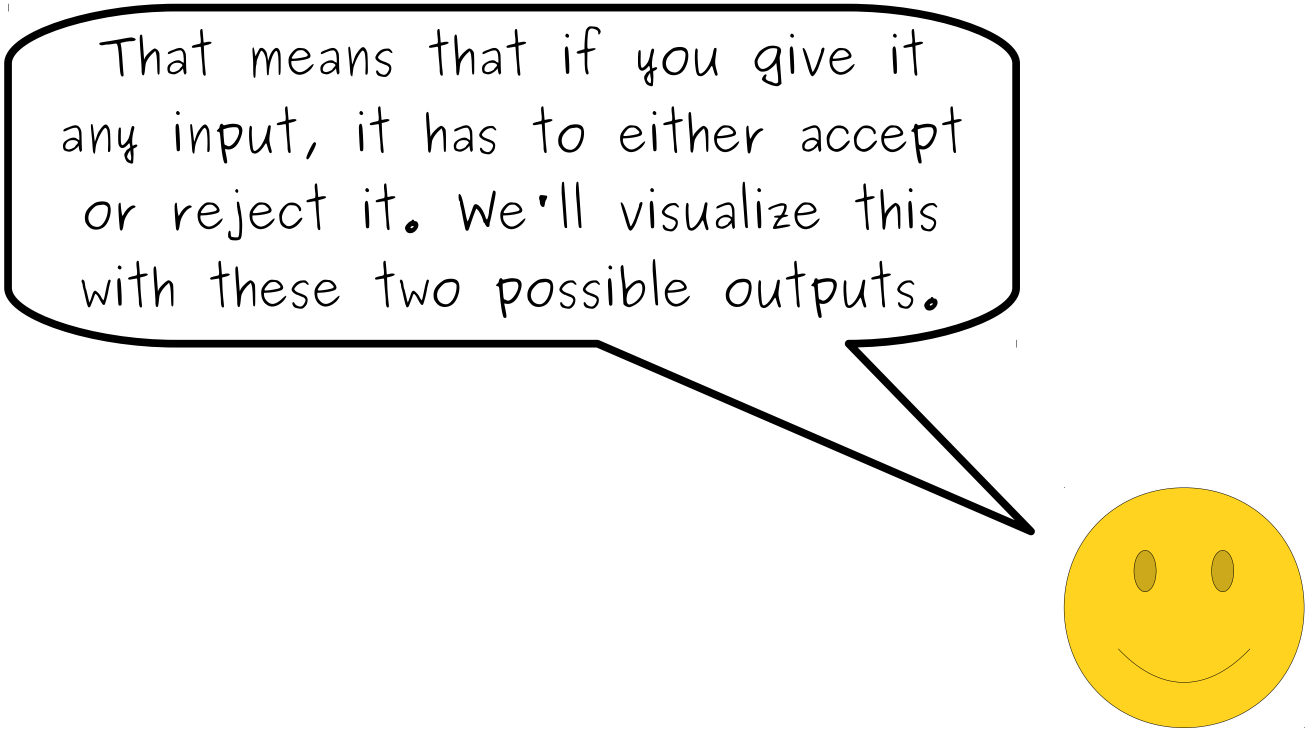

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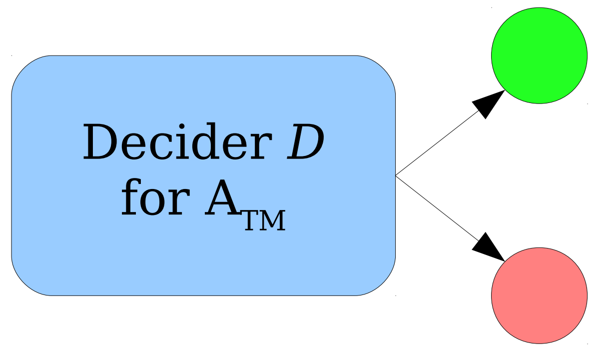

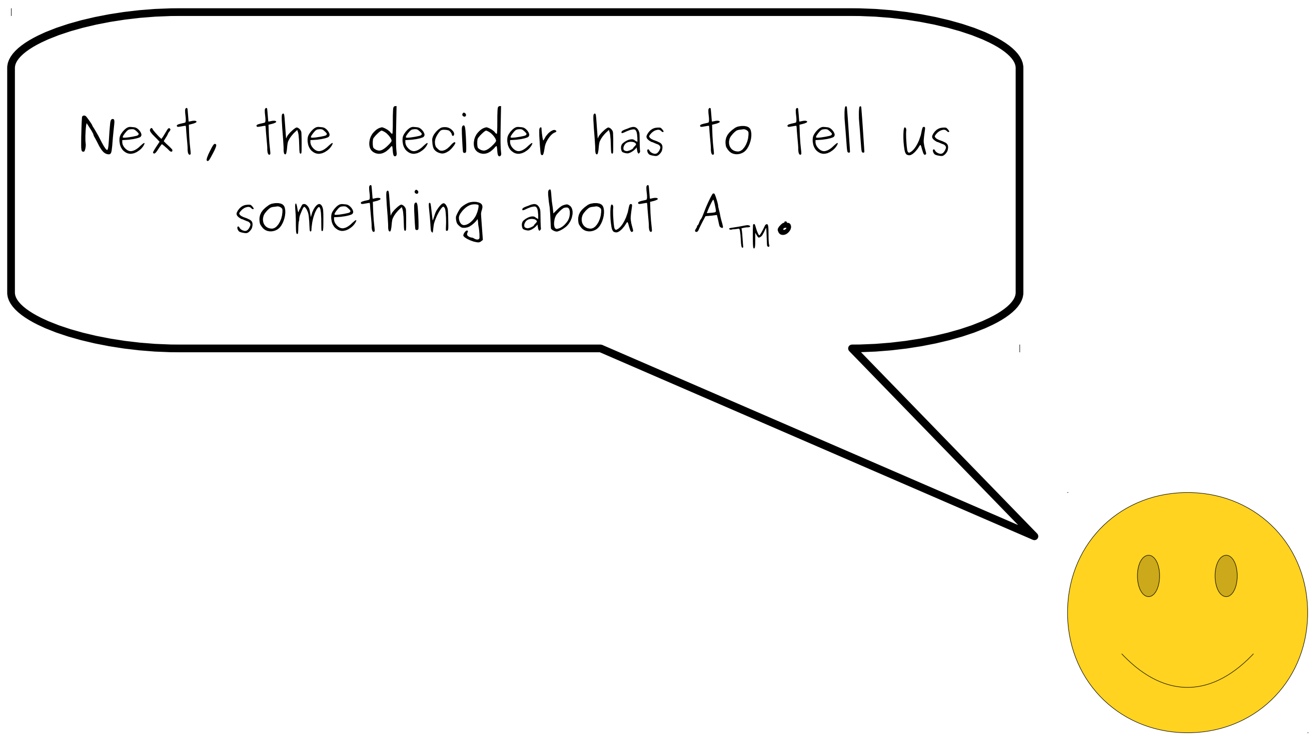

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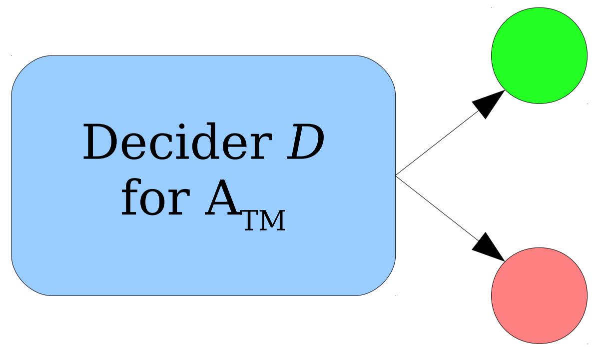
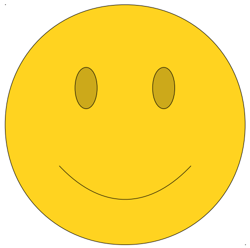
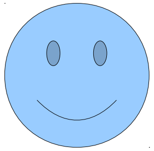


Next, the decider has to tell us something about A_{TM}.



As a reminder, A_{TM} is the language

$\{ \langle M, w \rangle \mid M \text{ is a TM and } M \text{ accepts } w \}$



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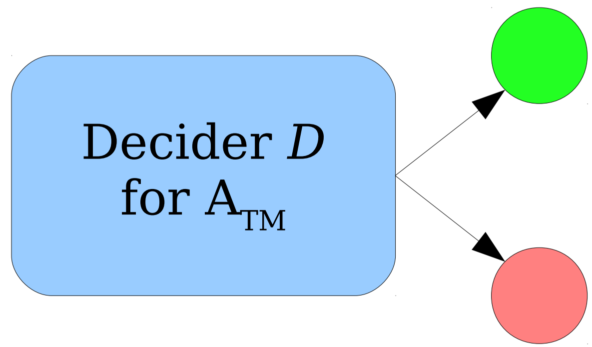
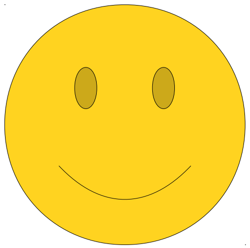
Contradiction!



Specifically, the decider D needs to take in an input and tell us whether that input is in A_{TM}



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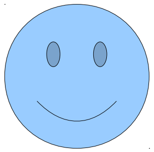

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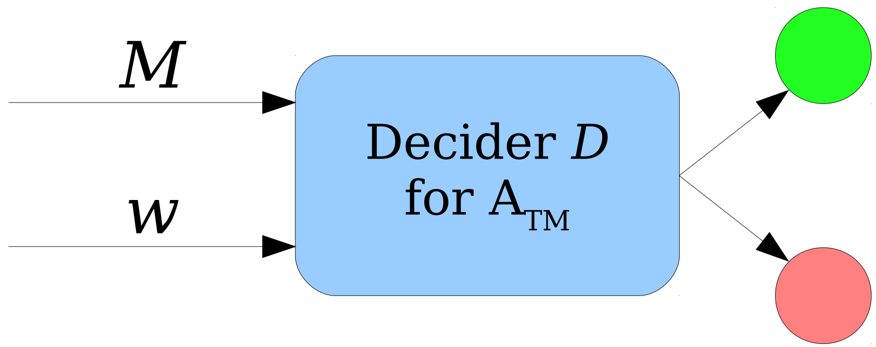

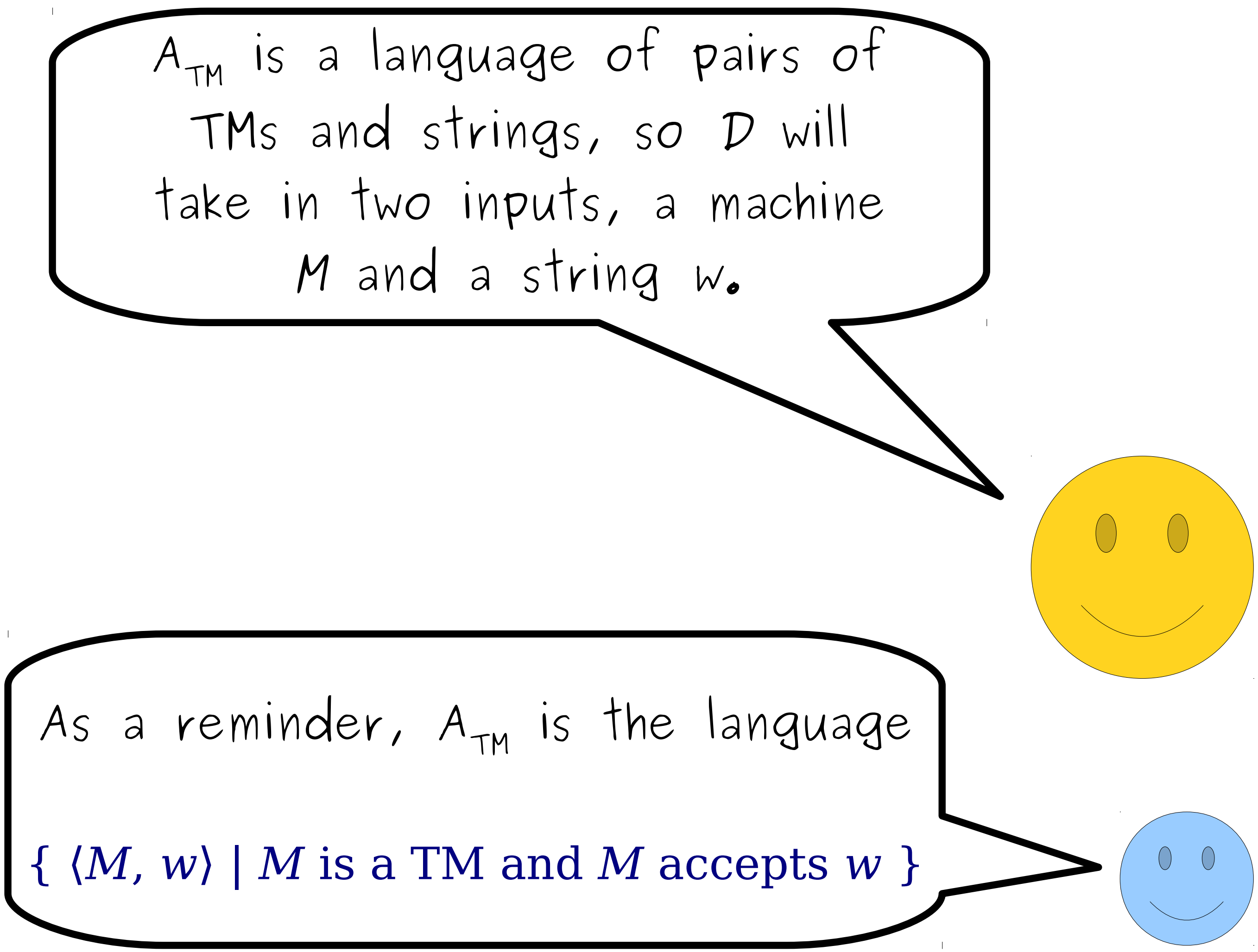

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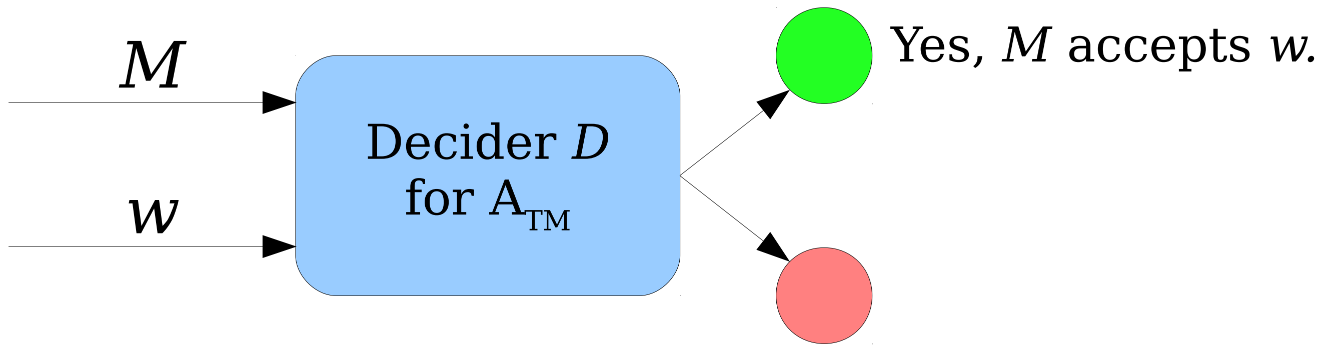

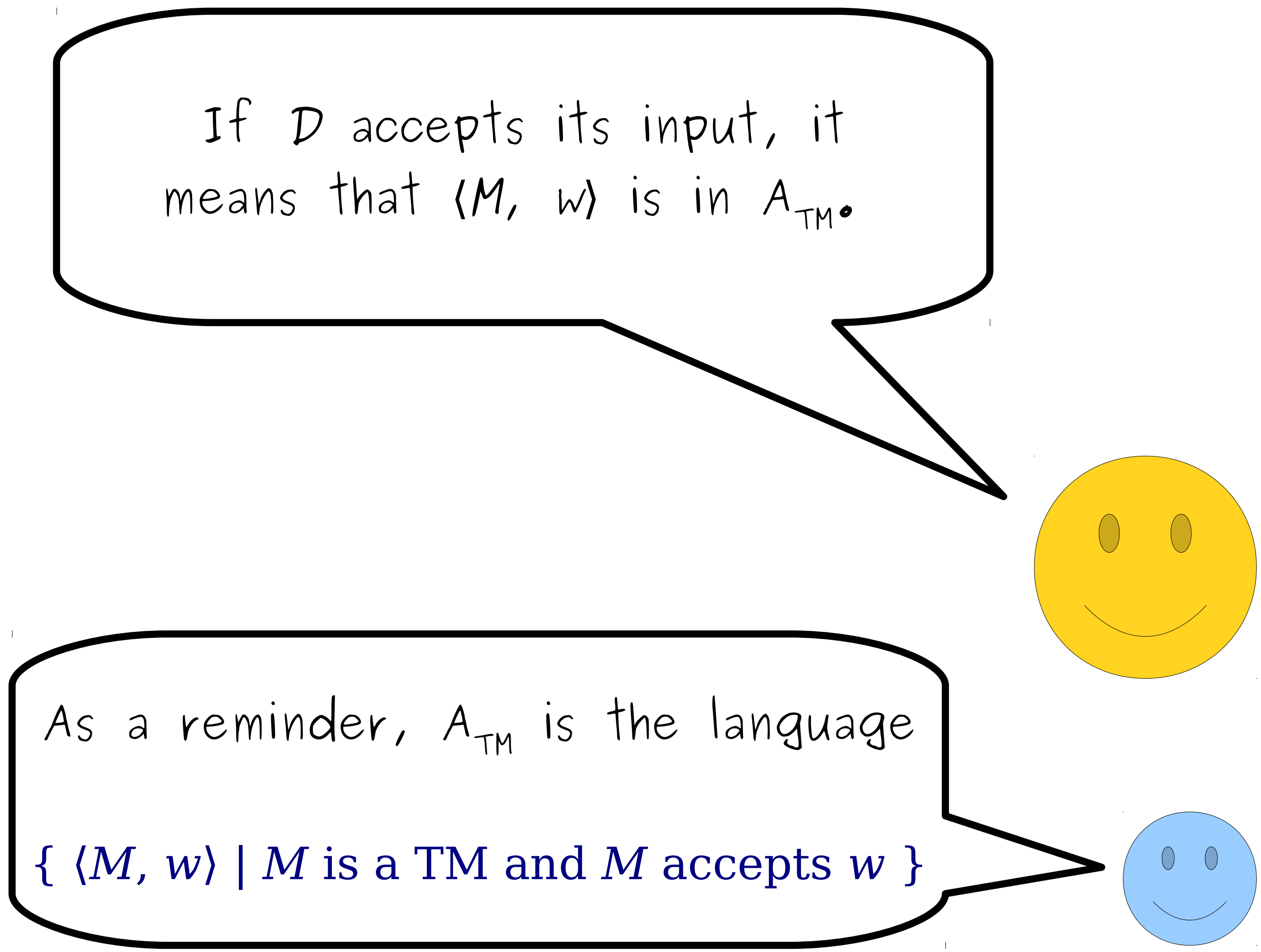

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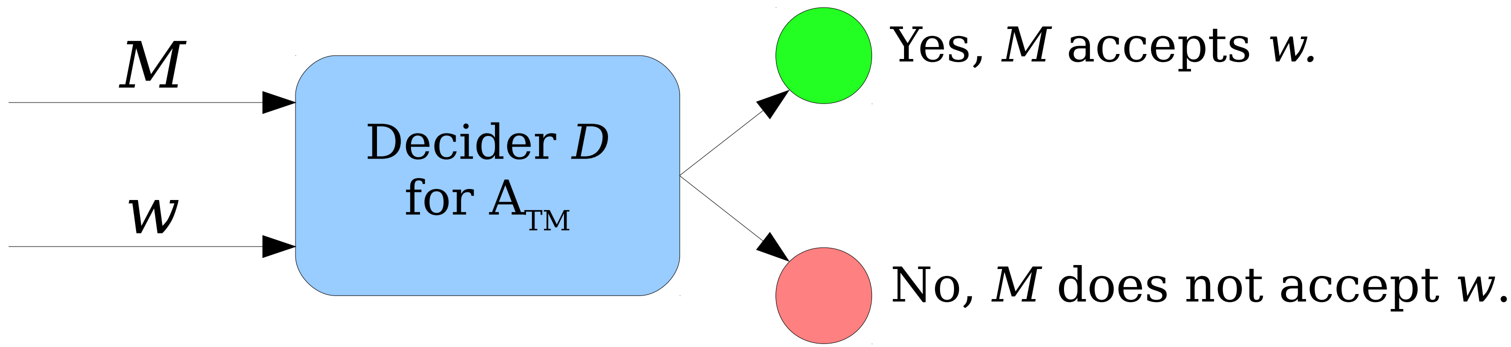

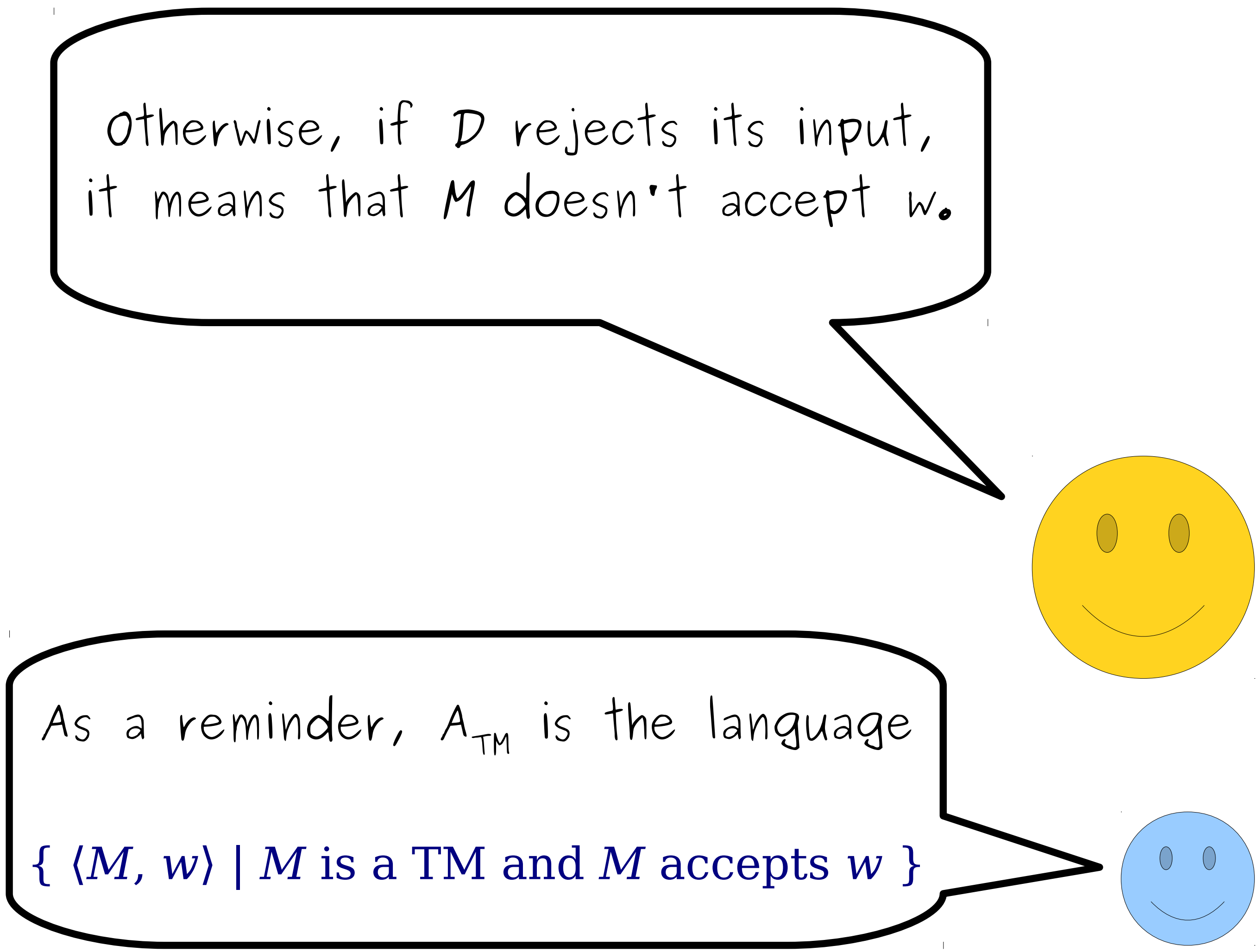

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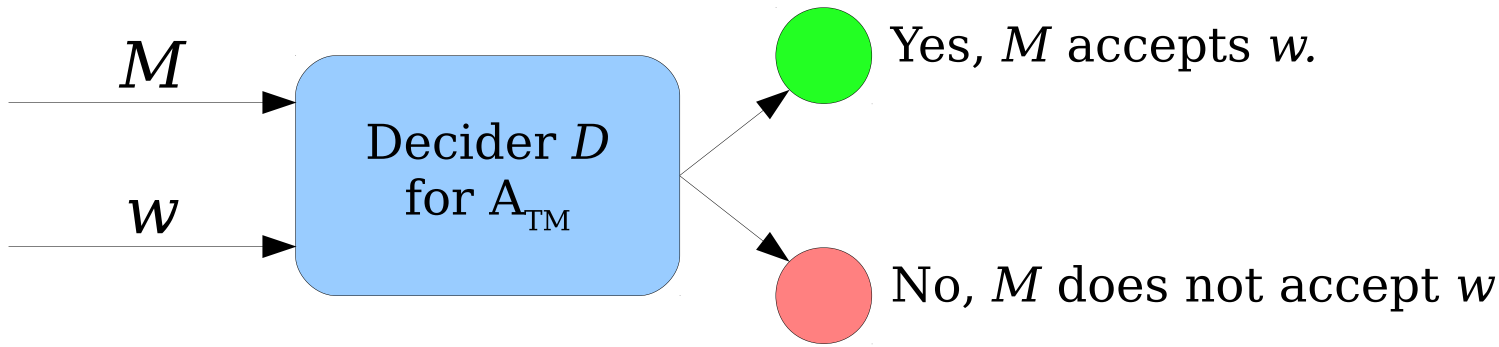

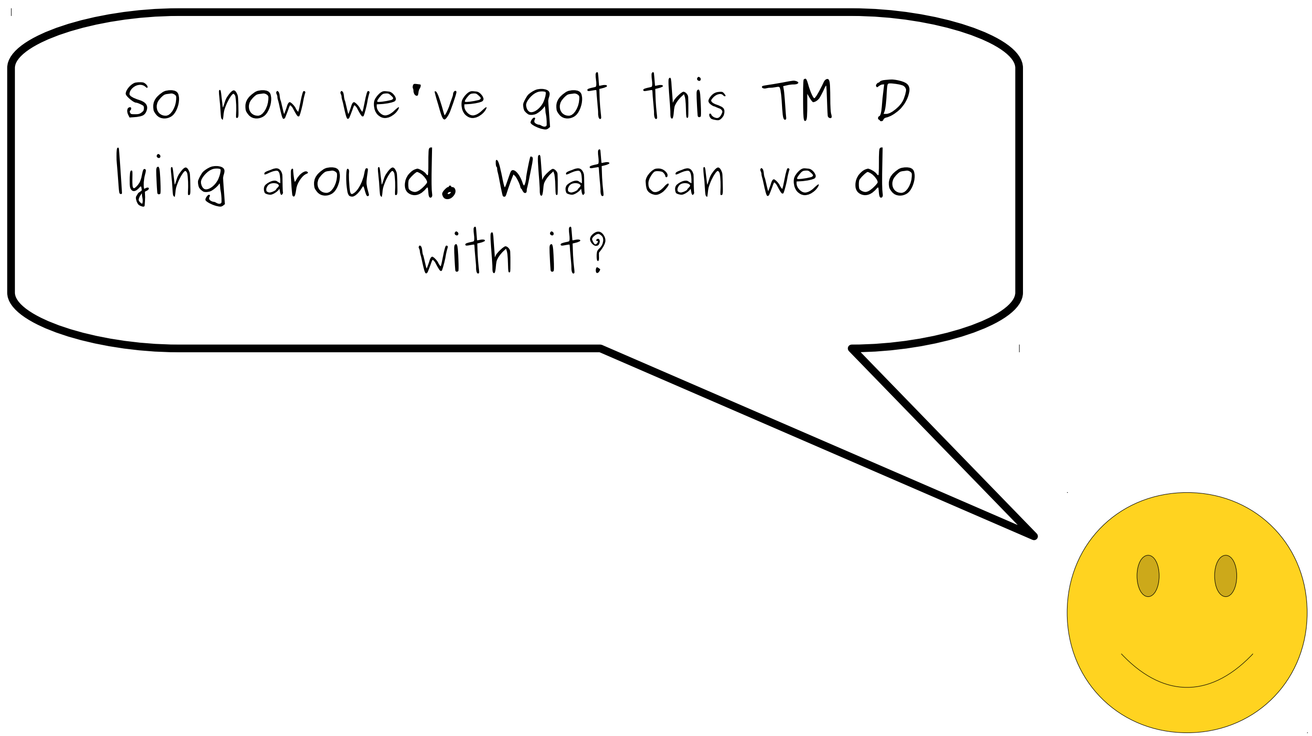

Contradiction!

$A_{TM} \in \mathbf{R}$

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There is a decider *D* for A_{TM}

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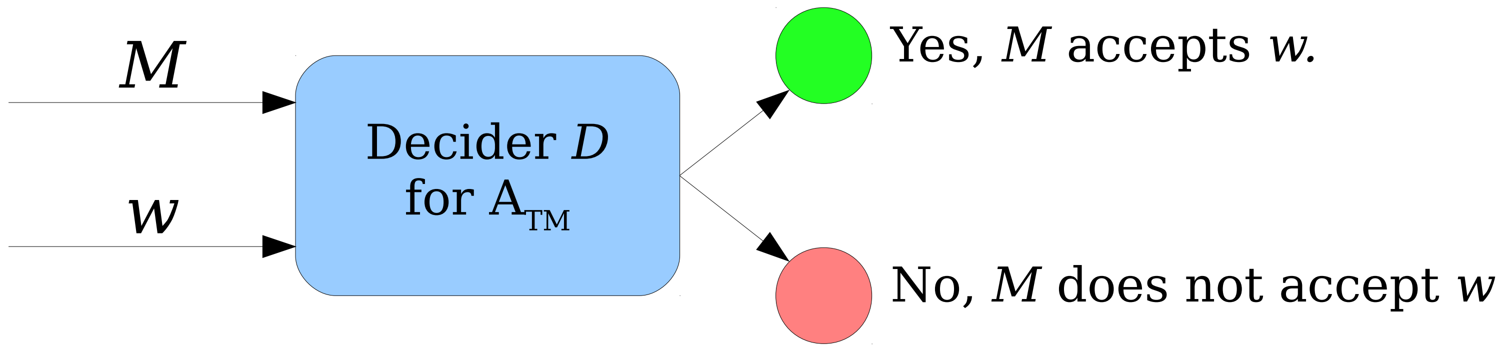
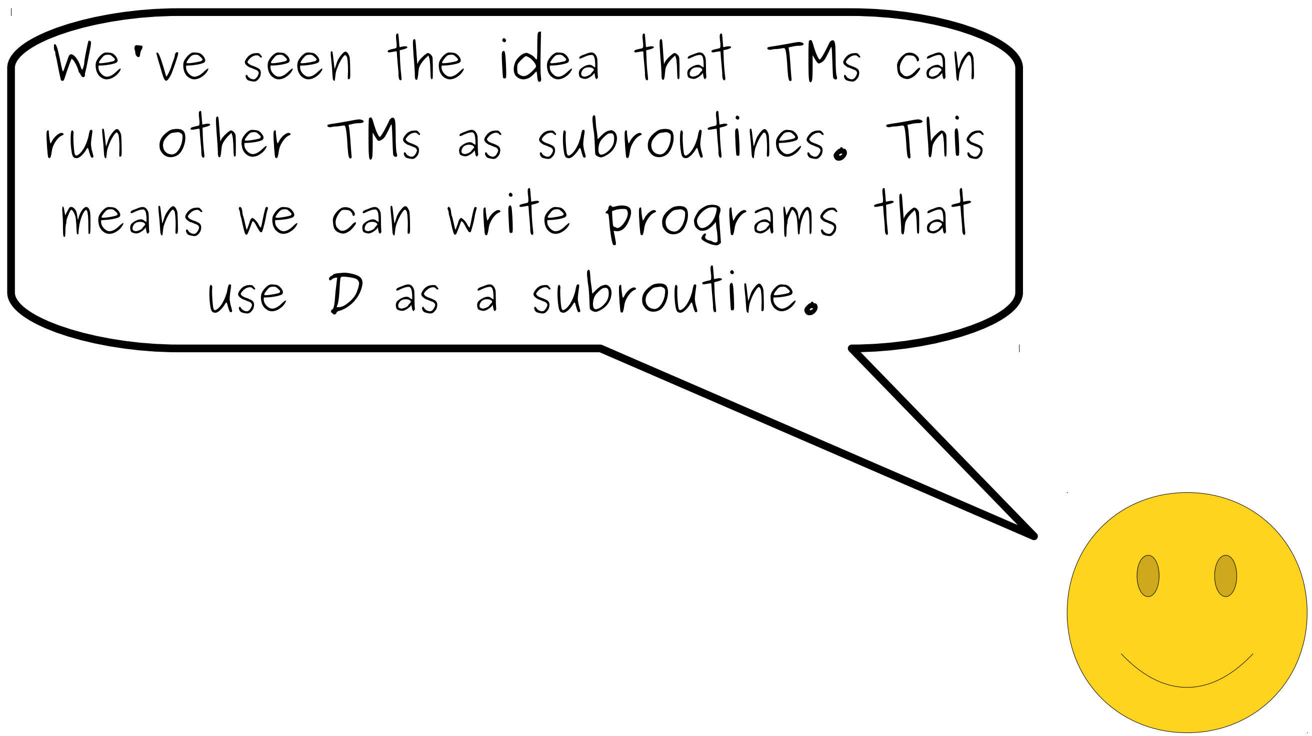
We can write programs that use *D* as a helper method

Contradiction!





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There is a decider D for A_{TM}

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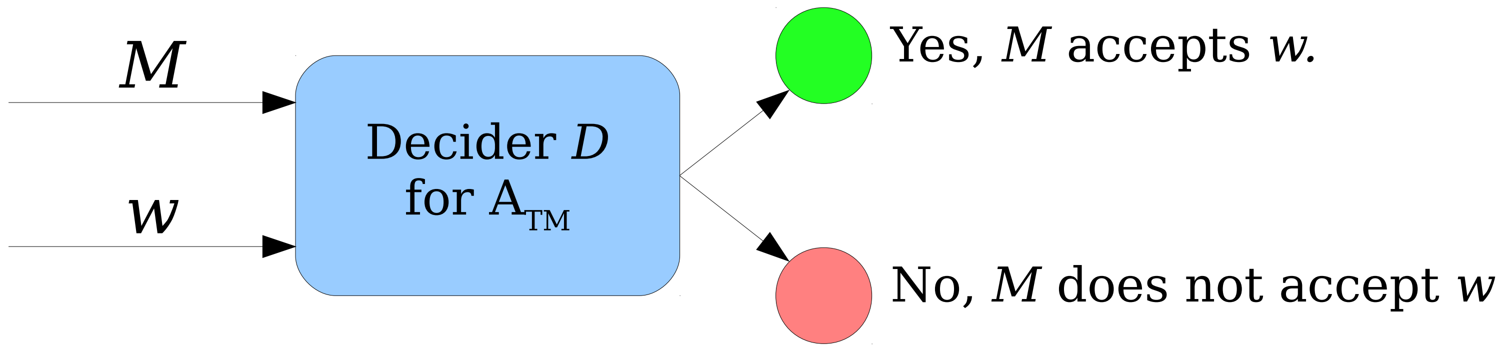
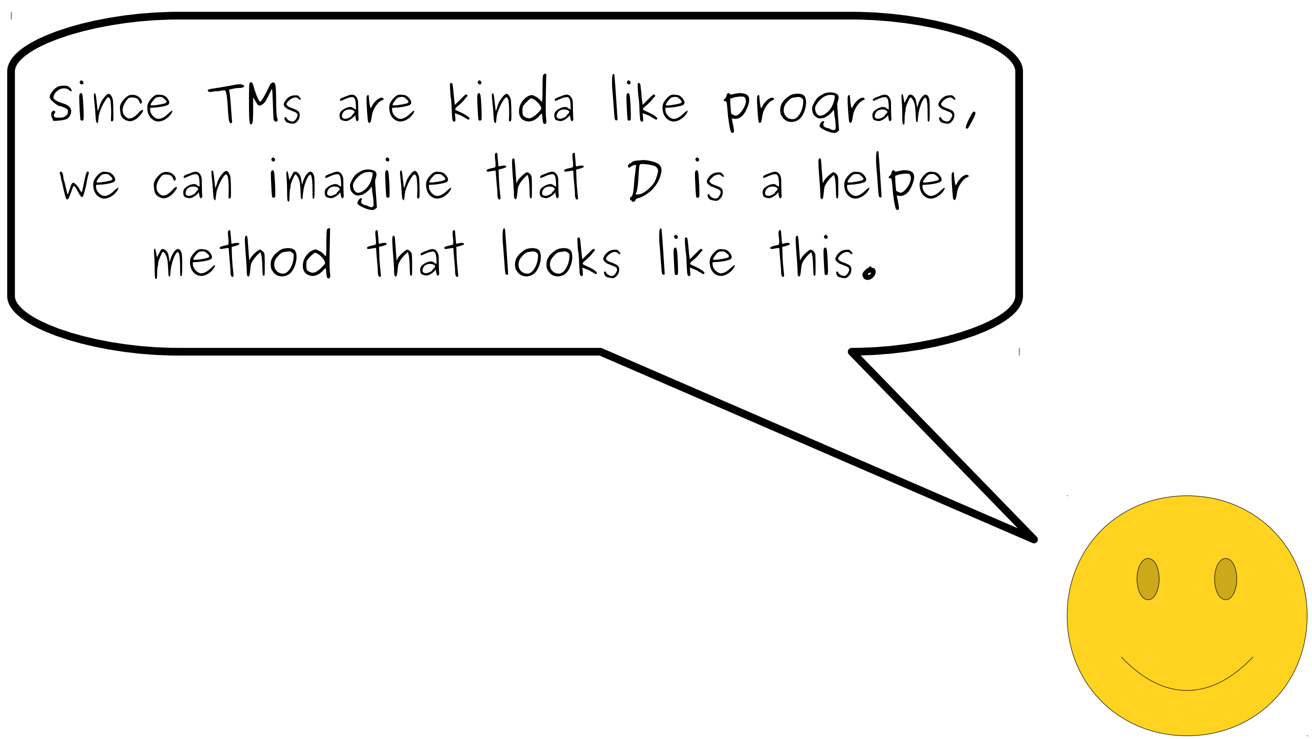
We can write programs that use D as a helper method

Contradiction!




```
bool willAccept(string program, string input)
```



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We can write programs that use *D* as a helper method

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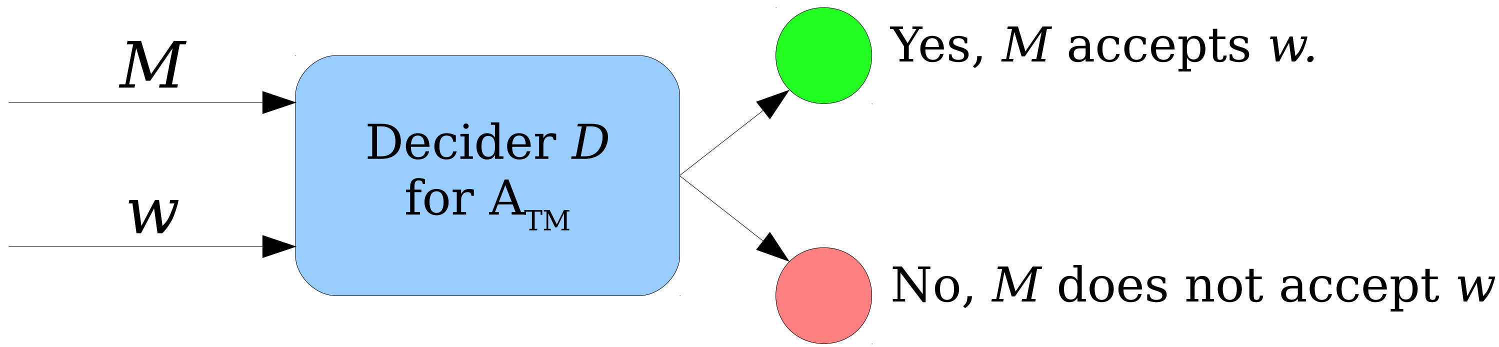


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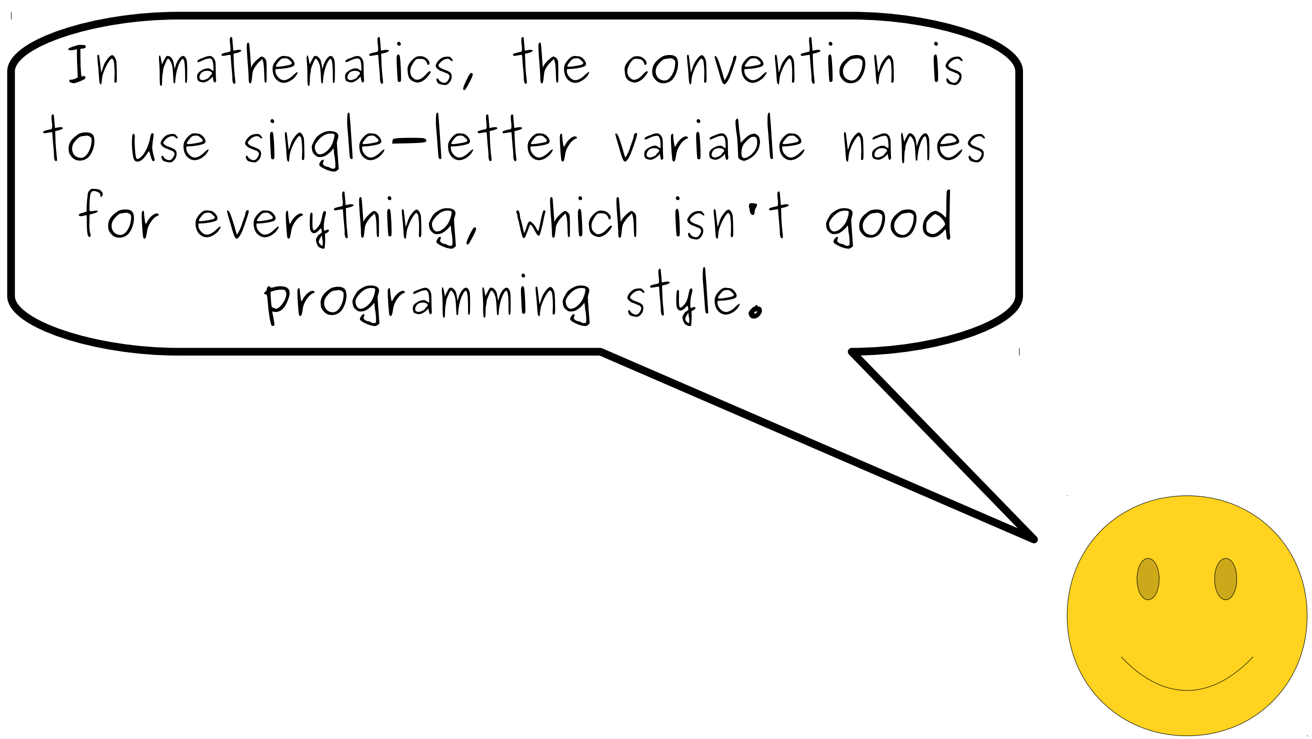

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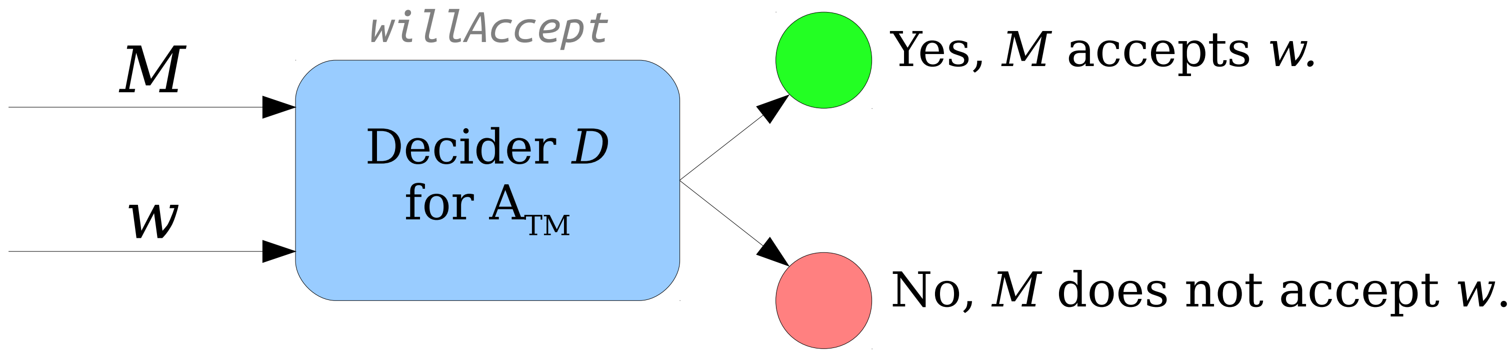


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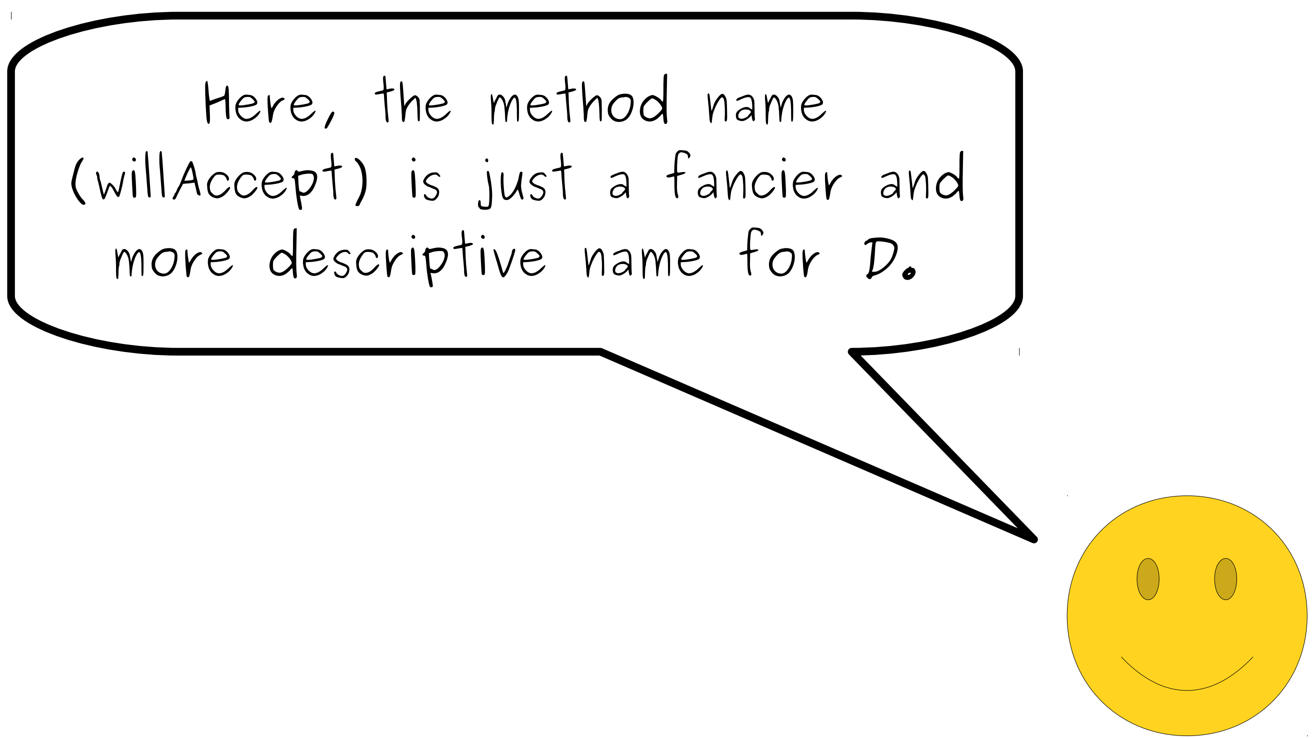

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There is a decider *D* for A_{TM}

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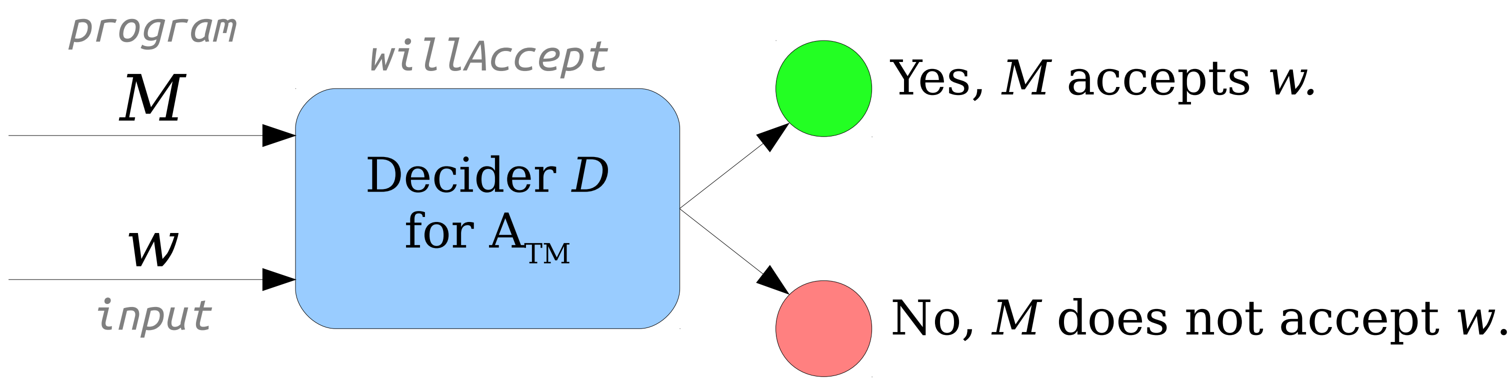
We can write programs that use *D* as a helper method

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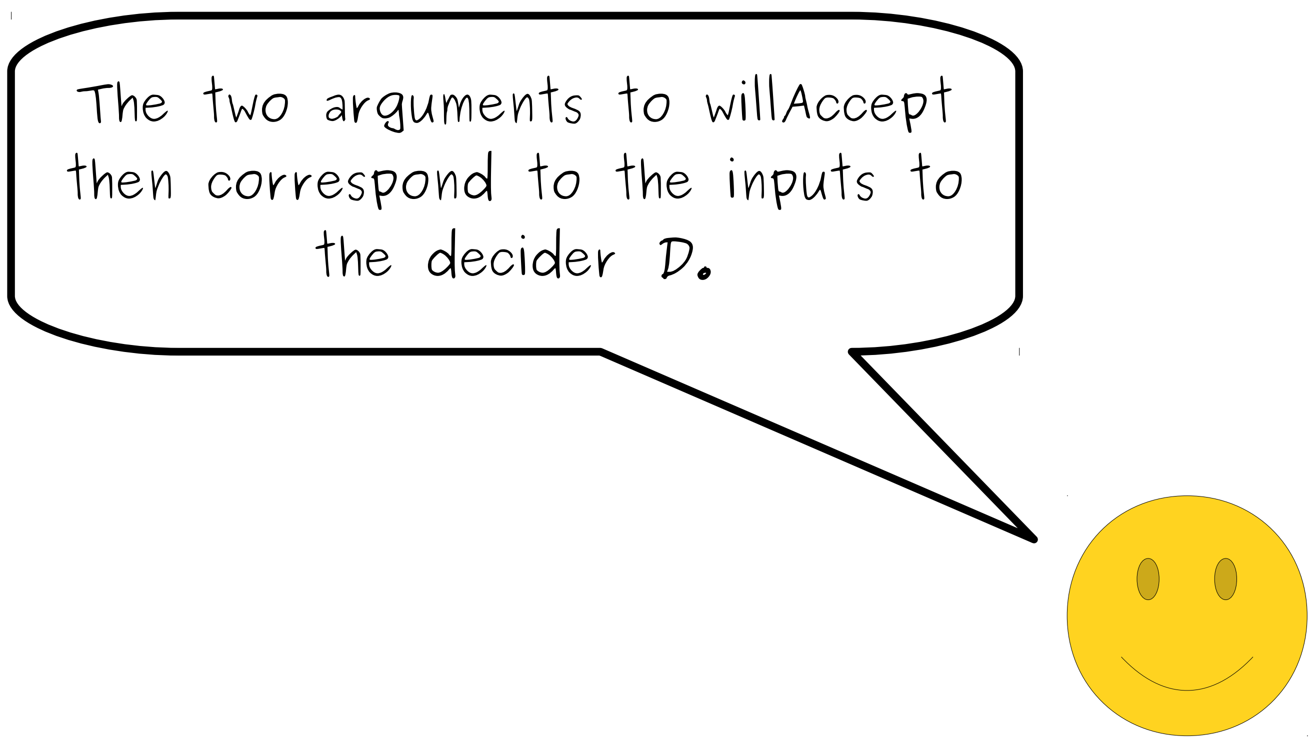

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We can write programs that use D as a helper method

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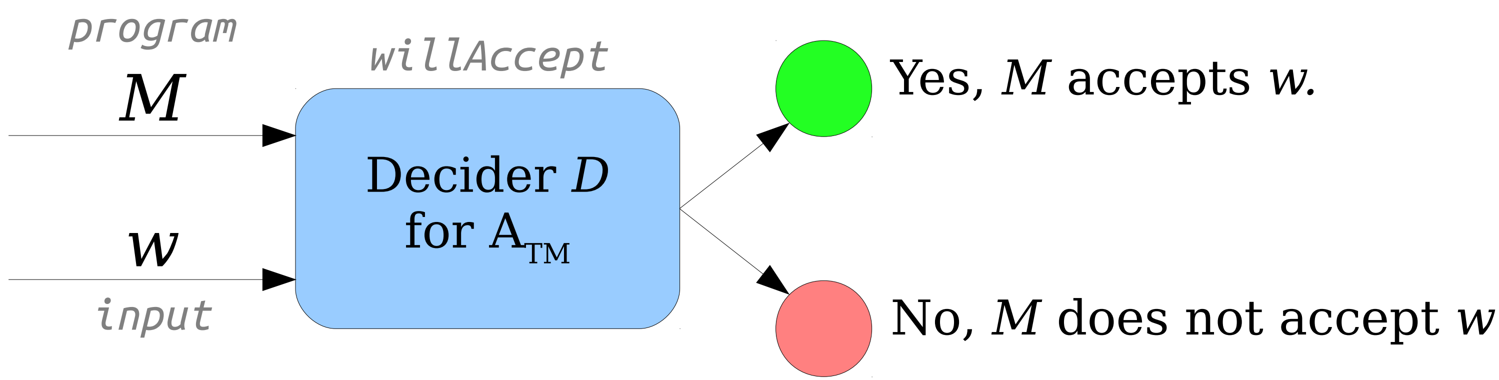


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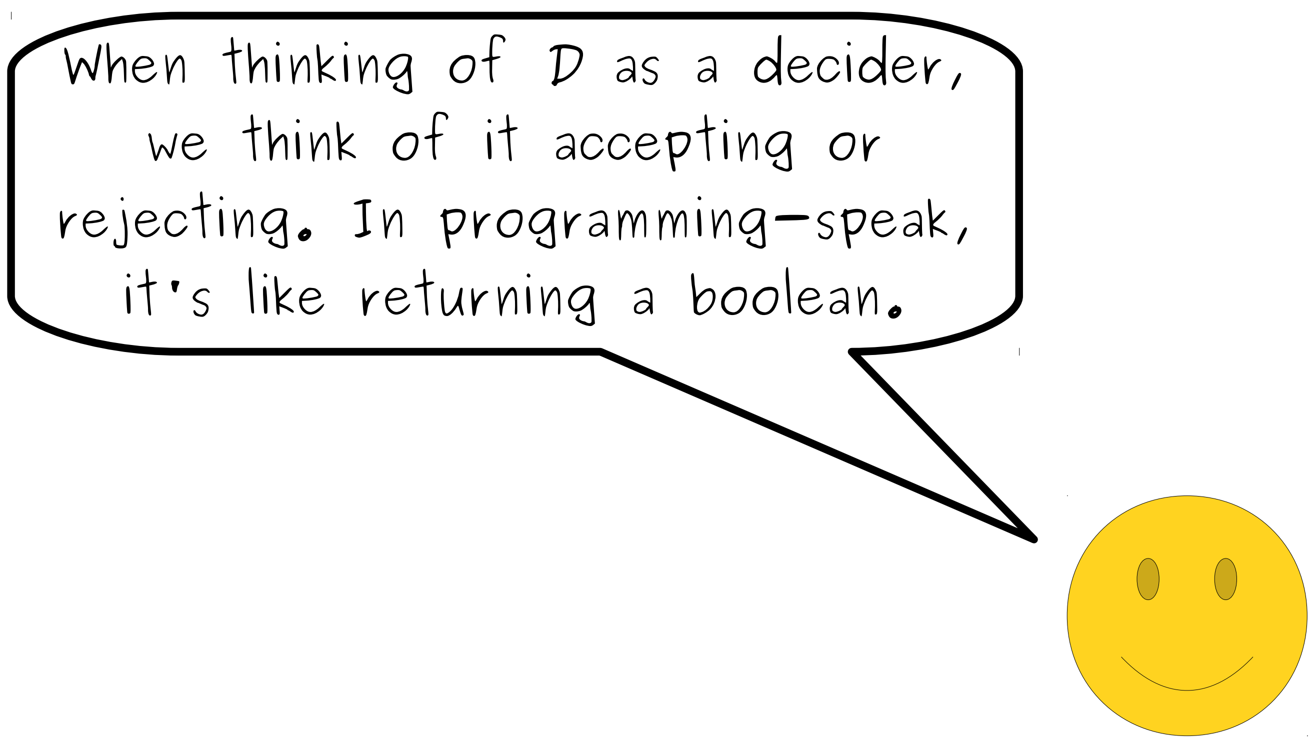

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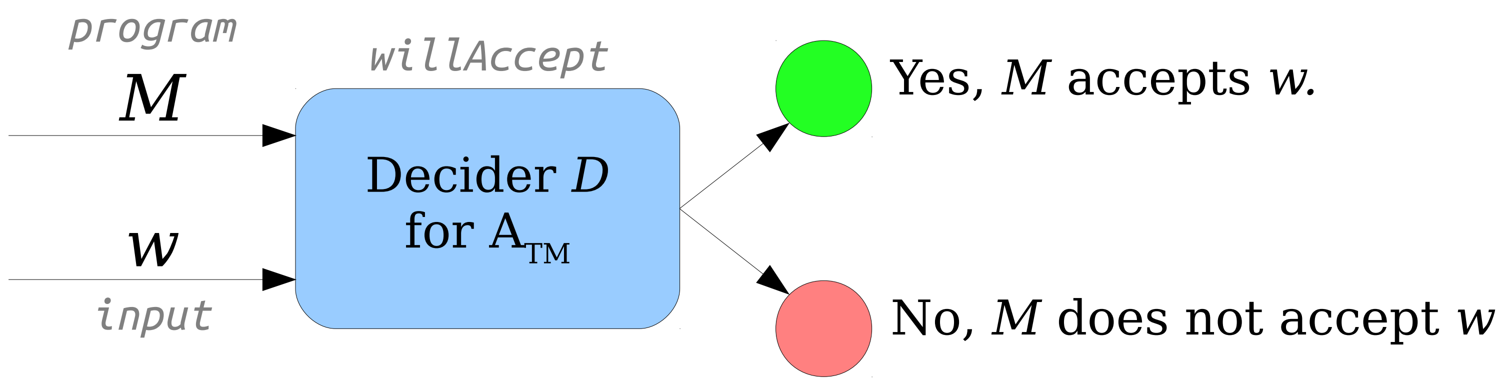


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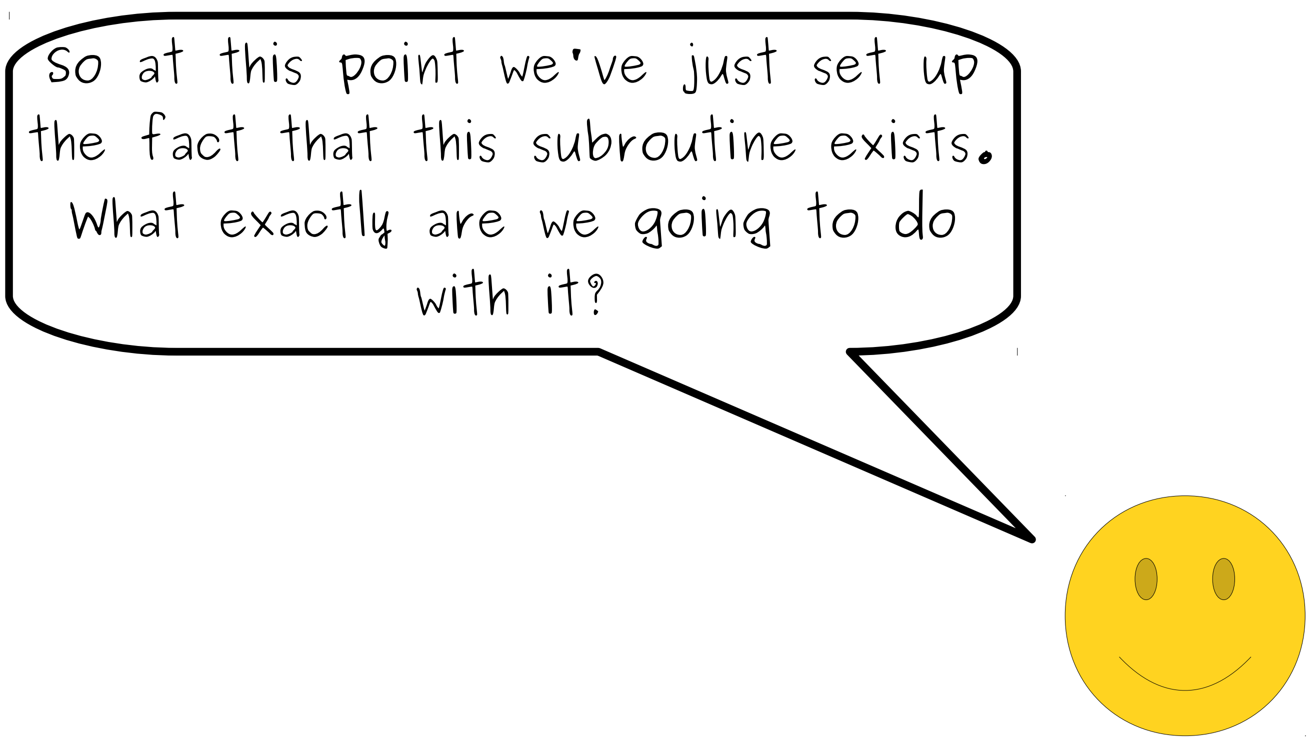

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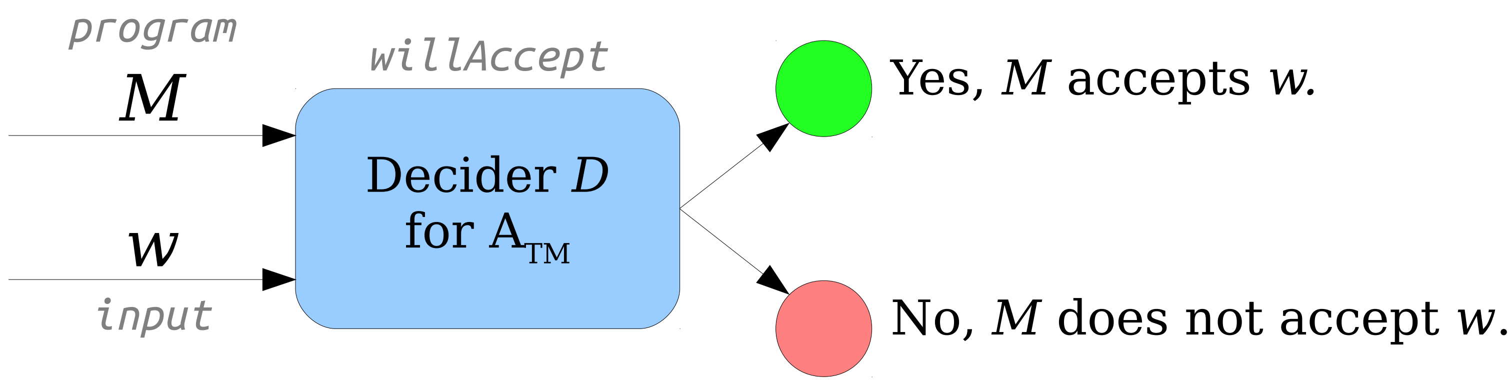


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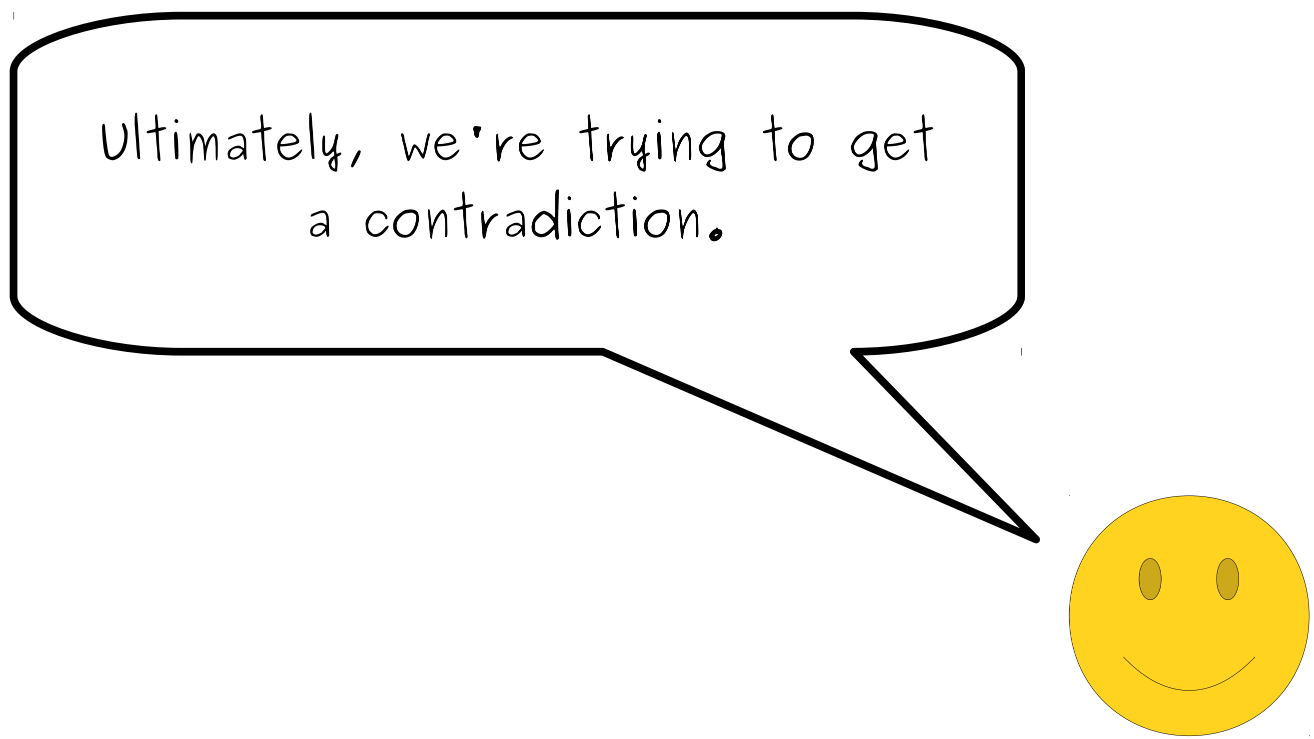

$A_{TM} \in \mathbf{R}$

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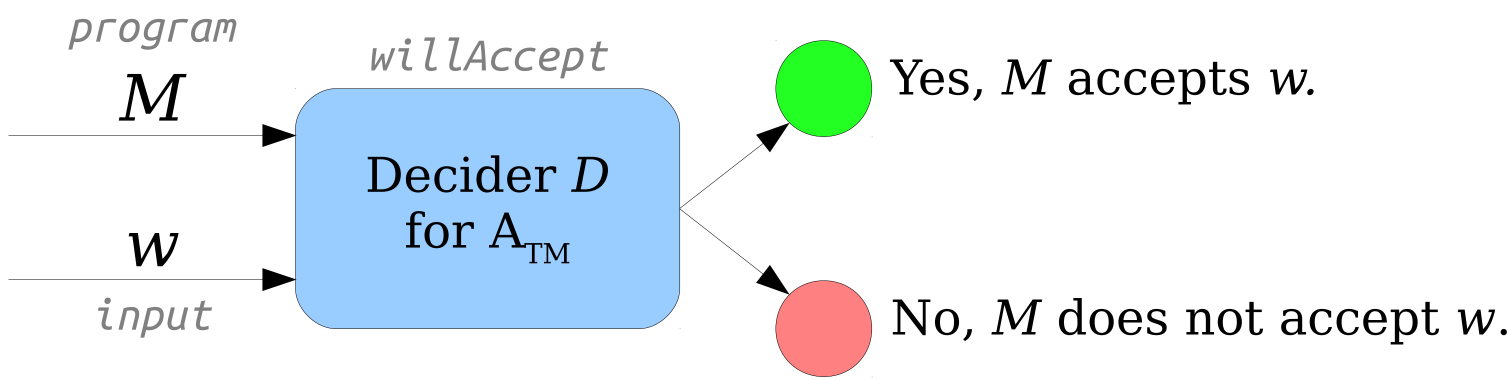
Program *P* accepts its input if and only if program *P* does not accept its input

Contradiction!

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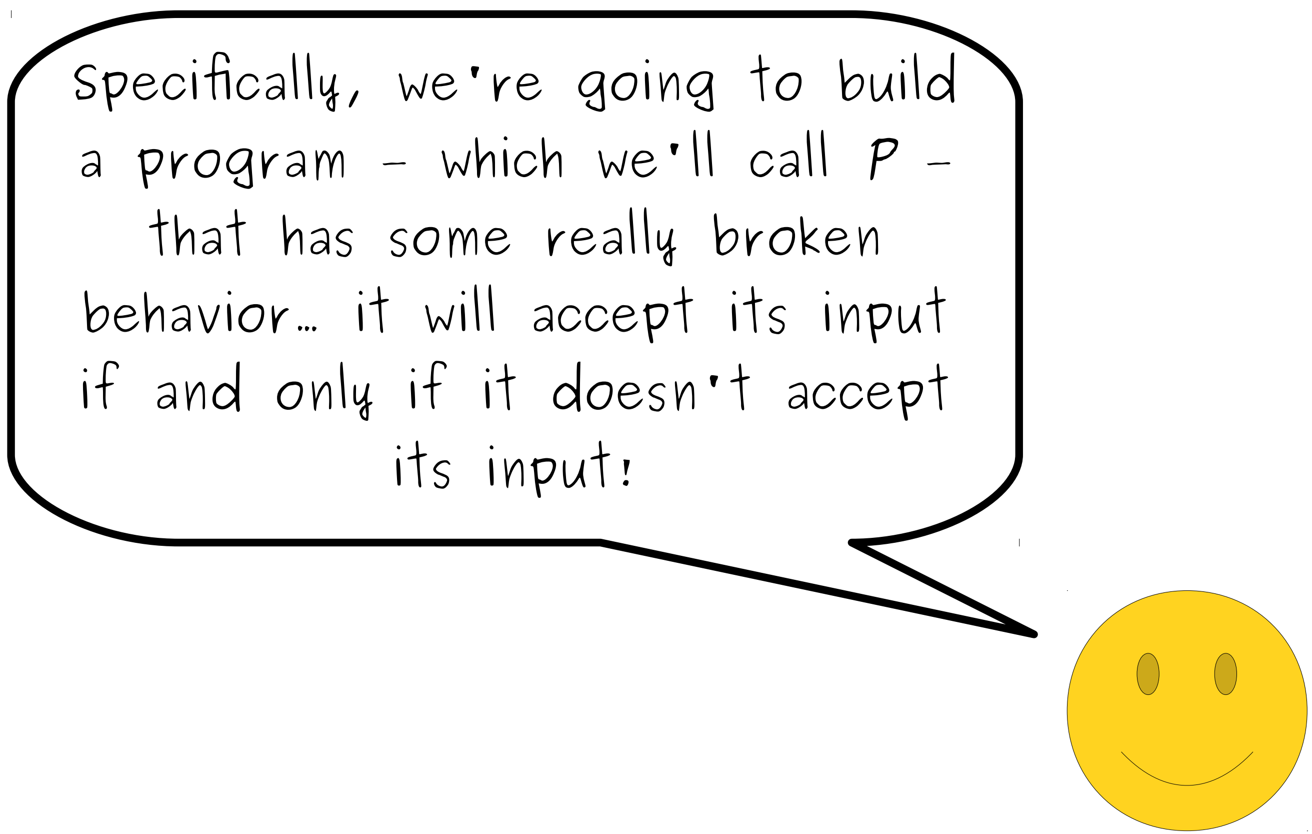

$A_{TM} \in \mathbf{R}$

There is a decider D for A_{TM}

We can write programs that use D as a helper method

Program P accepts its input if and only if program P does not accept its input

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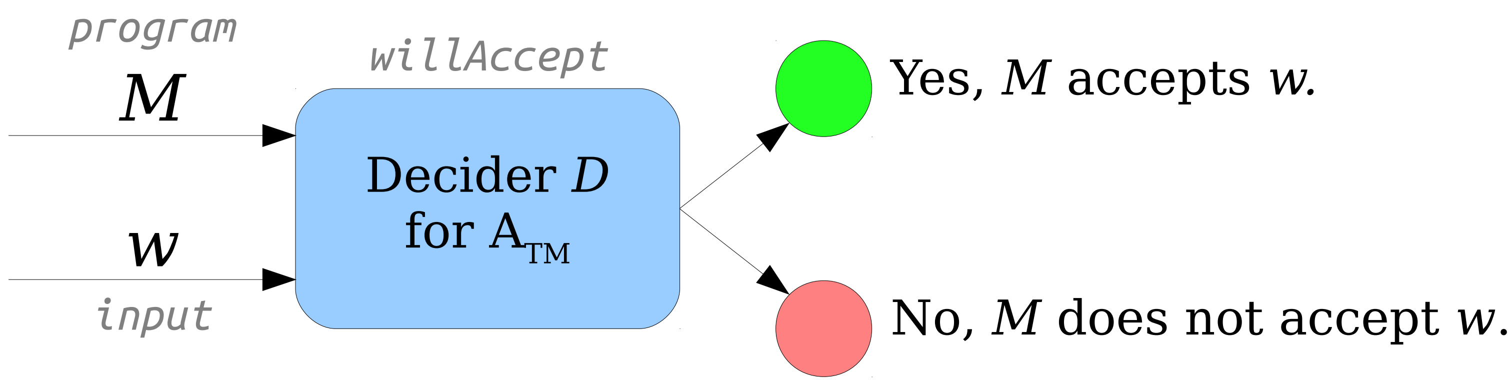


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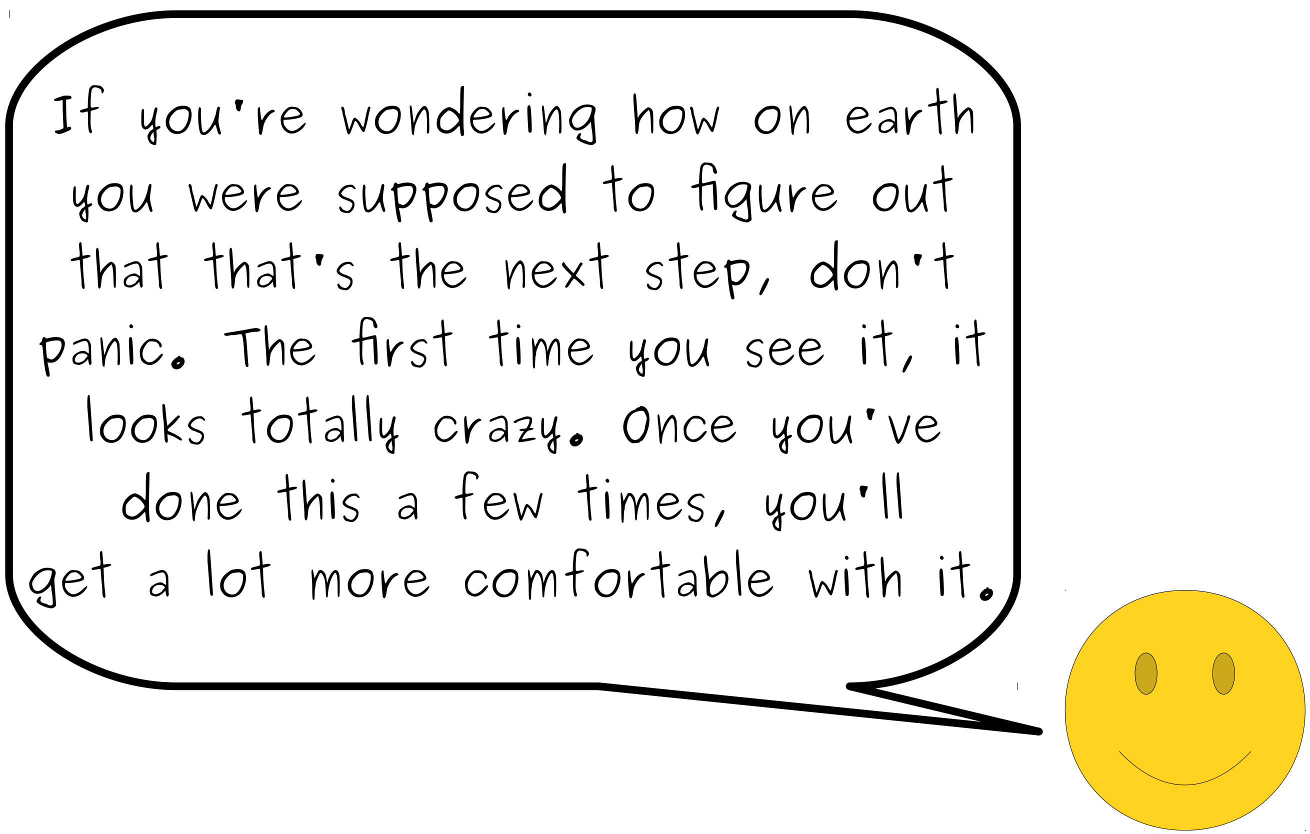

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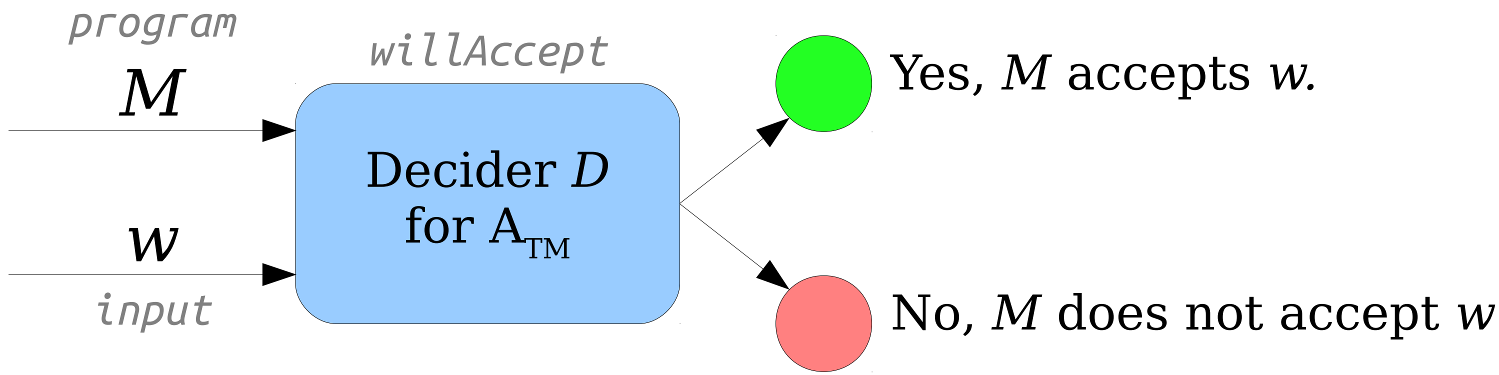


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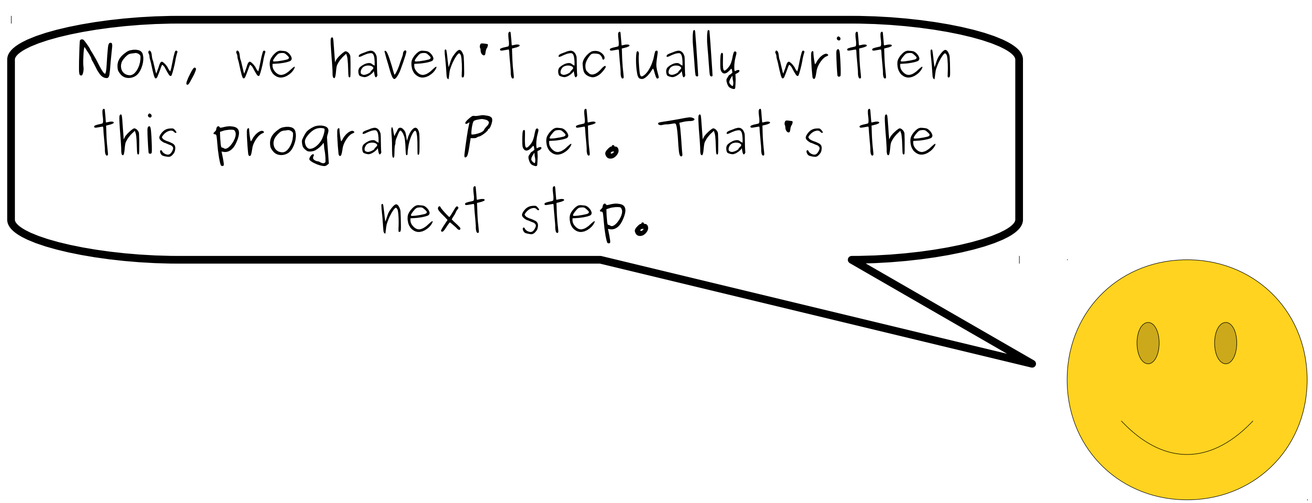

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We can write programs that use *D* as a helper method

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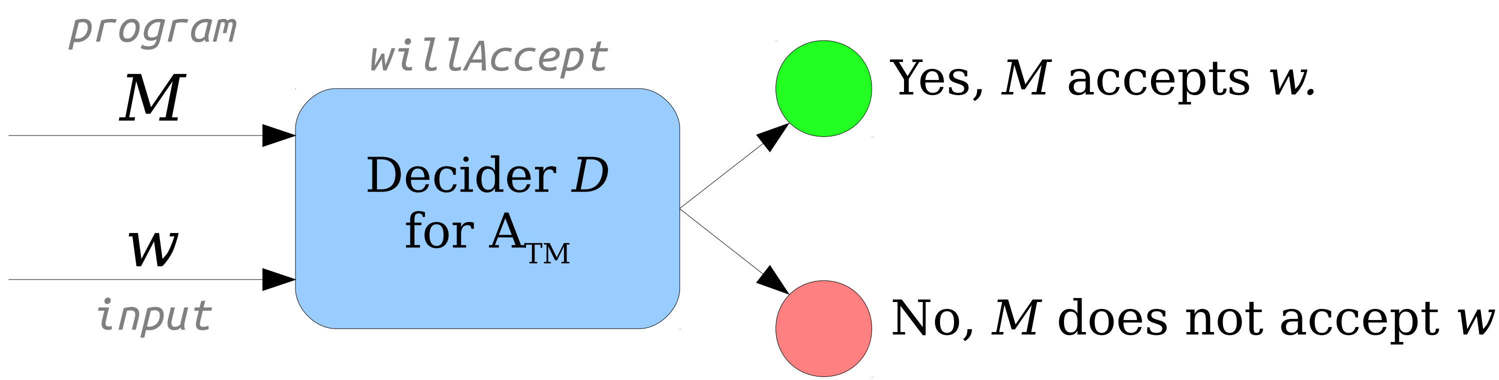


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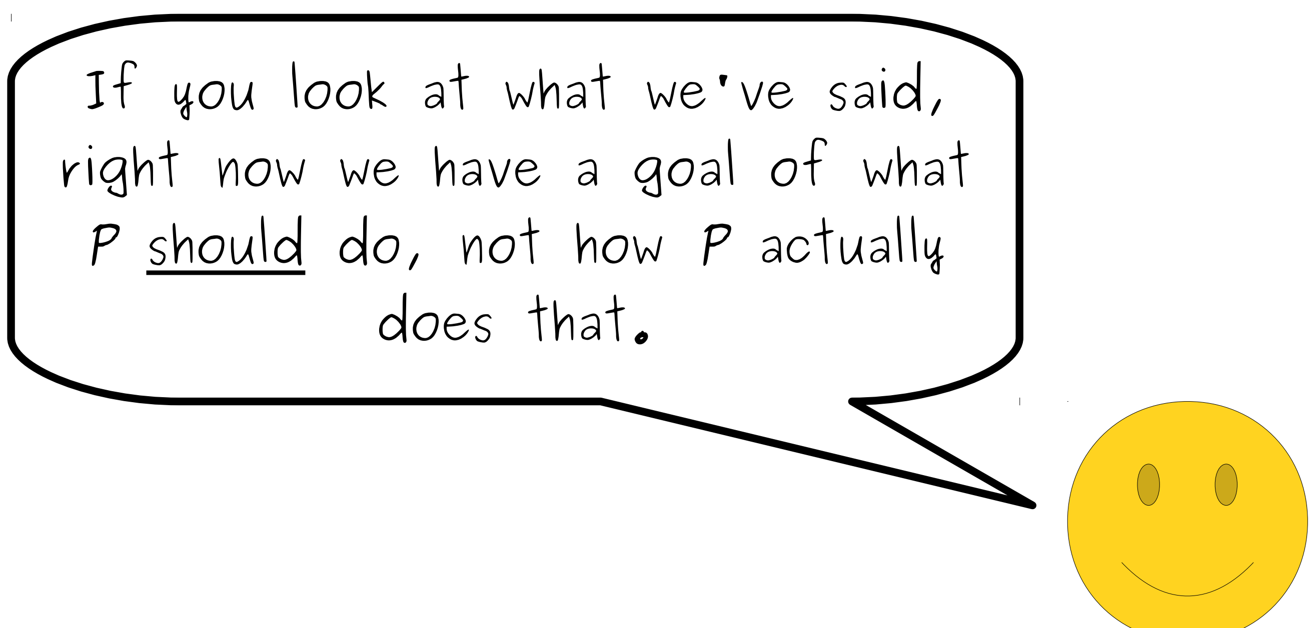

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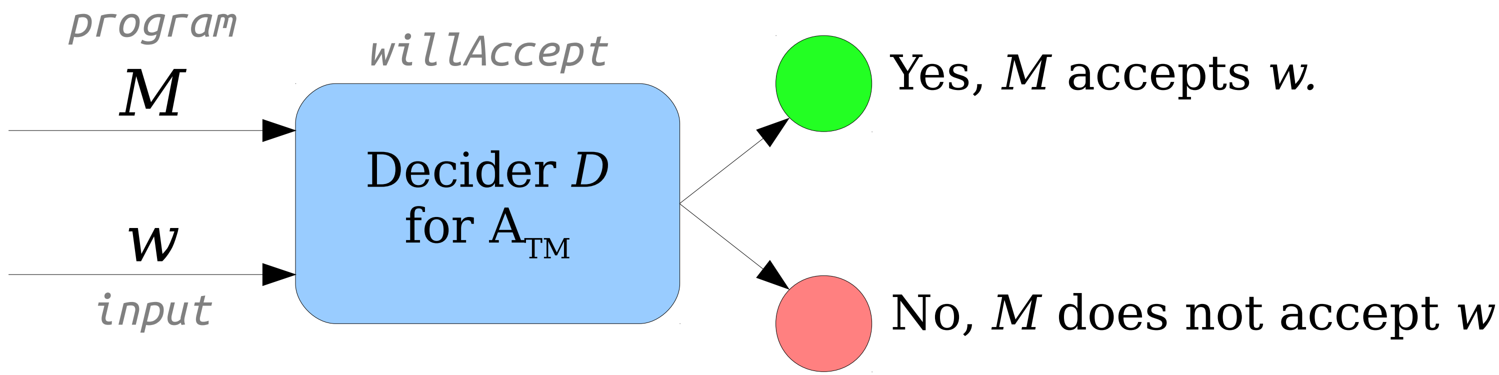


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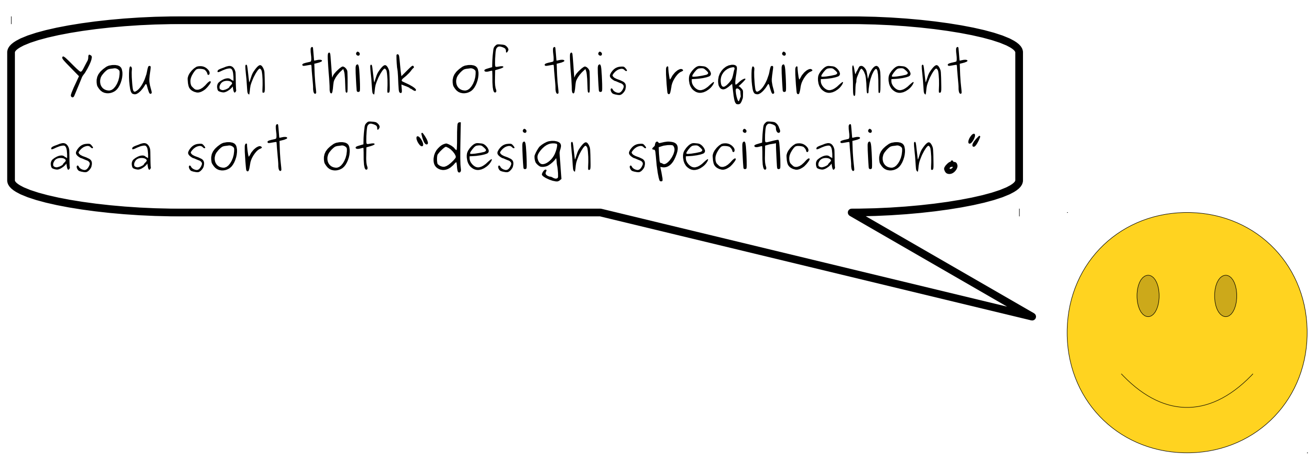

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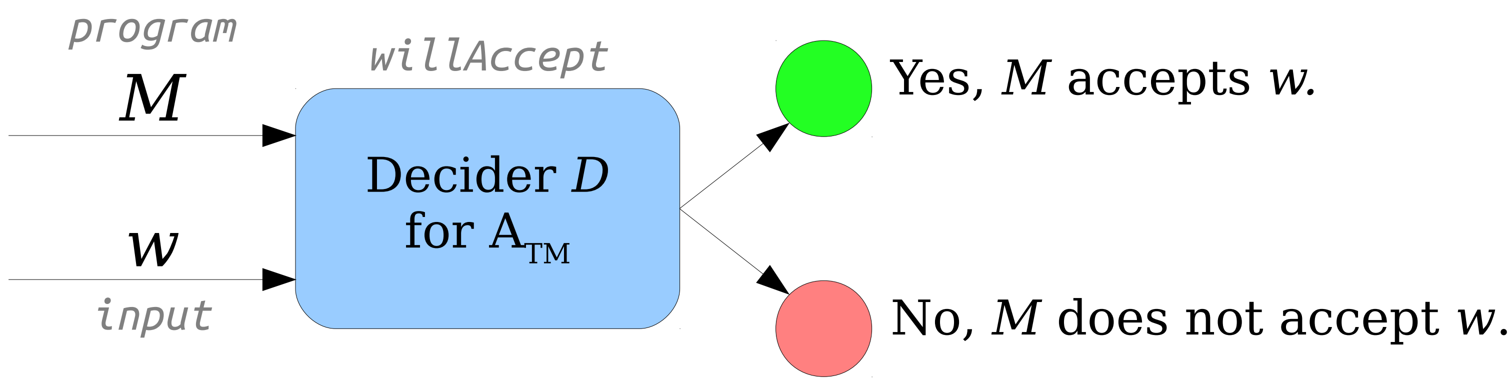


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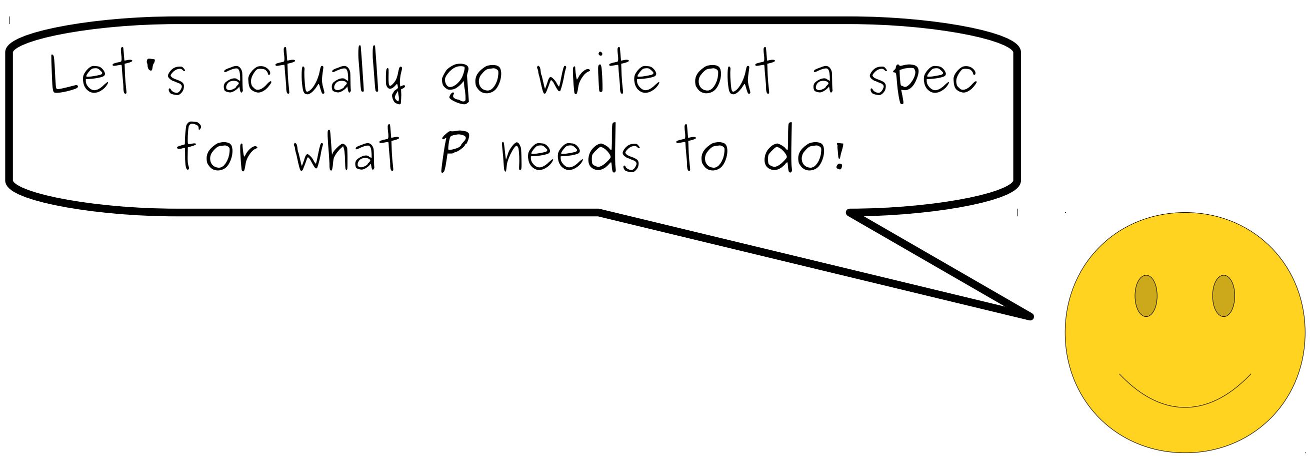

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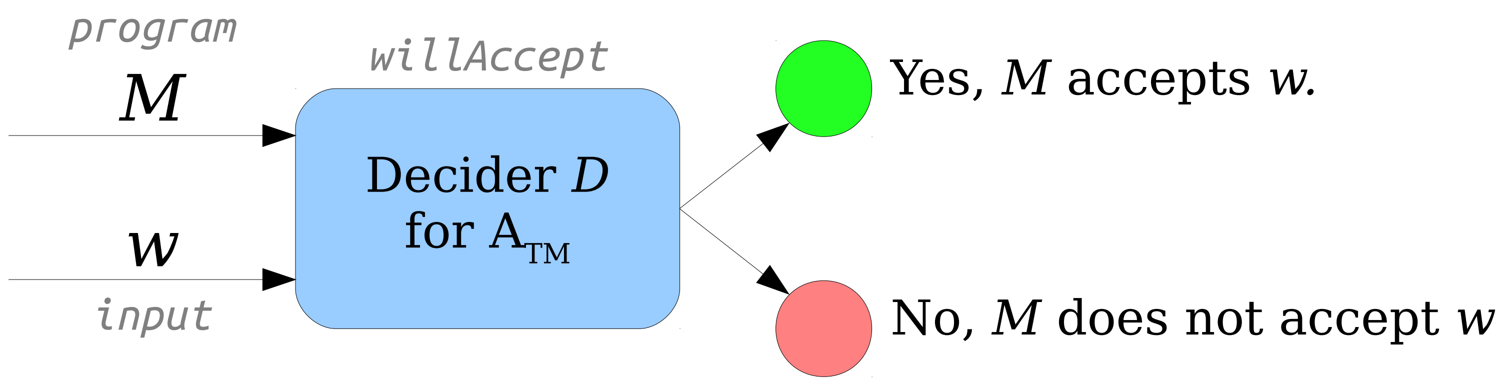


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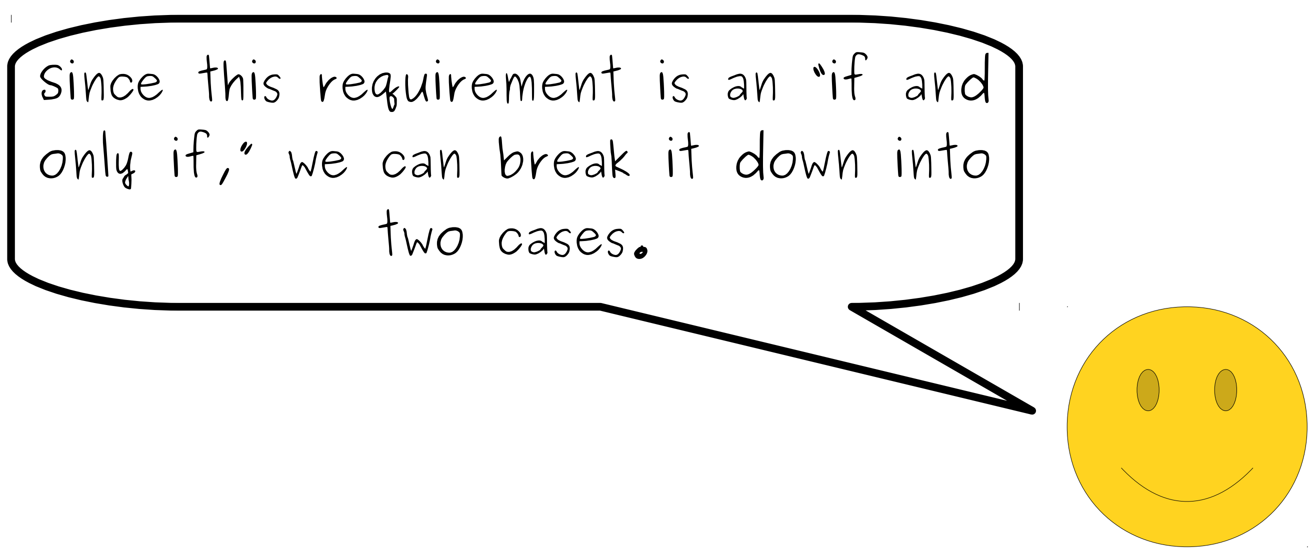

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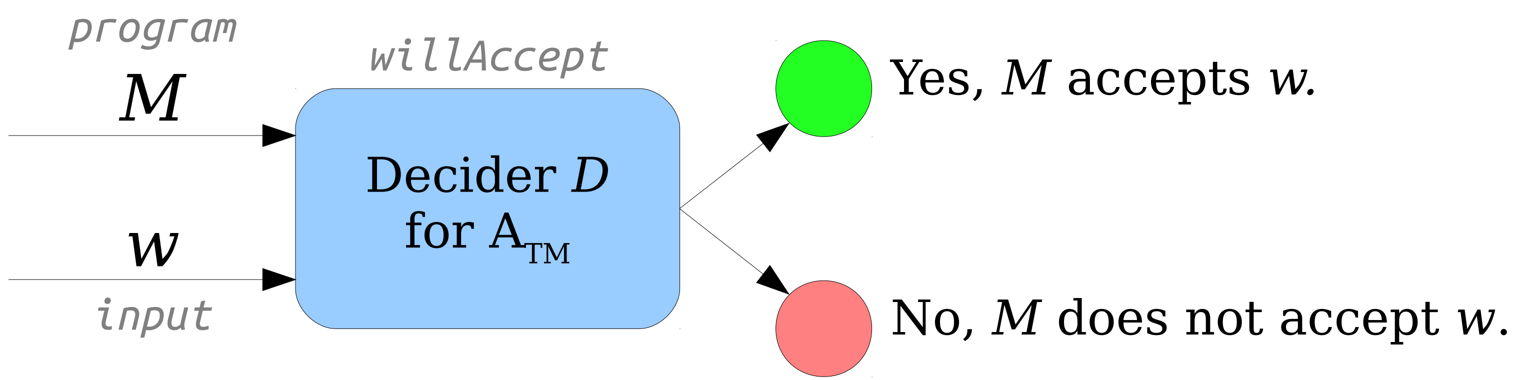


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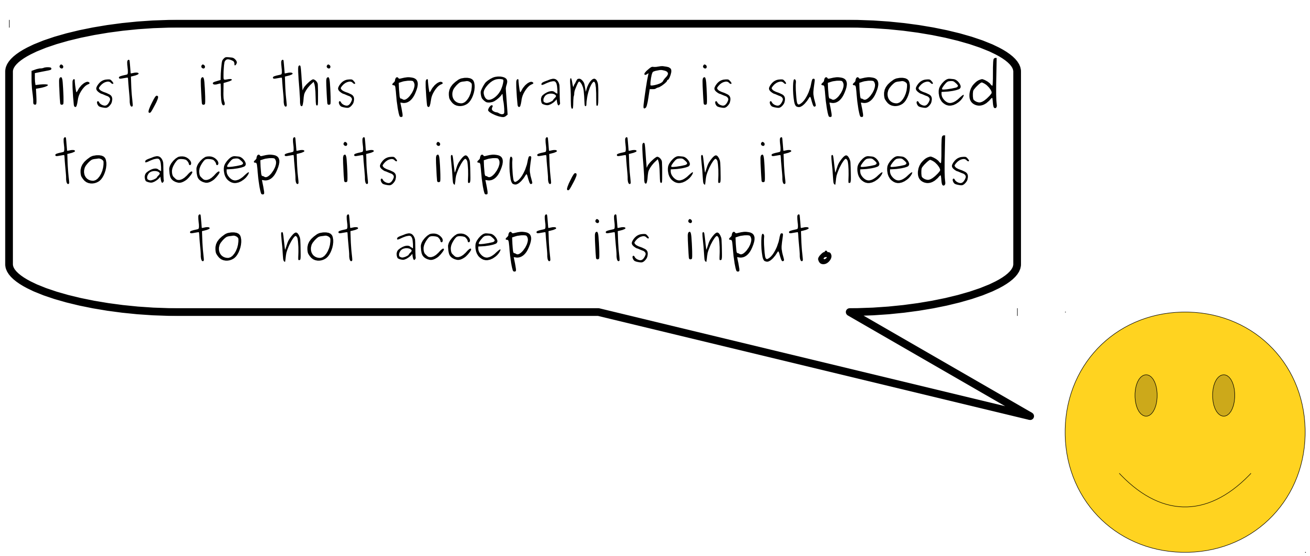

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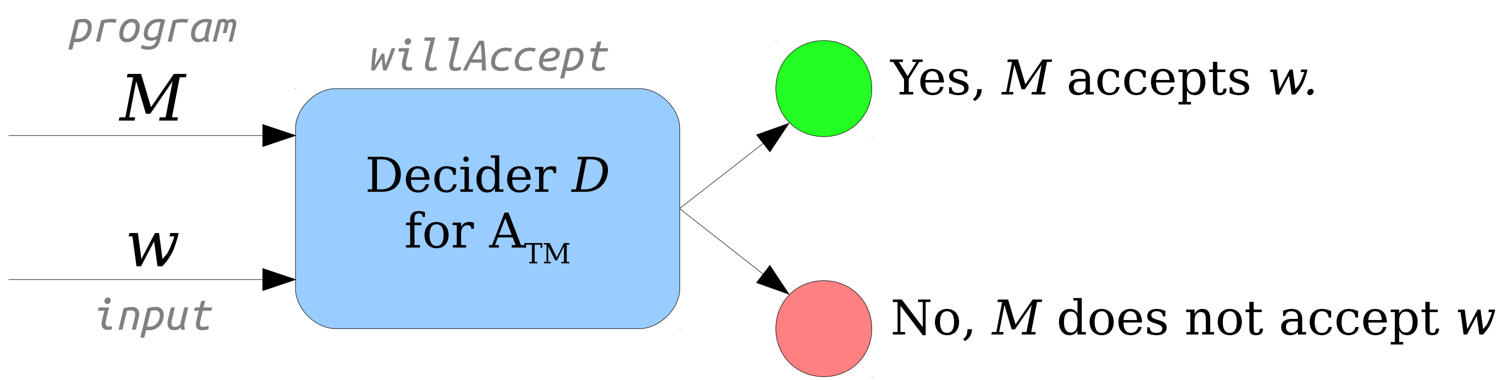


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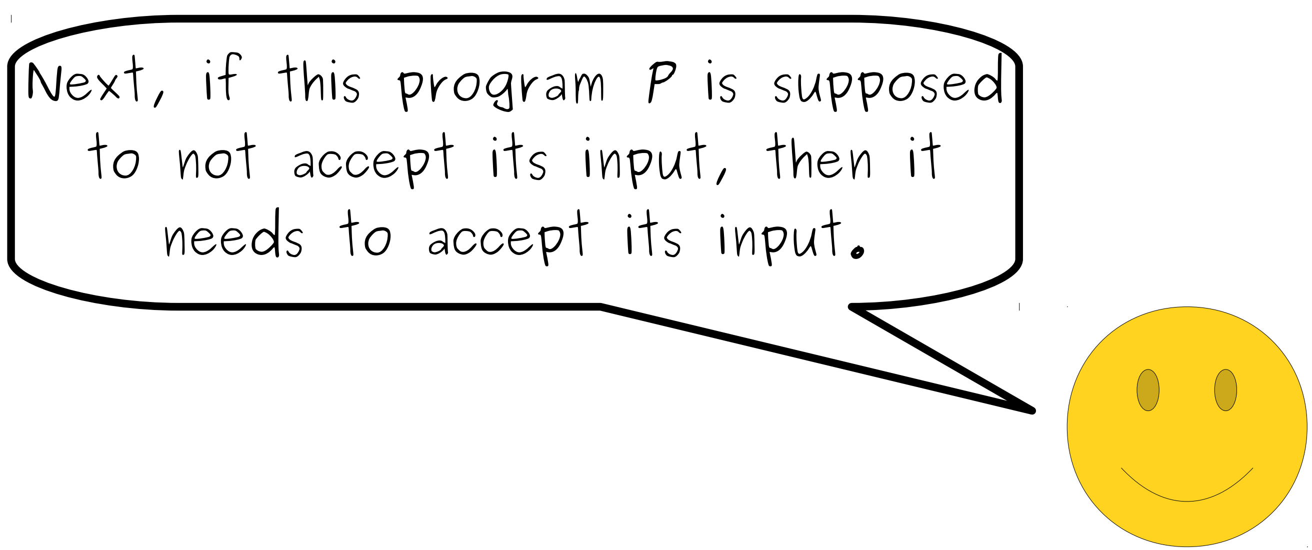

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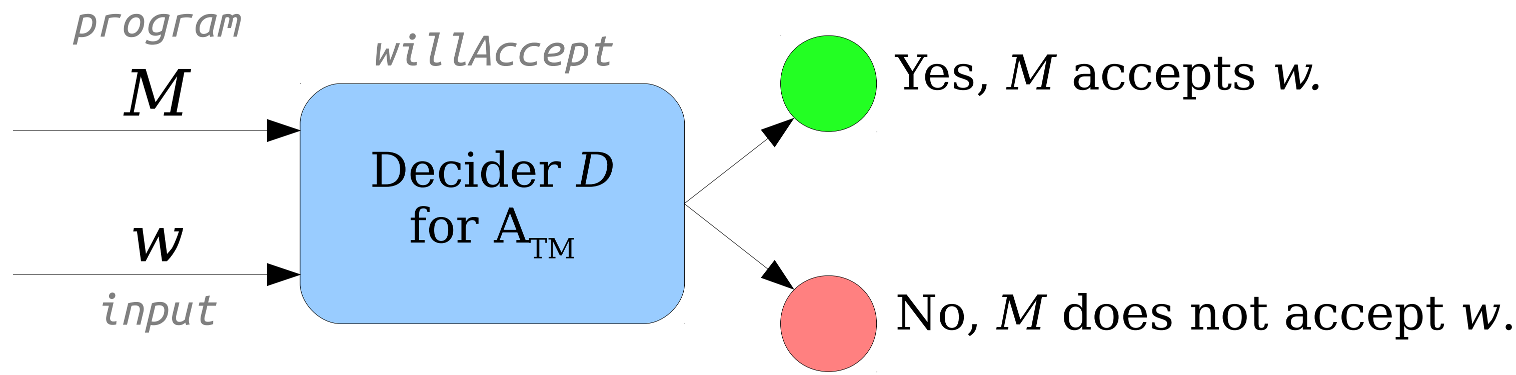


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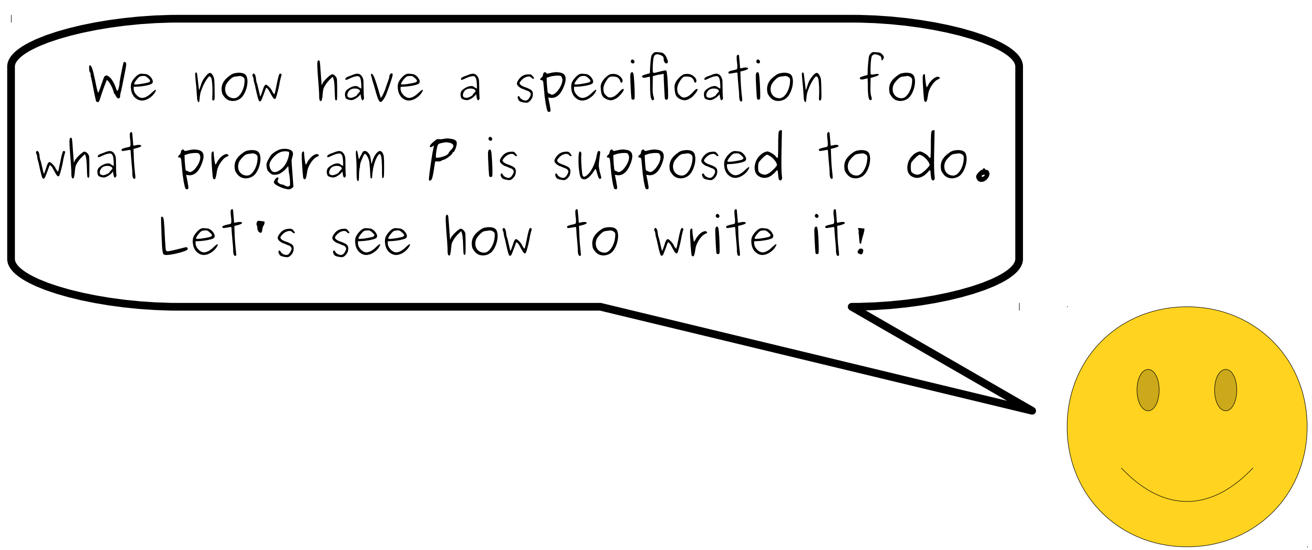

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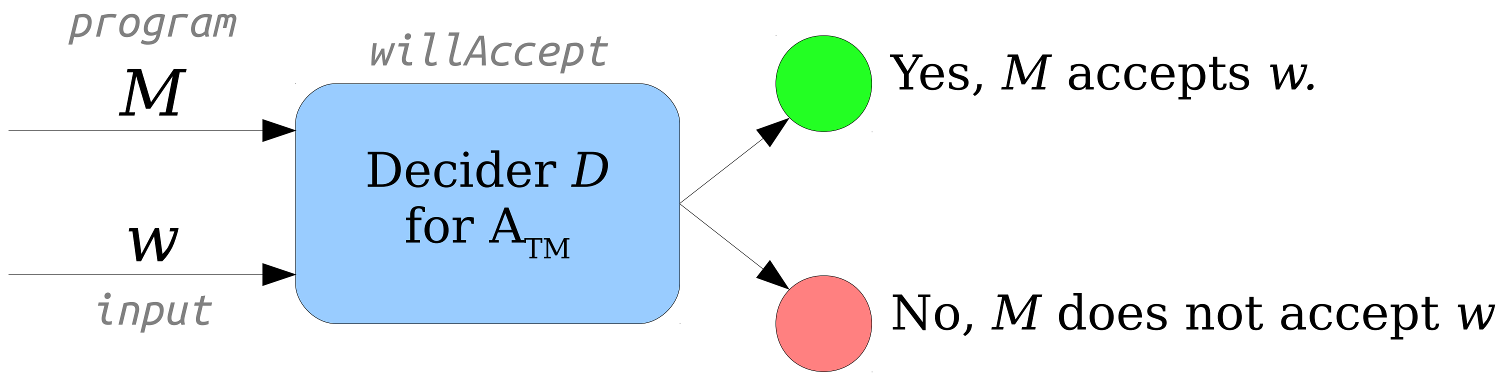


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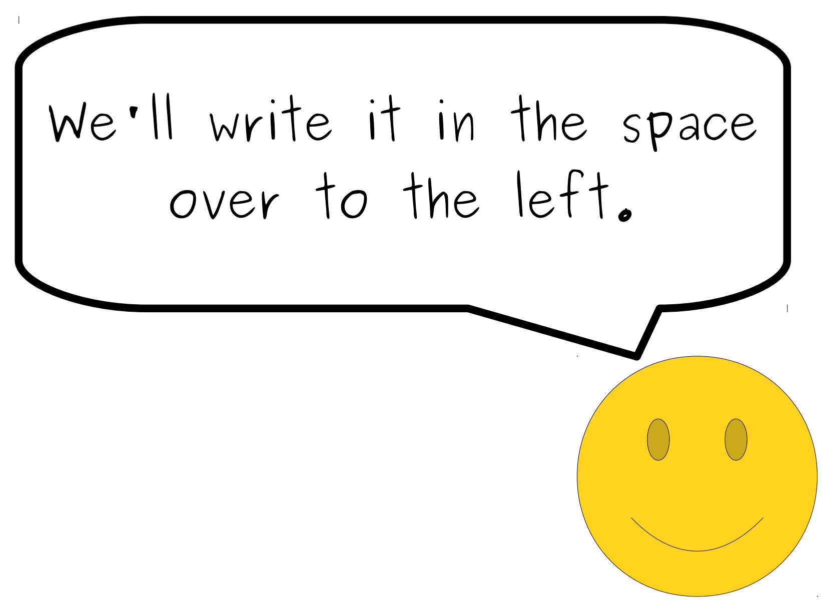

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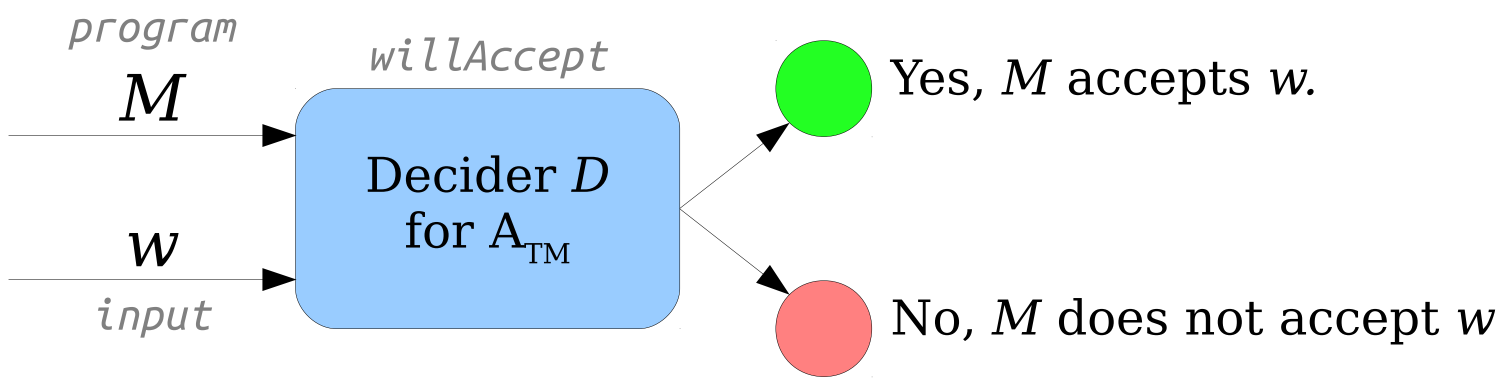


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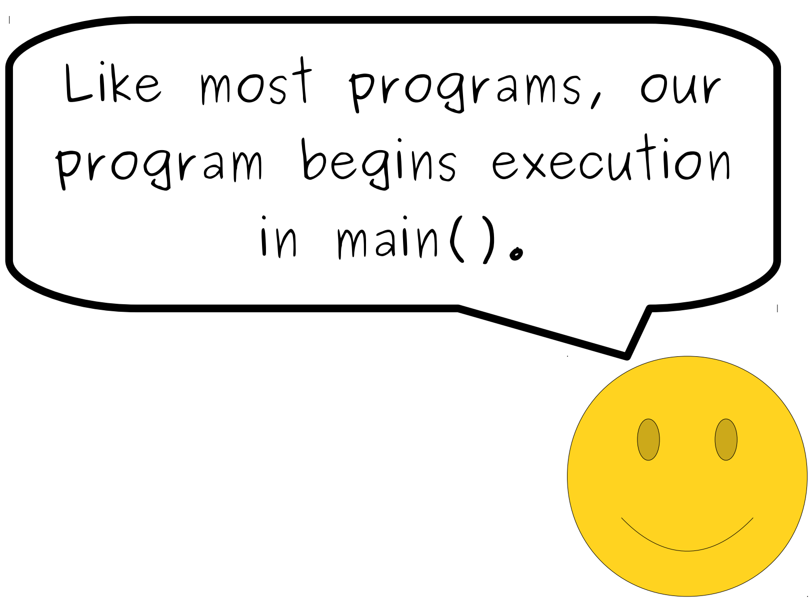

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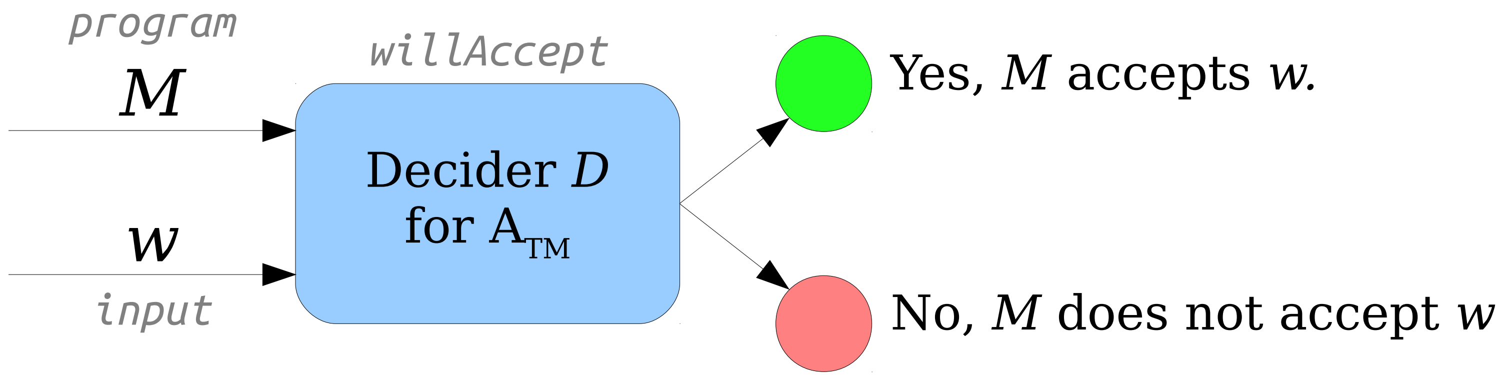


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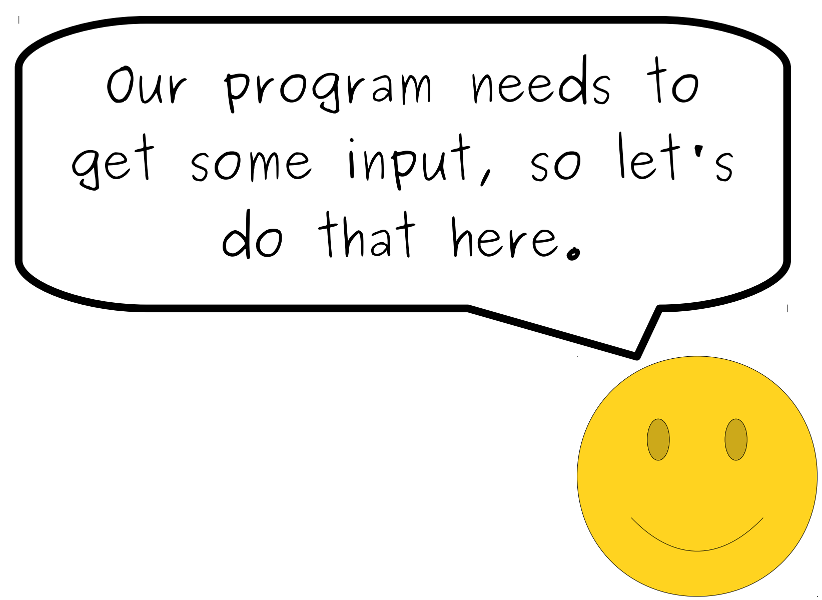

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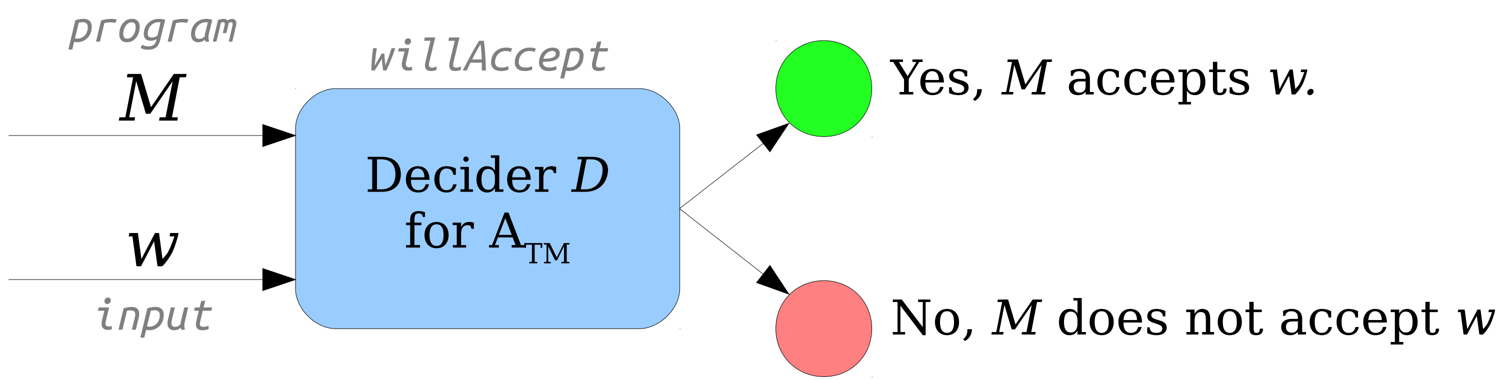


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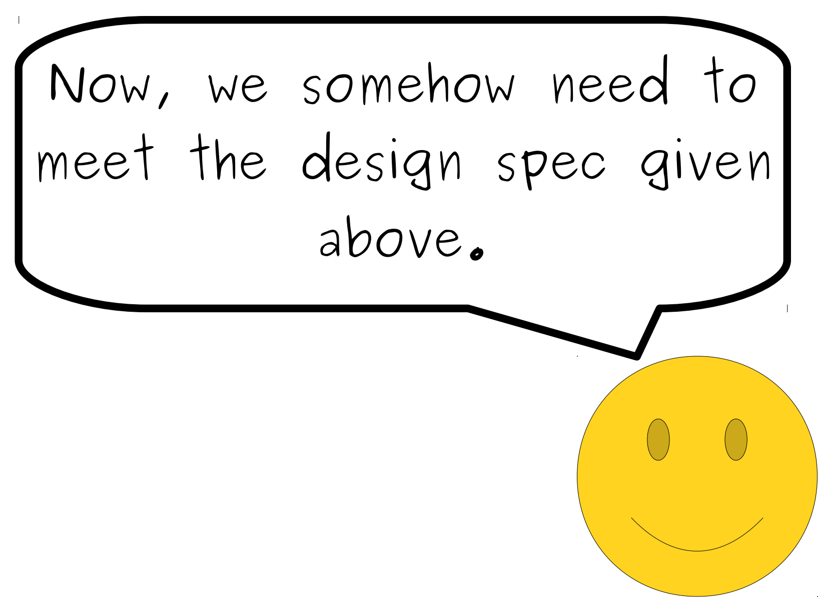

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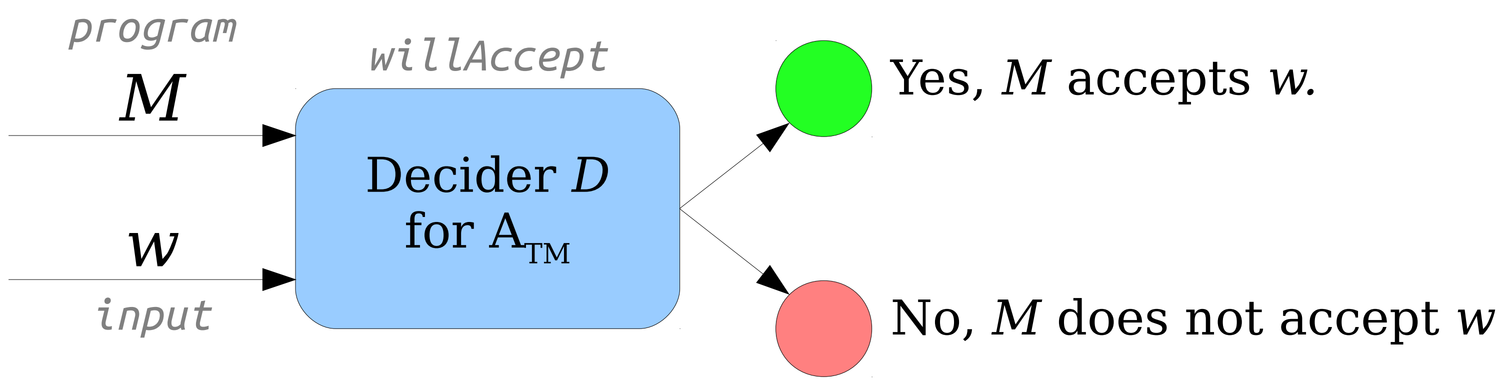


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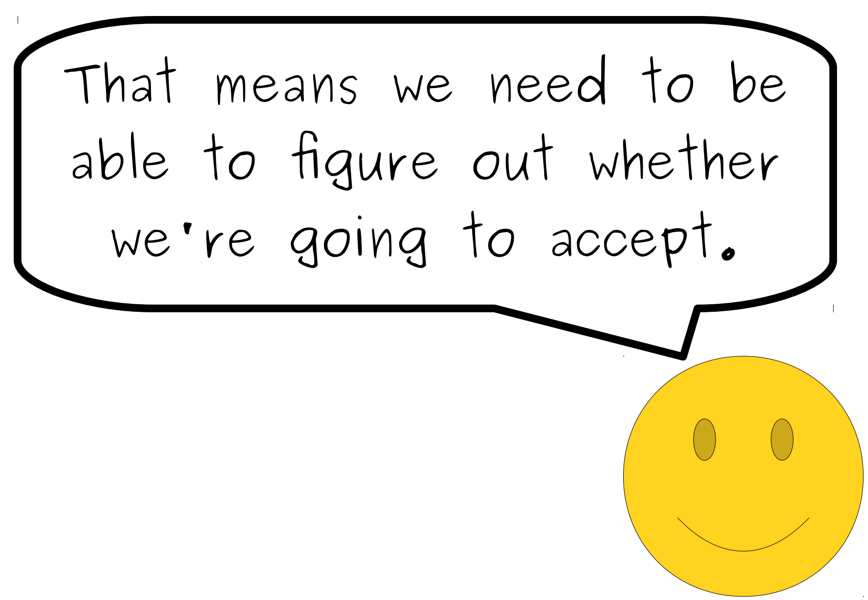

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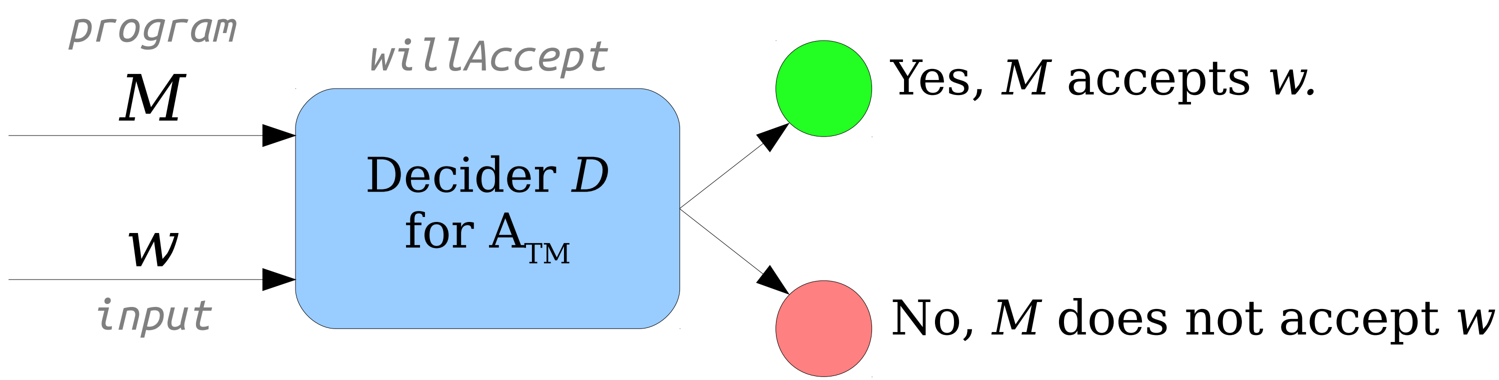


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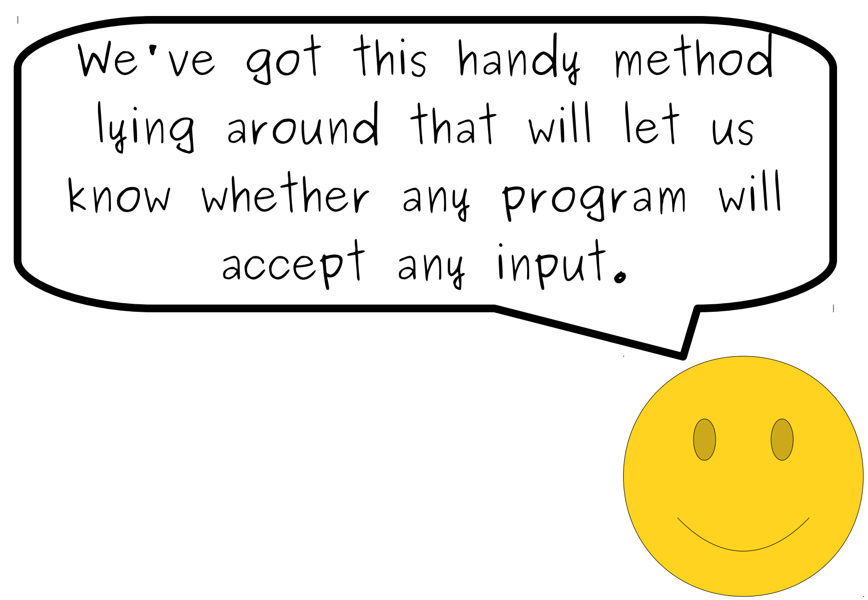

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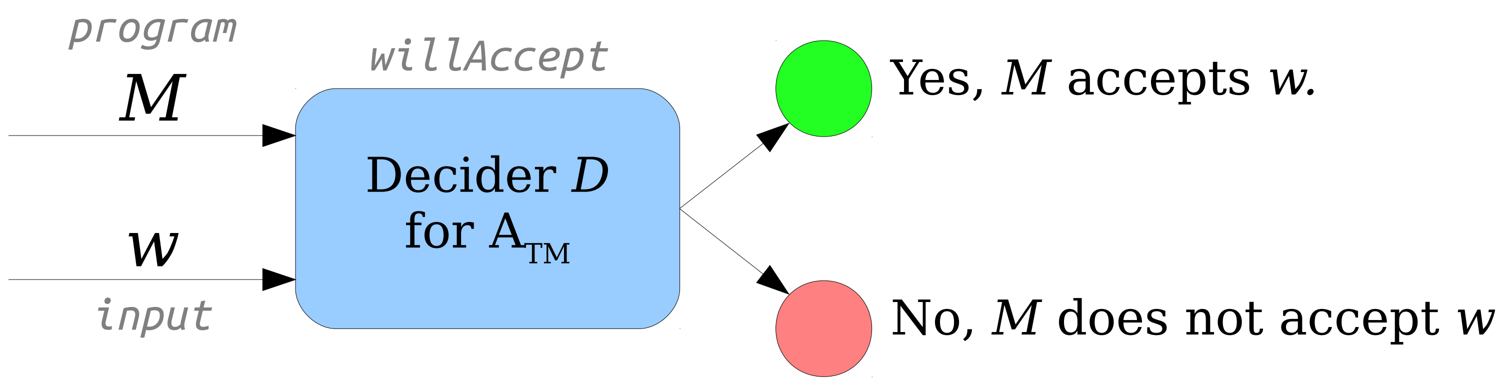


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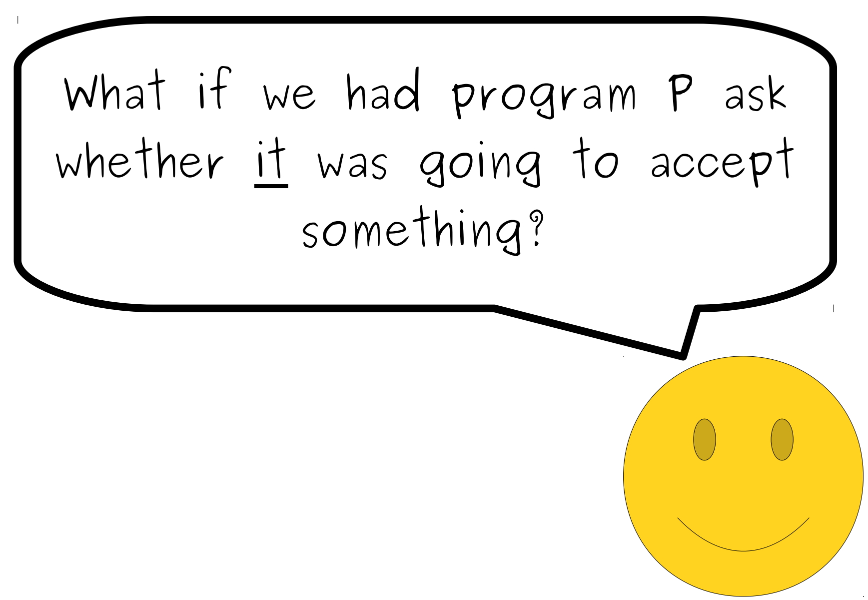

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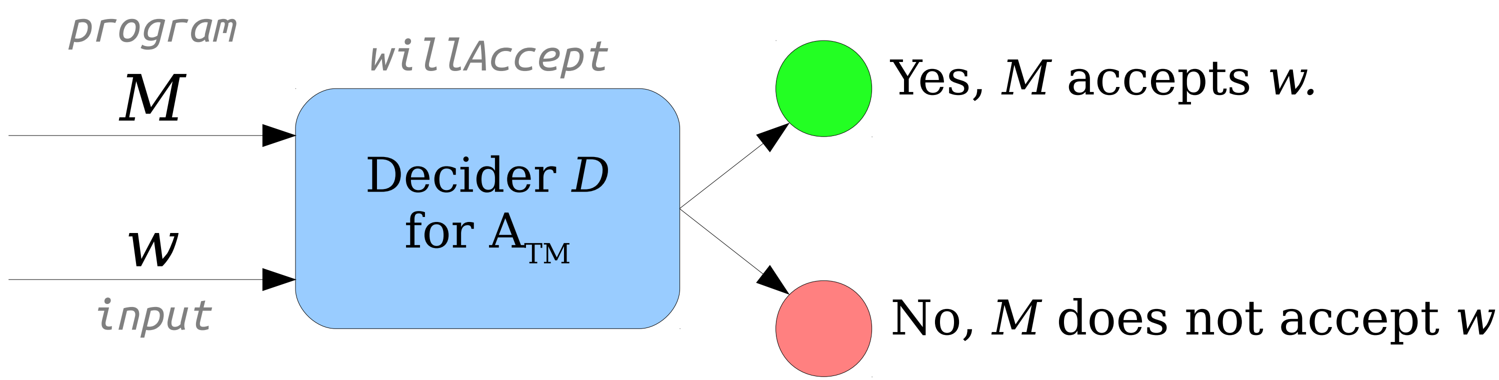


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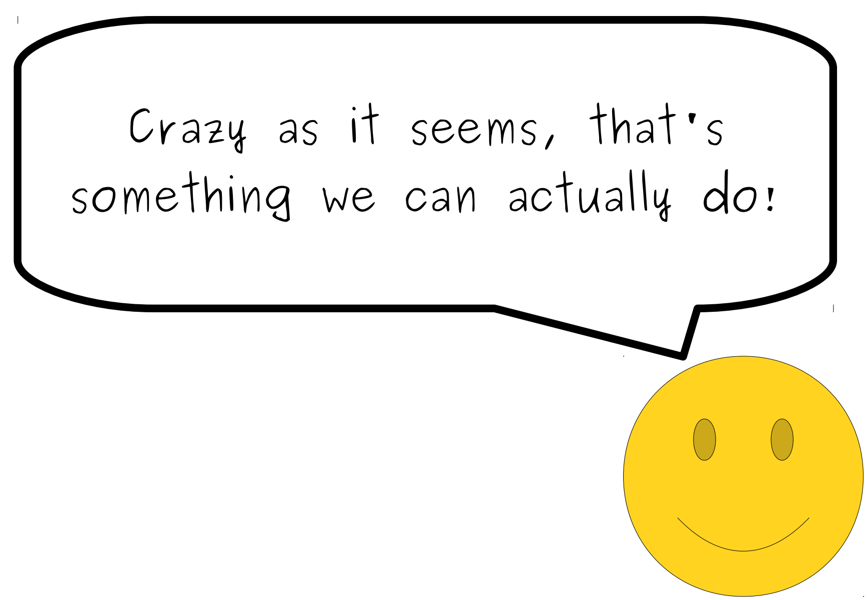

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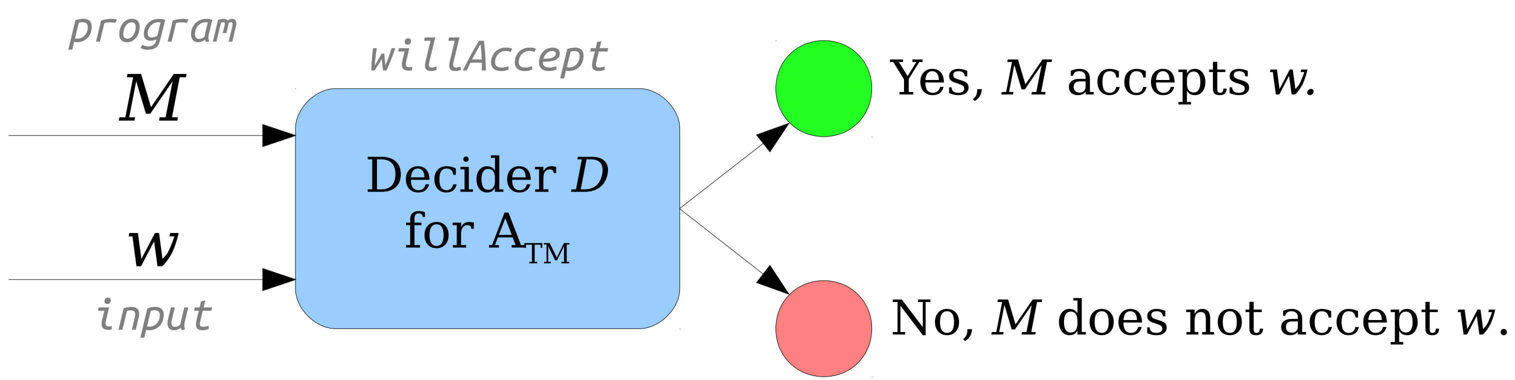


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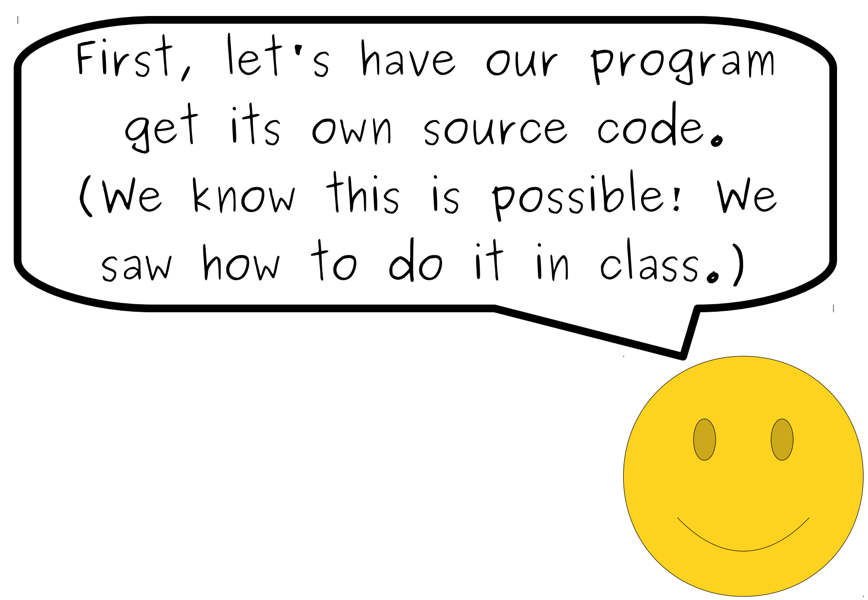

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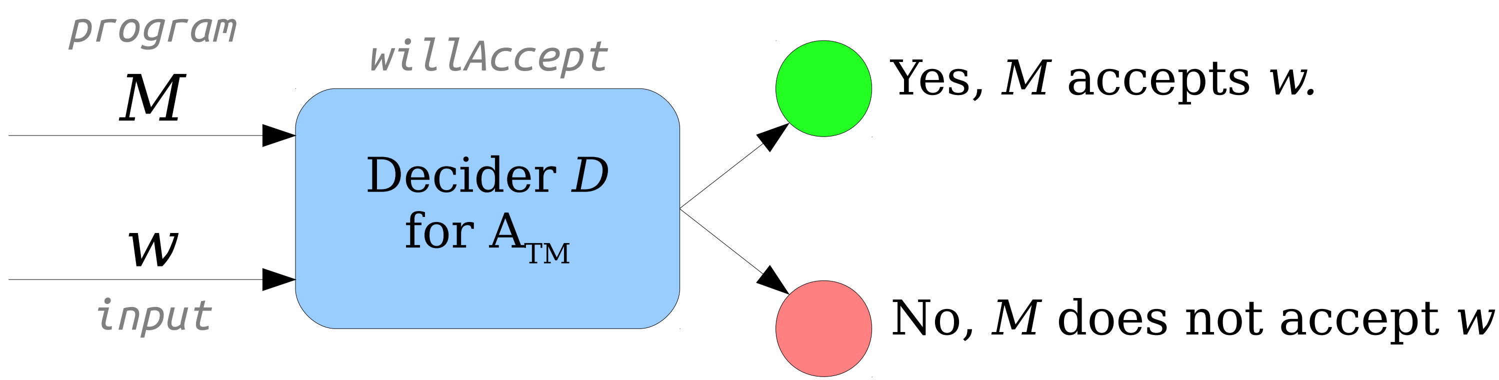


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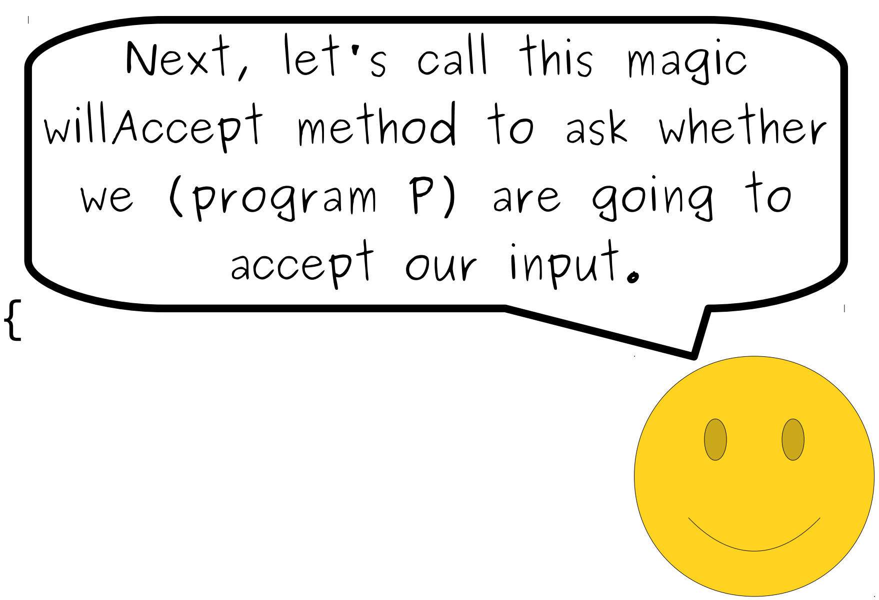

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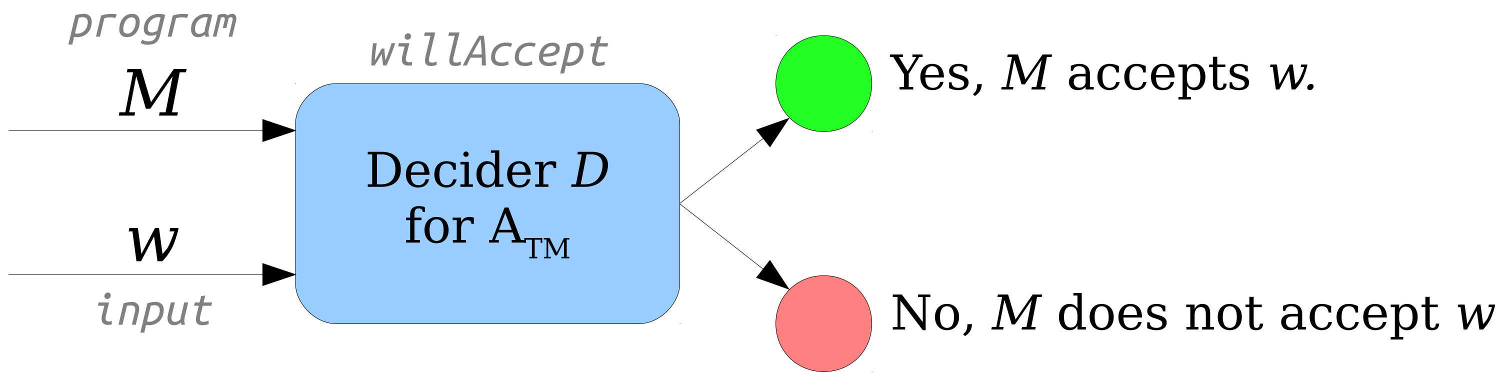


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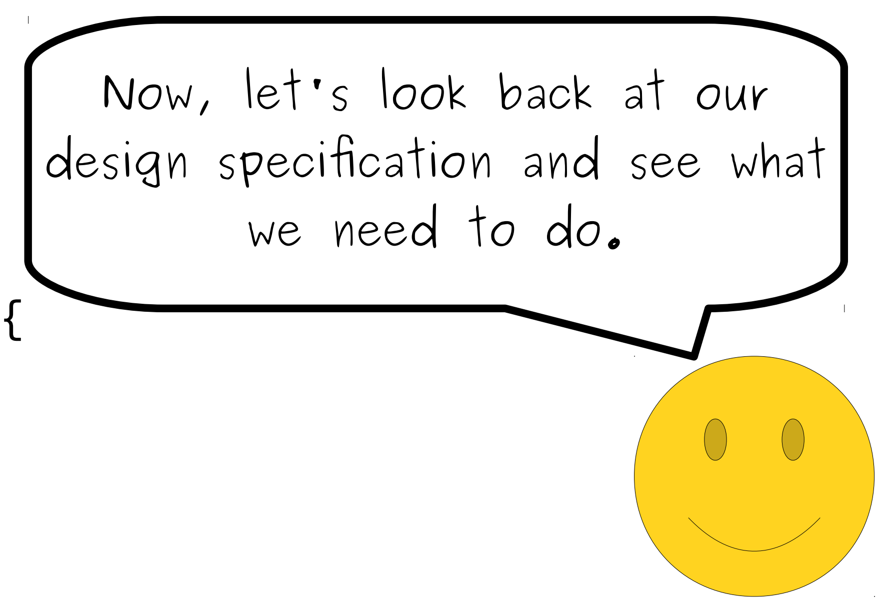

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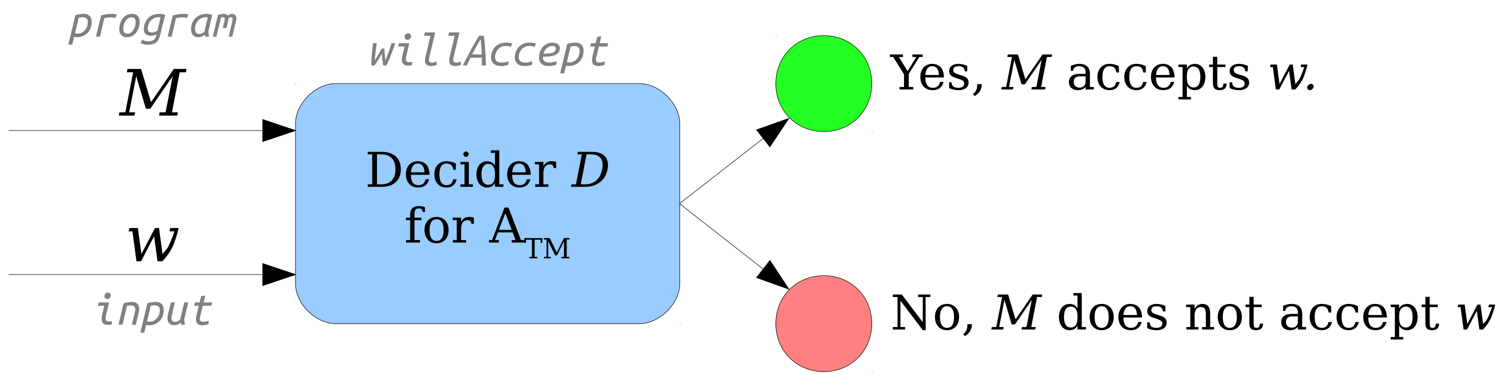


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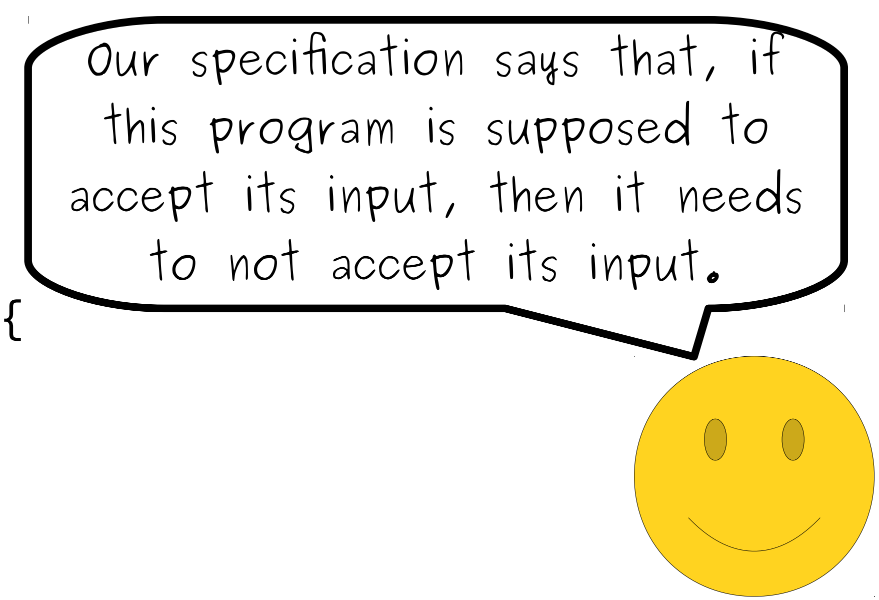

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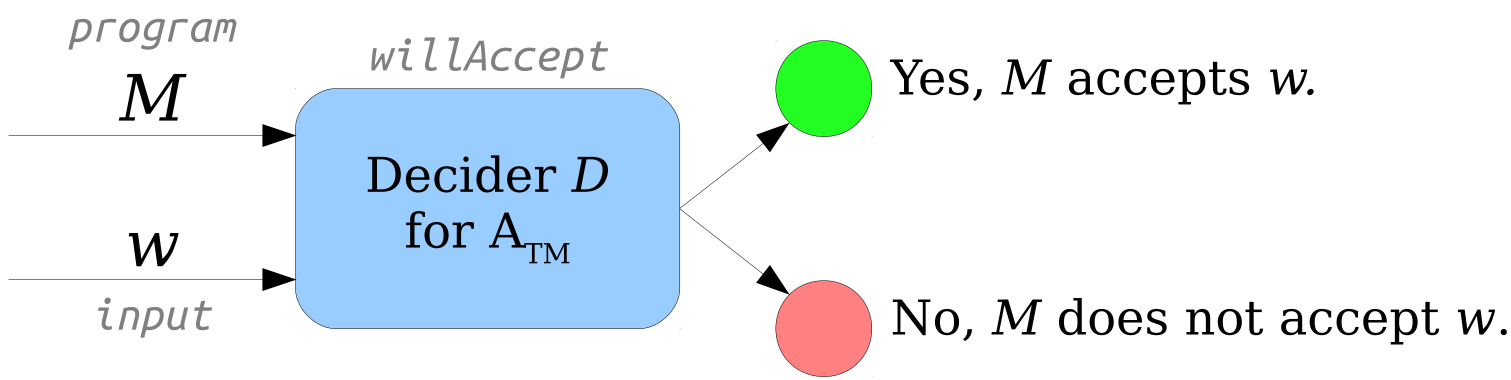


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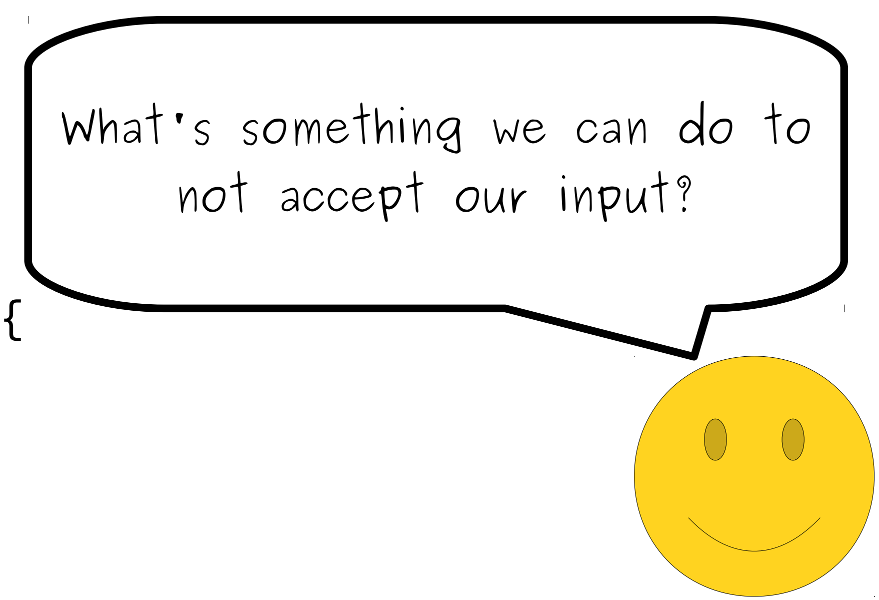

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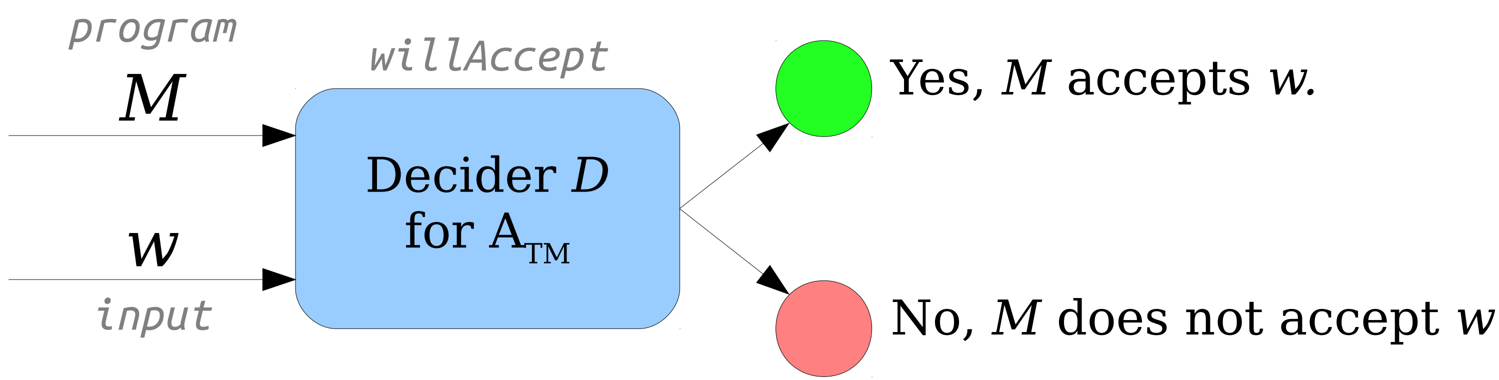


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int main() {
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  if (willAccept(me, input)) {
    reject();
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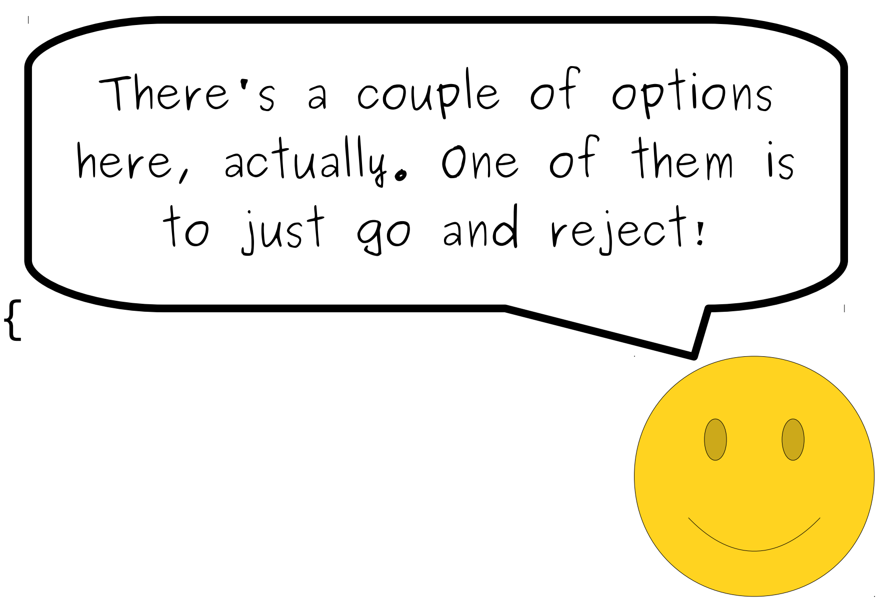

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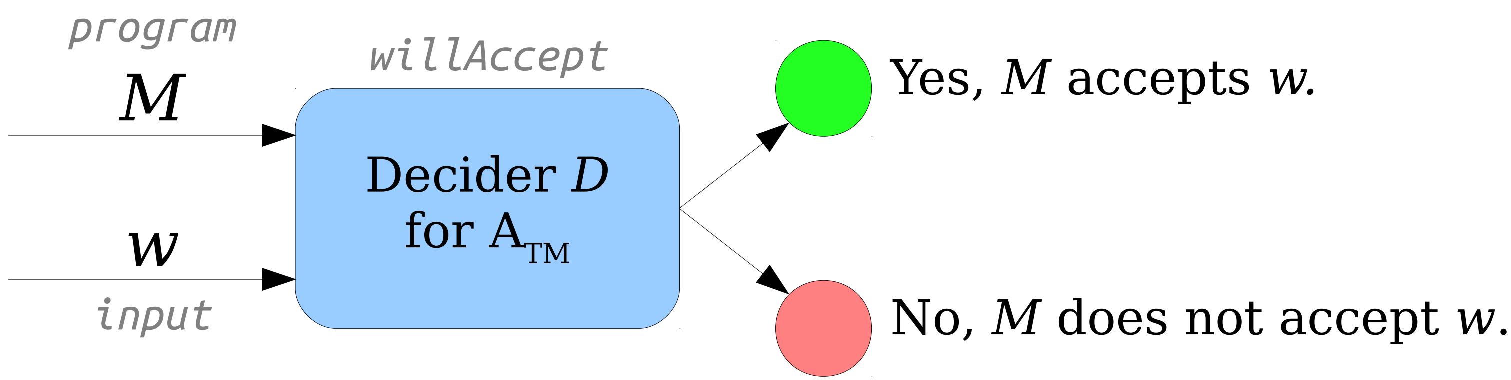


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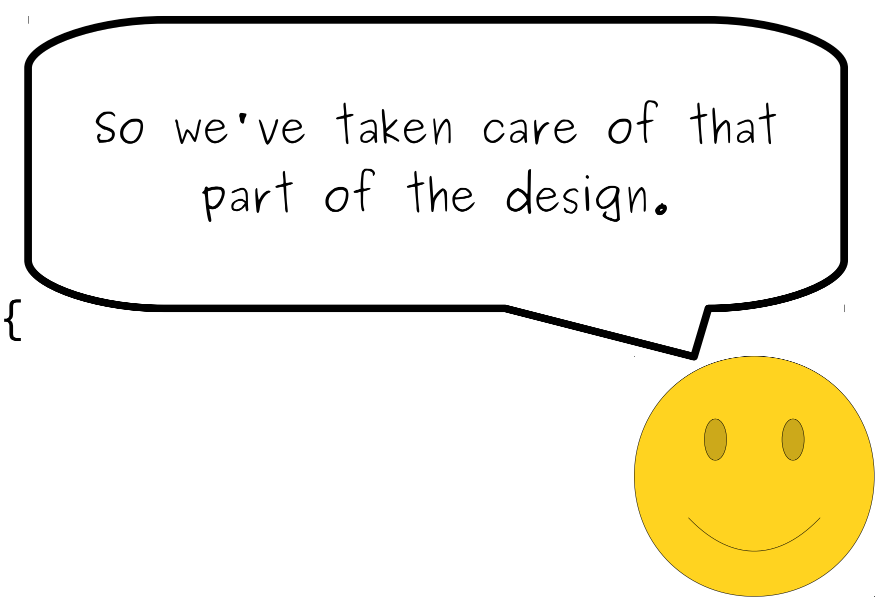

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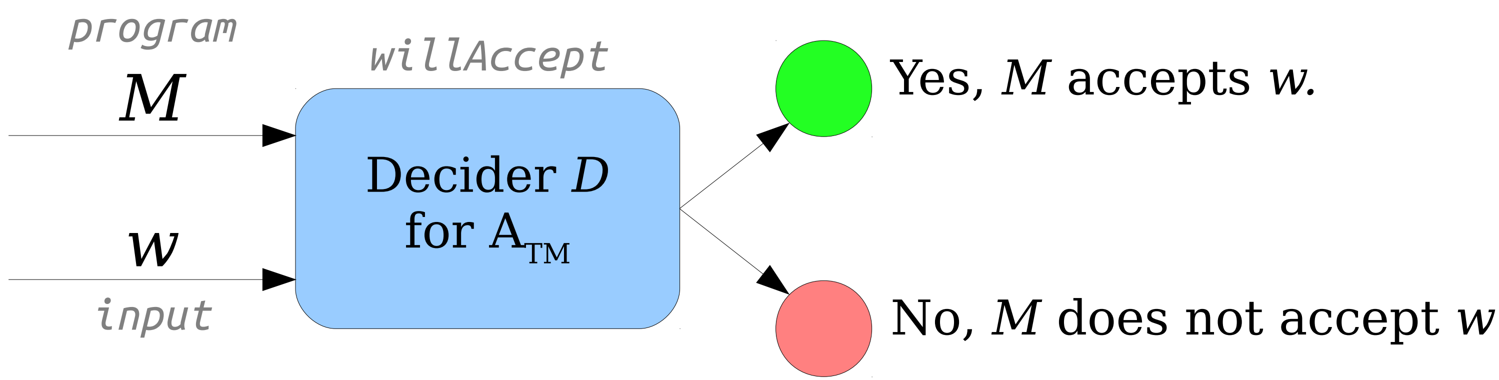


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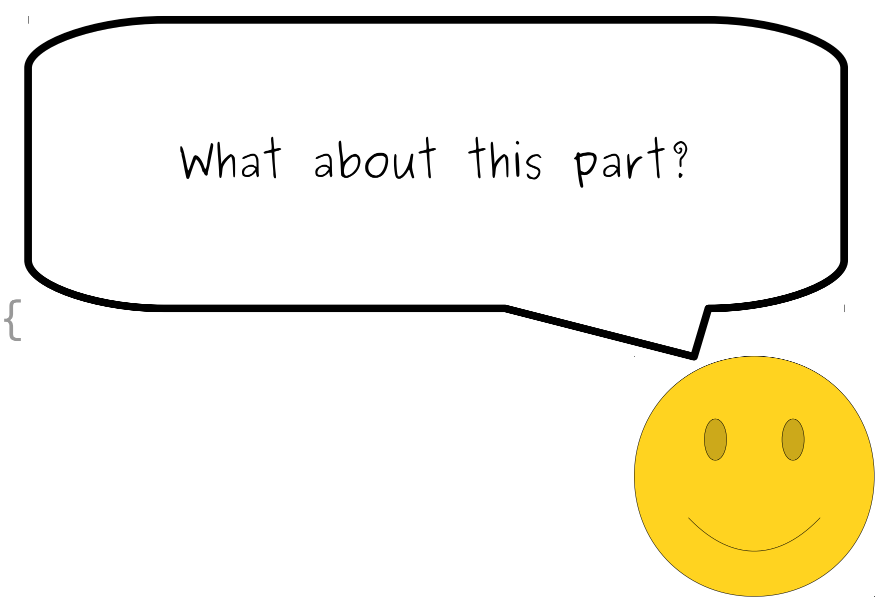

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Program P accepts its input if and only if program P does not accept its input

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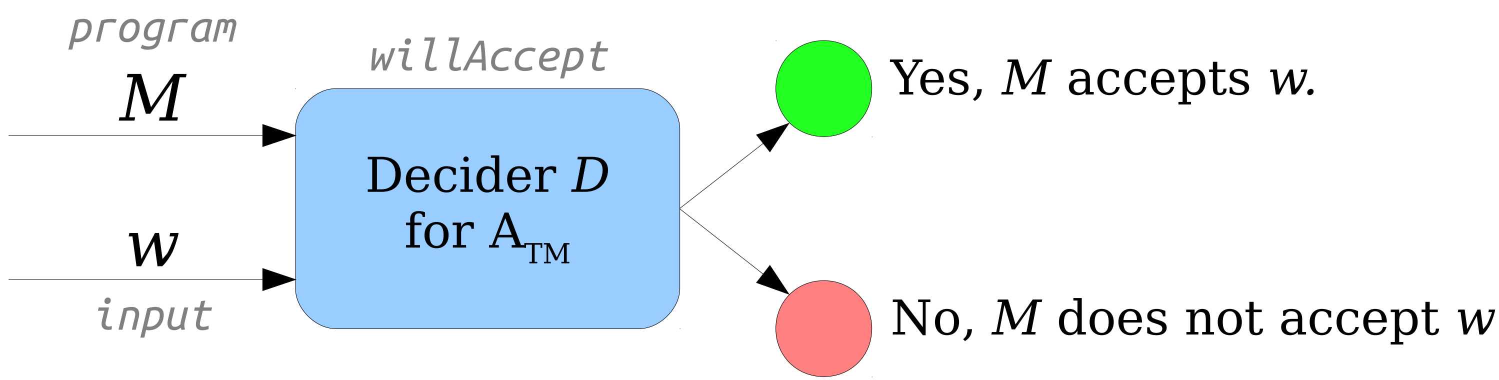


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bool willAccept(string program, string input)
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Program P design specification:

✓ If P accepts its input, then
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If P does not accept its input, then
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```
// Program P

int main() {
  string input = getInput();
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  if (willAccept(me, input)) {
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  } else {

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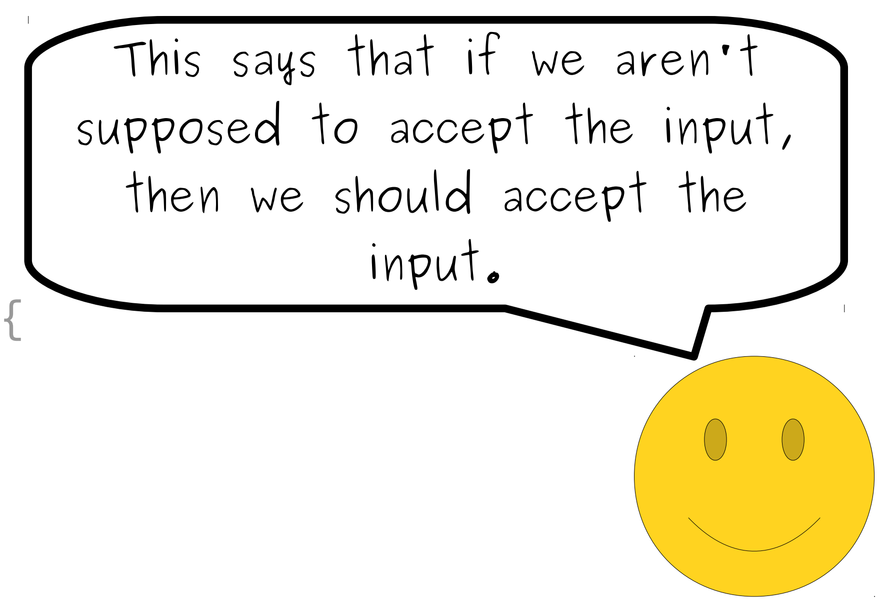

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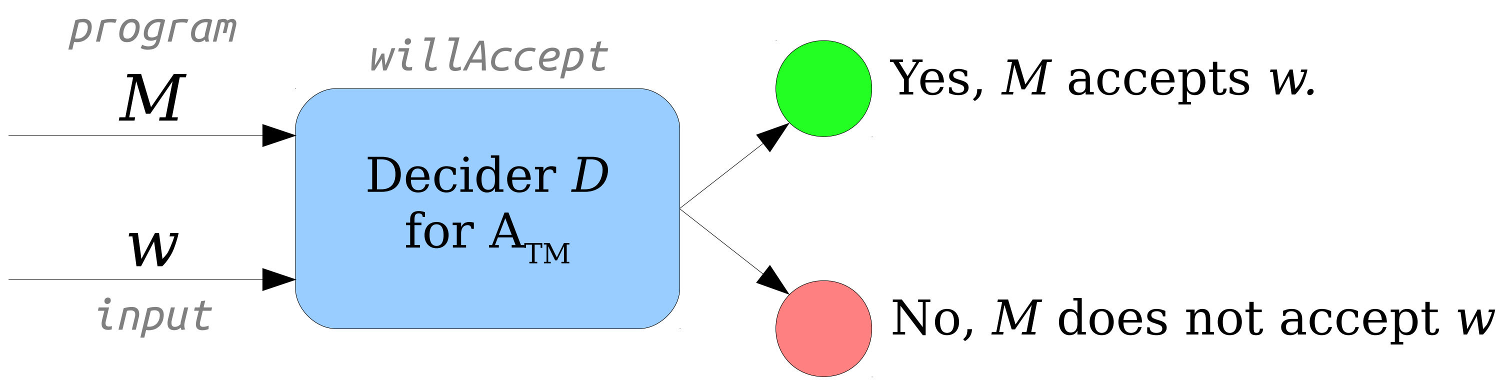


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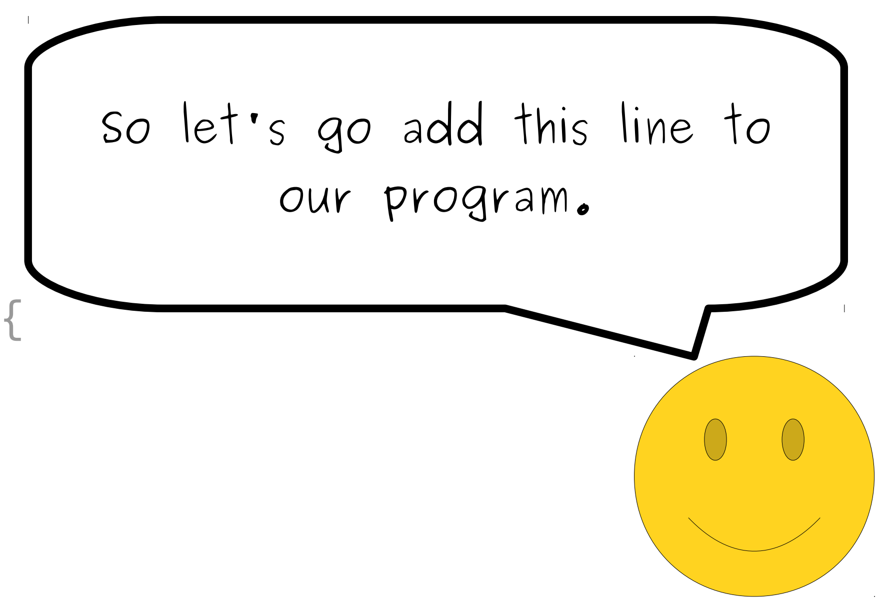

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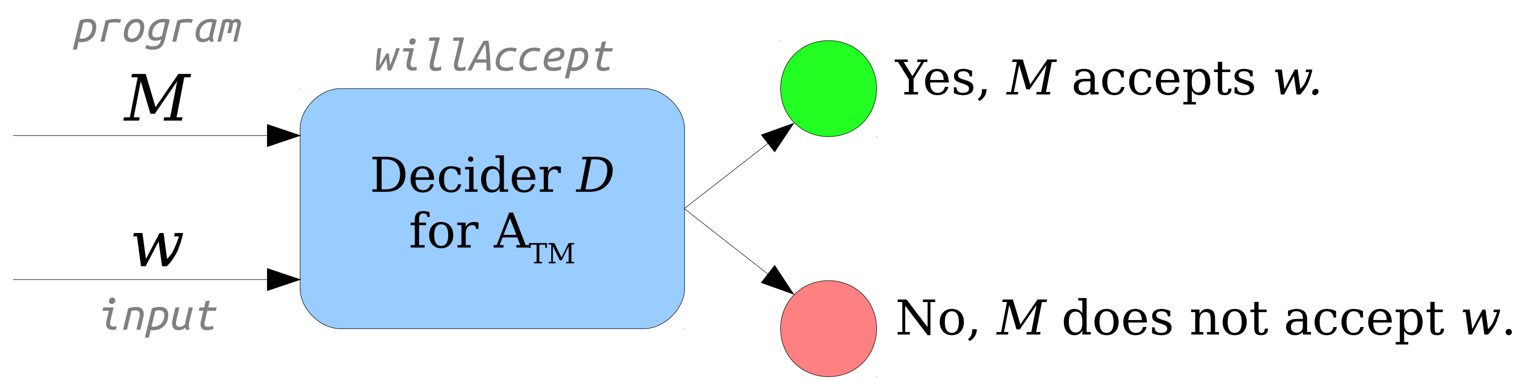


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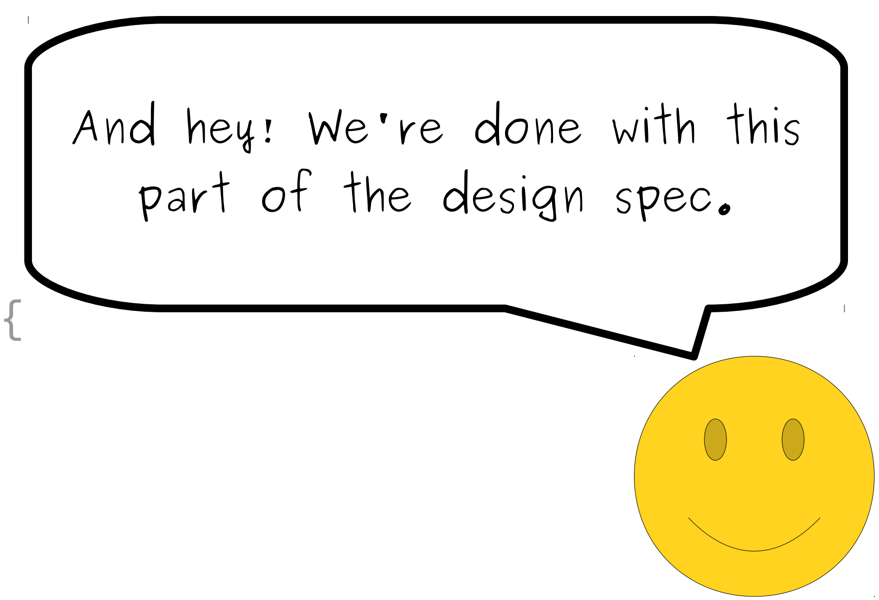

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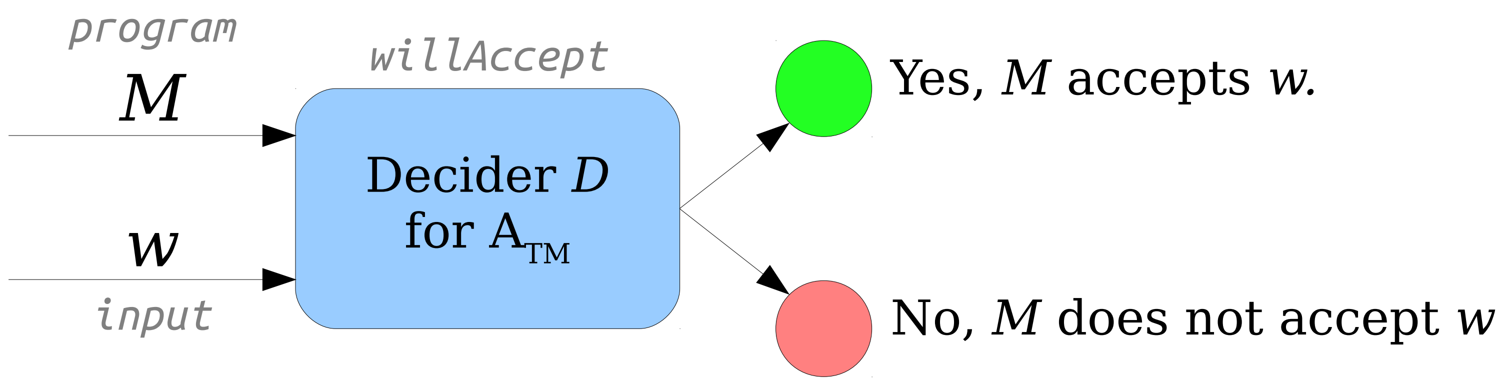


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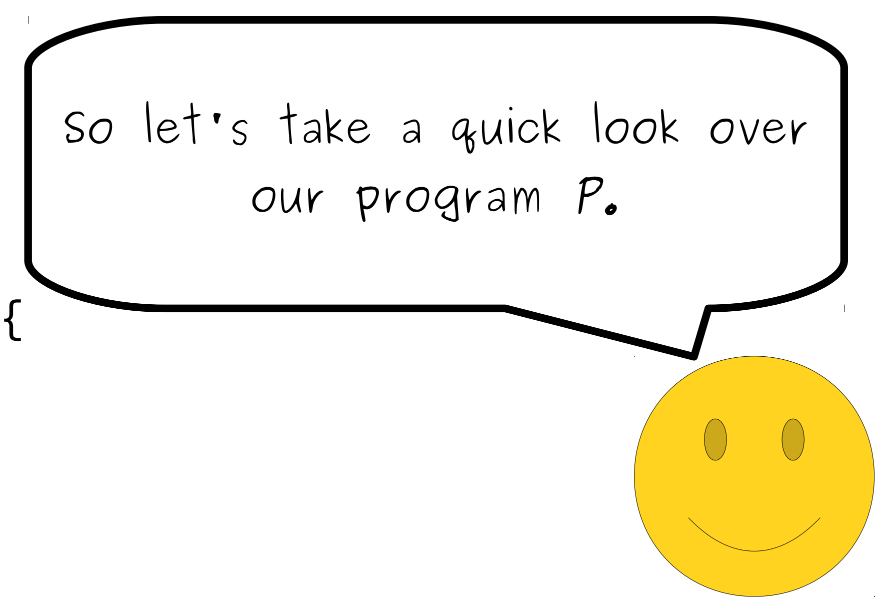

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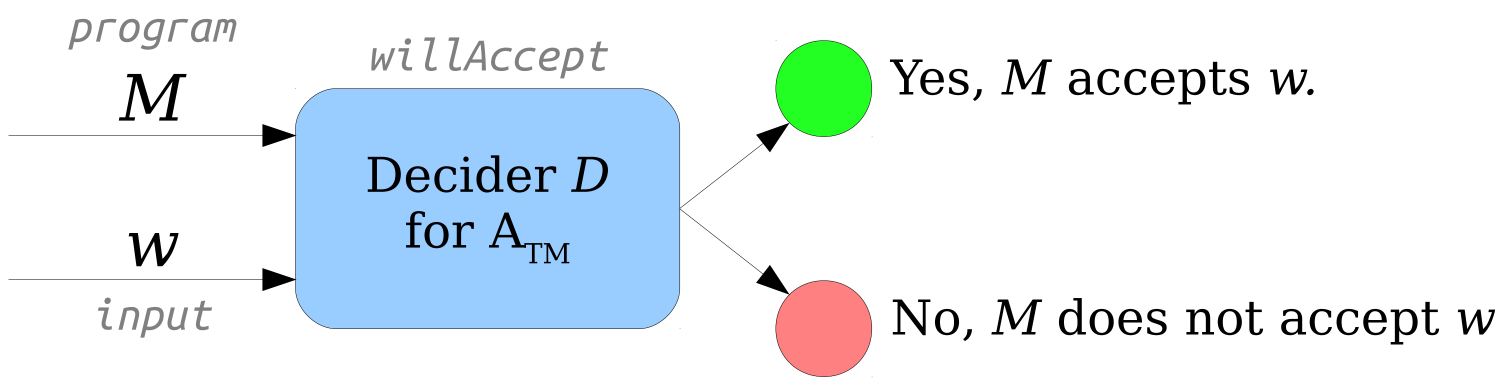


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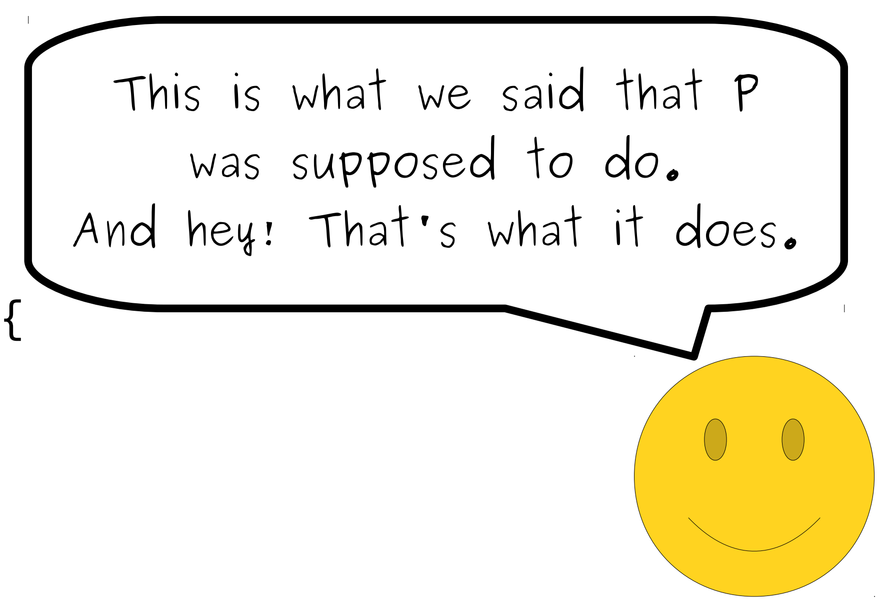

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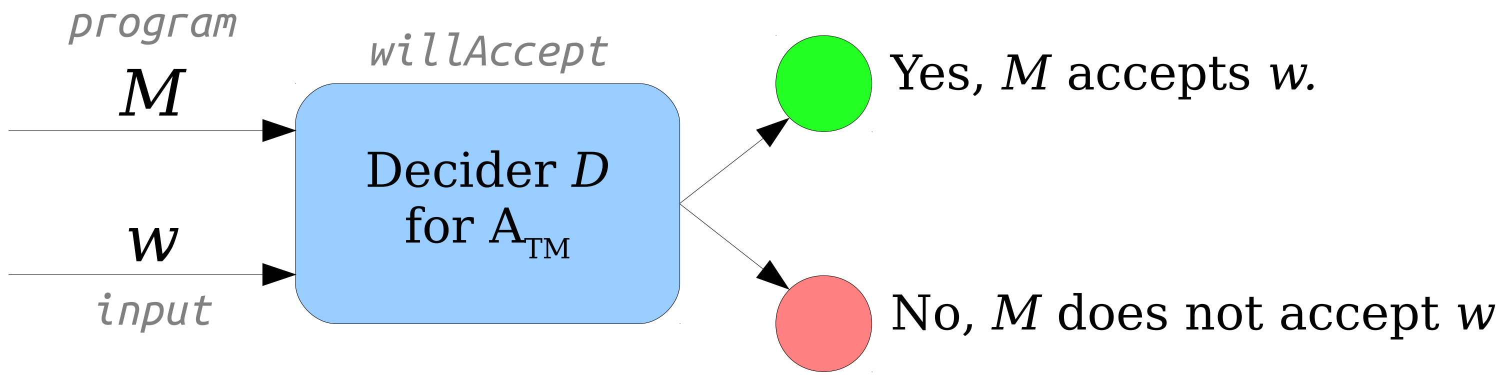


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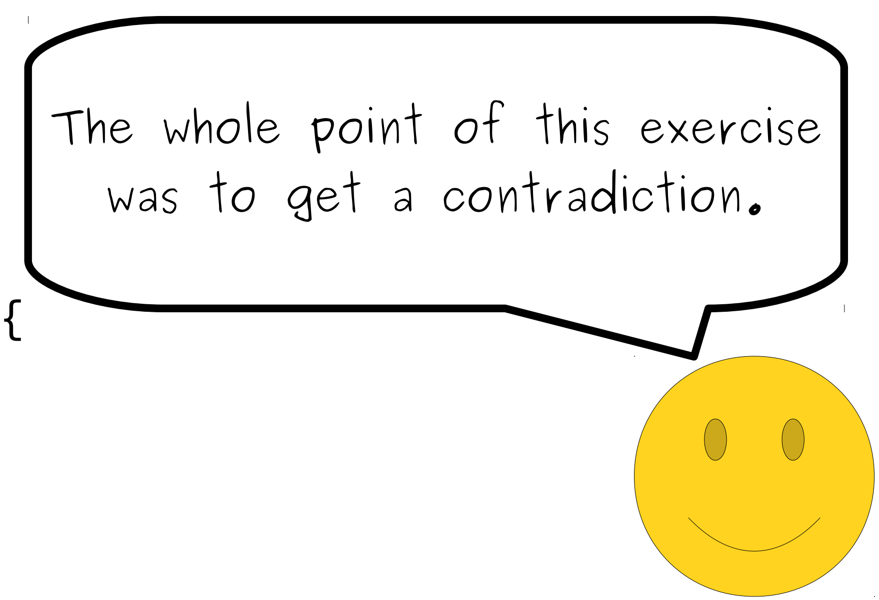

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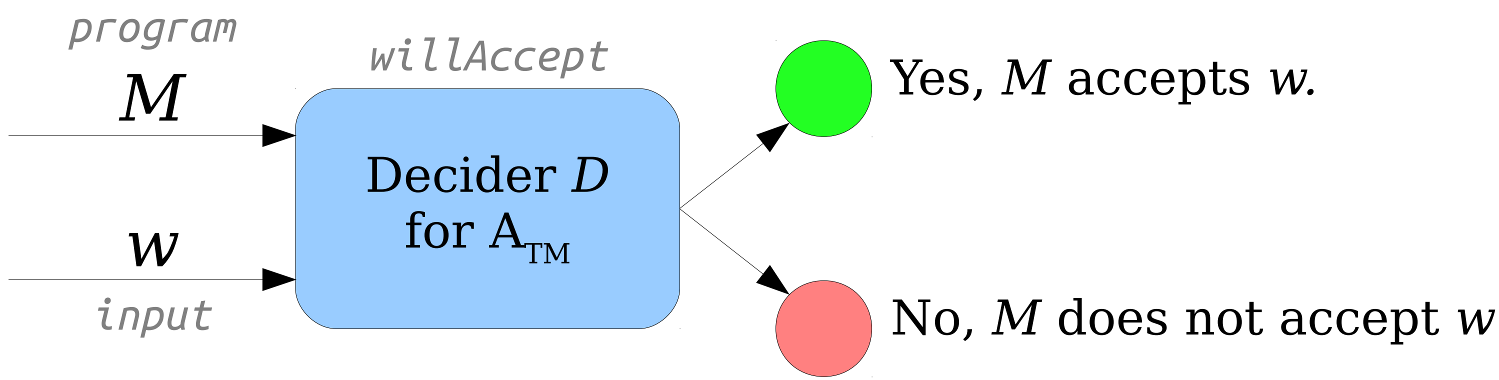


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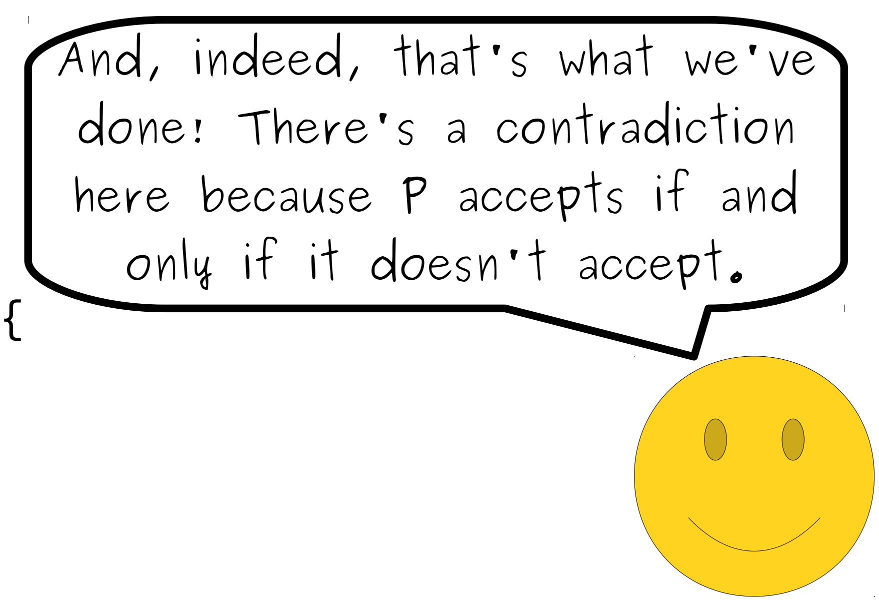

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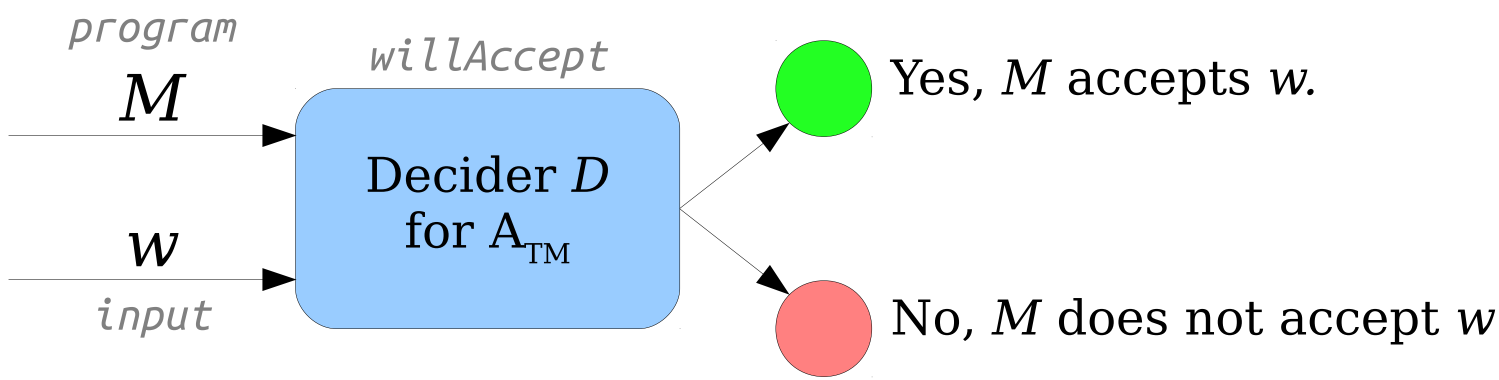


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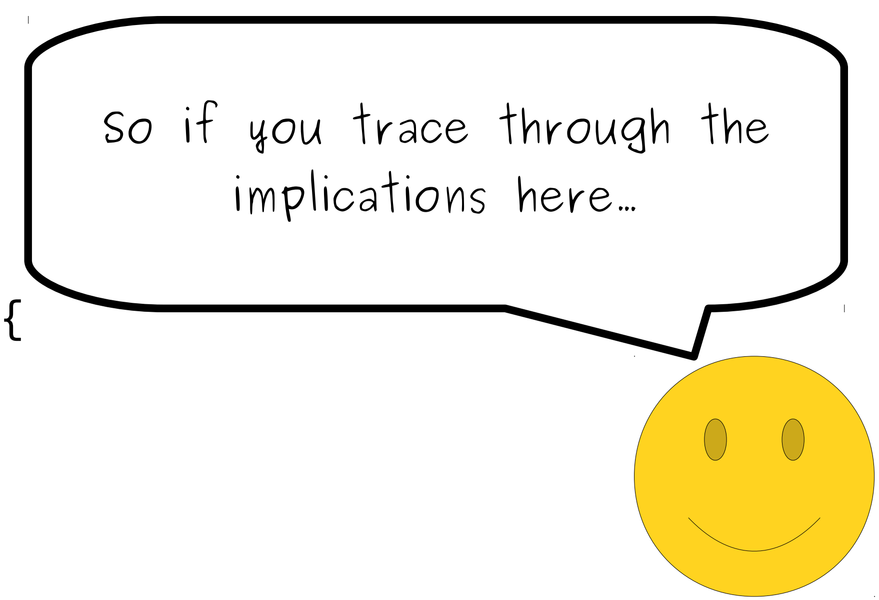

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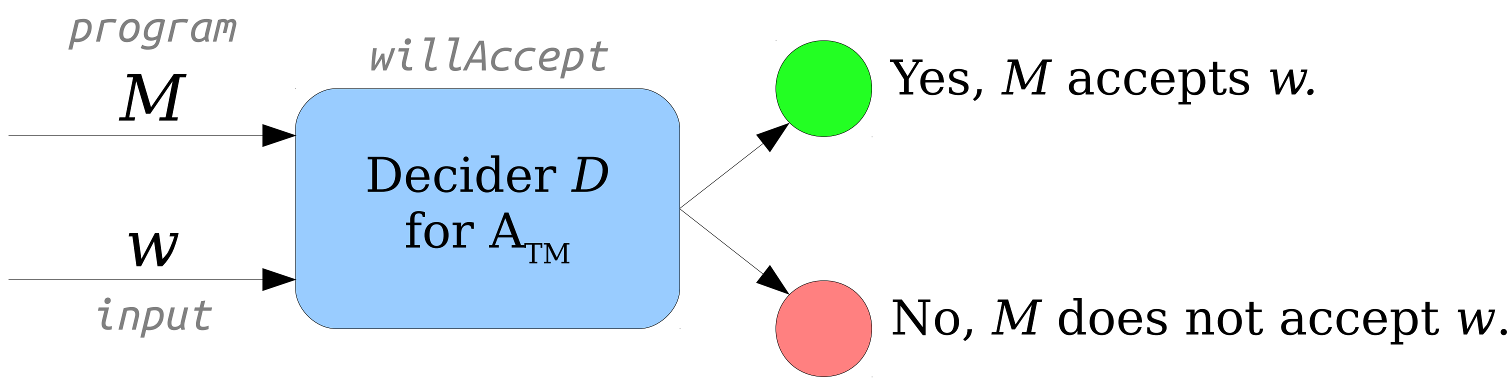


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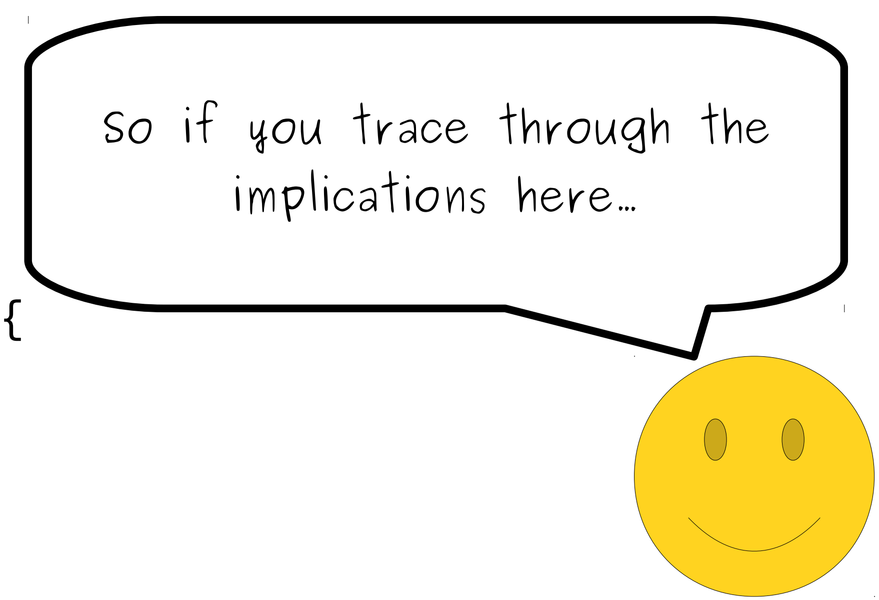

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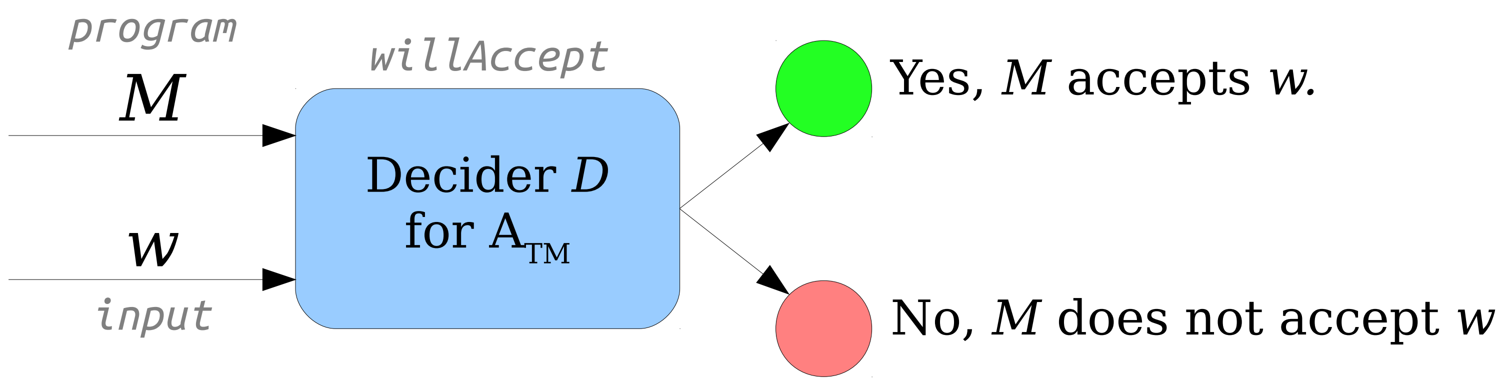


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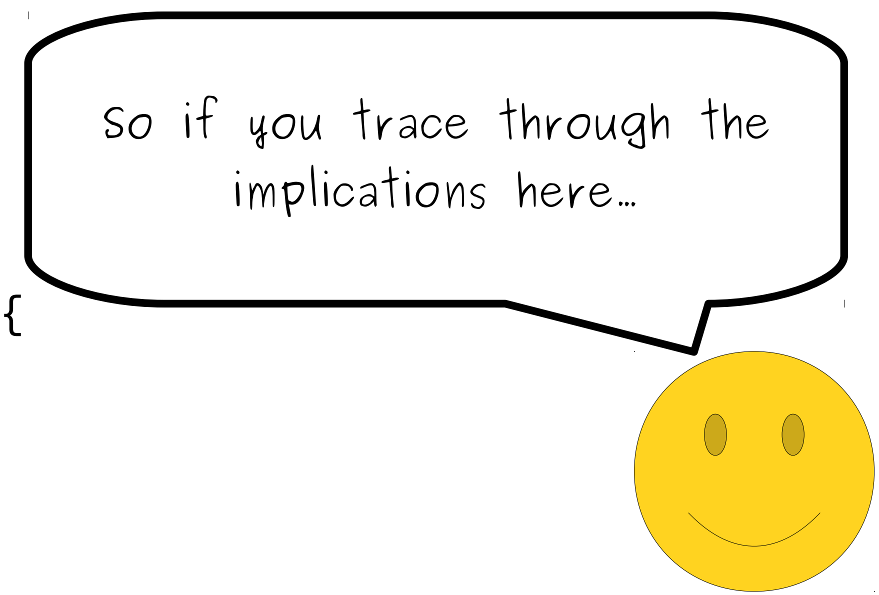

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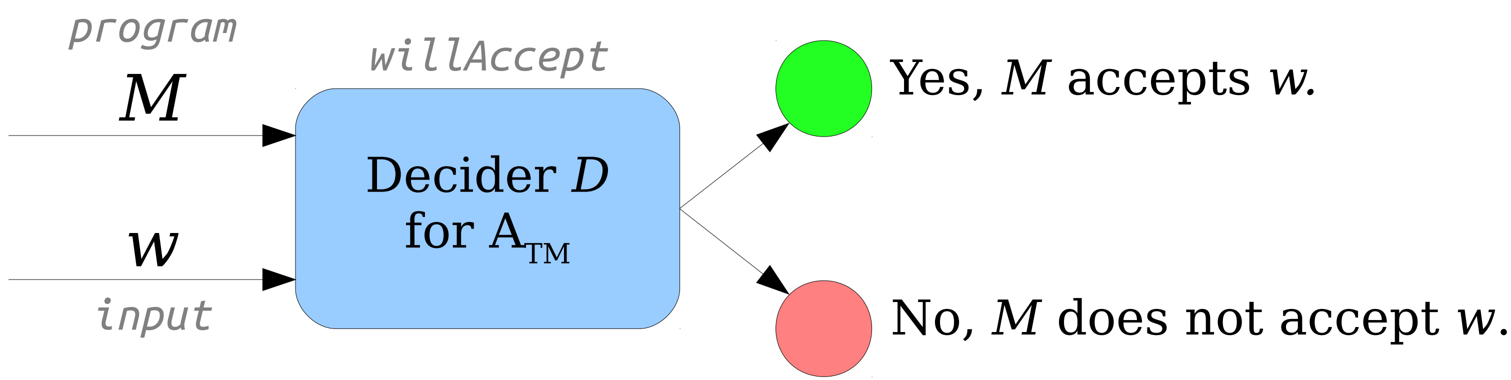


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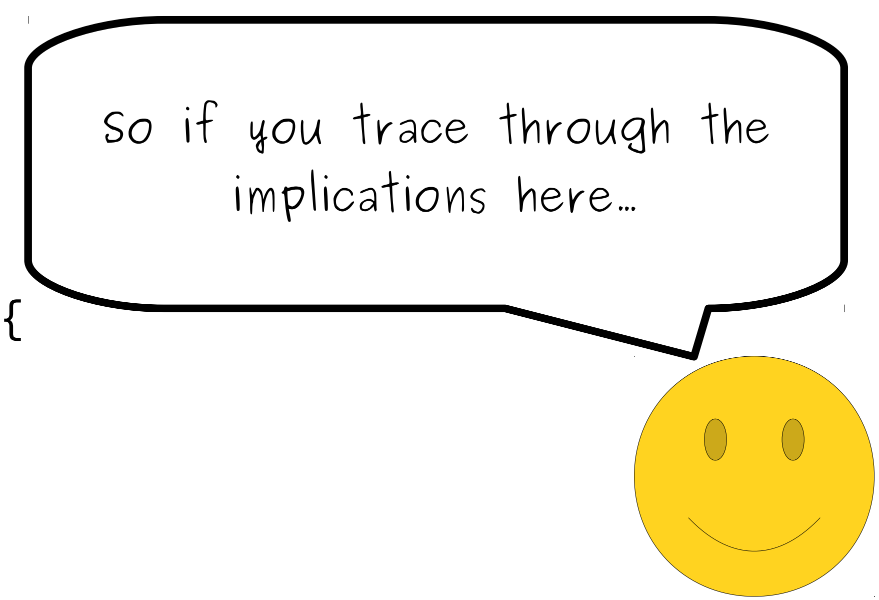

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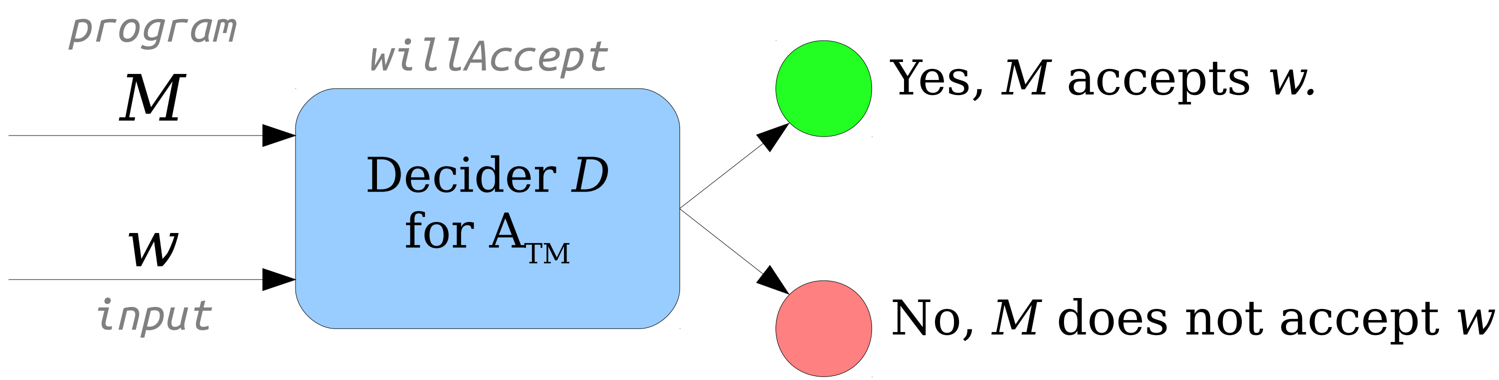


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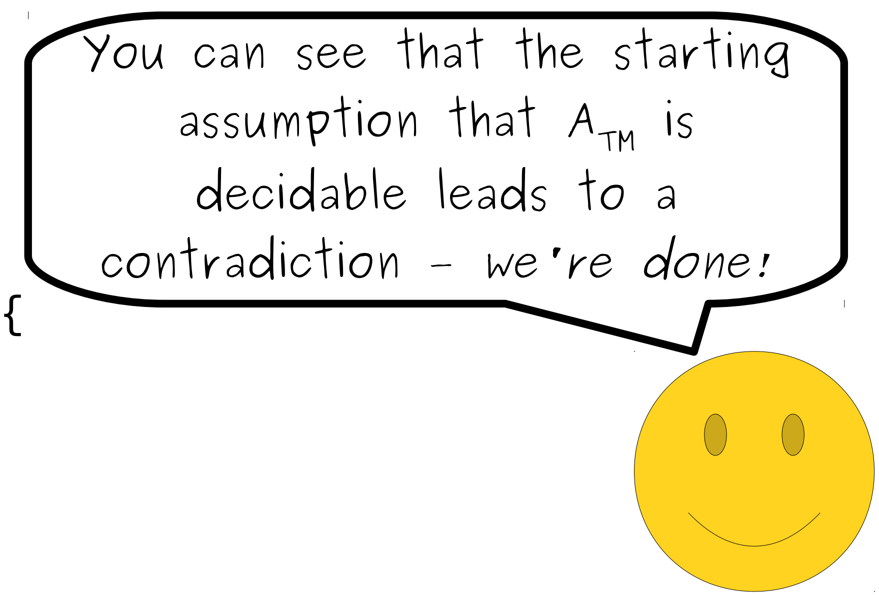

What does this program do?

```
bool willAccept(string program, string input) {
   /* … some implementation … */
}

int main() {
   string me = mySource();
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   if (willAccept(me, input)) {
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   }
}
```

Try running this program on any input.
What happens if

… this program accepts its input?
It rejects the input!

… this program doesn't accept its input?
It accepts the input!

What does this program do?

```
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```

Take a look at it more closely.

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        accept();
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```

Recognize this code? Now you know where it comes from!

Try running this program on any input. What happens if

… this program accepts its input?
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We created it to get these contradictions.

Try running this program on any input. What happens if

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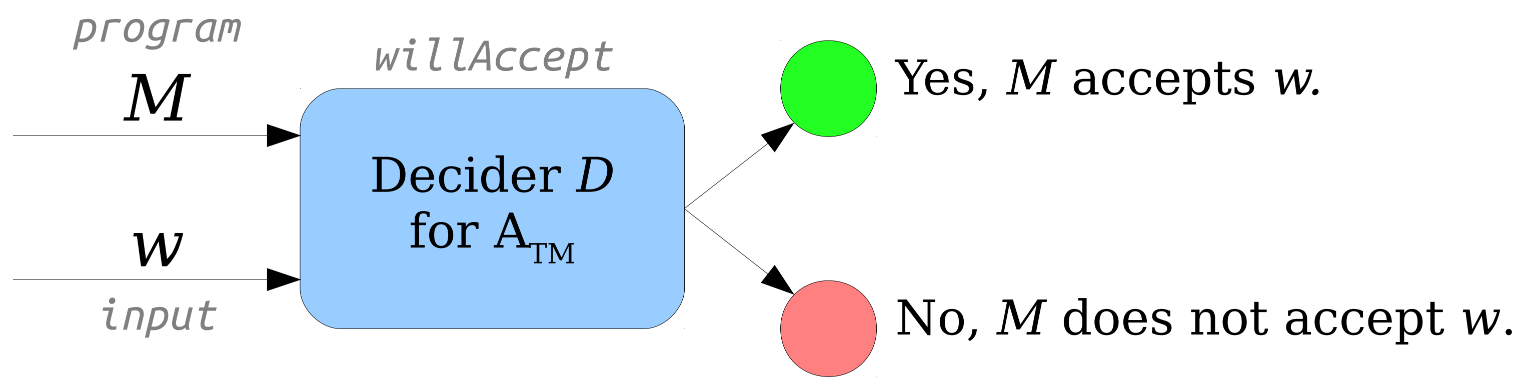


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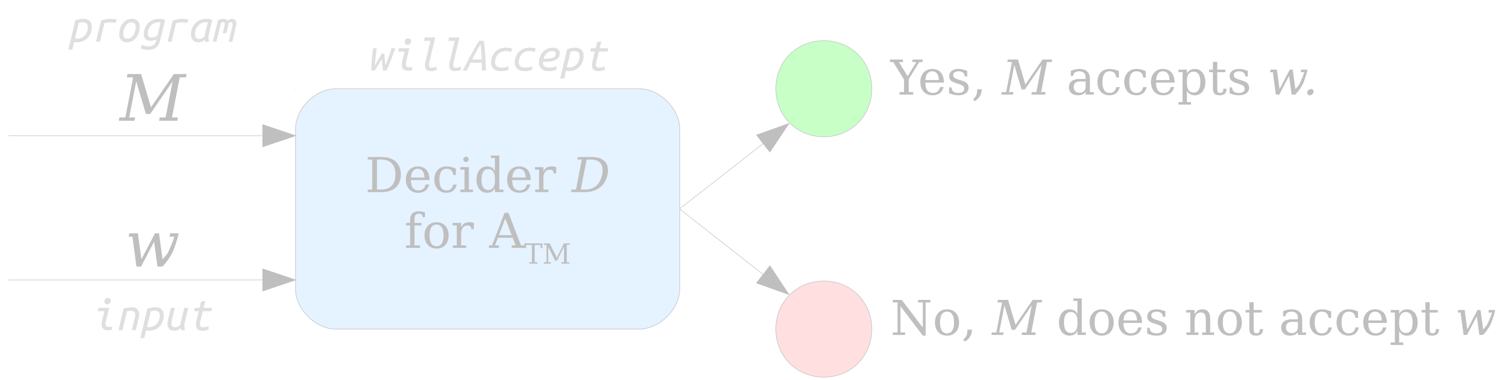


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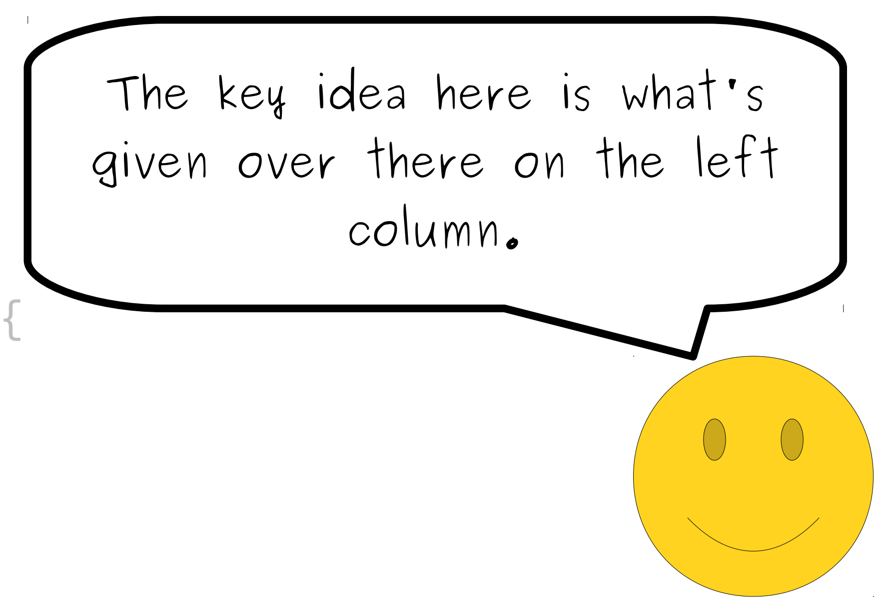

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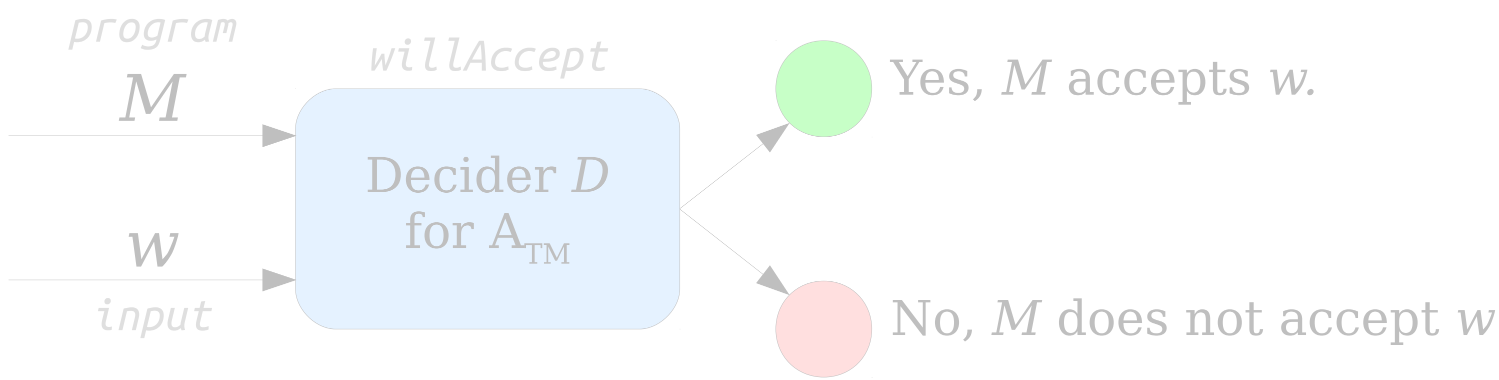


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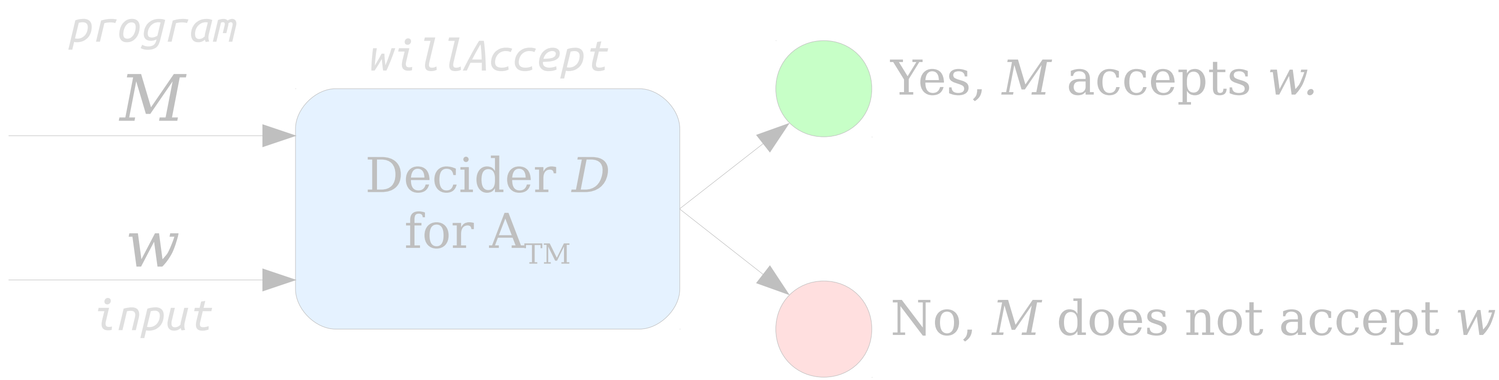


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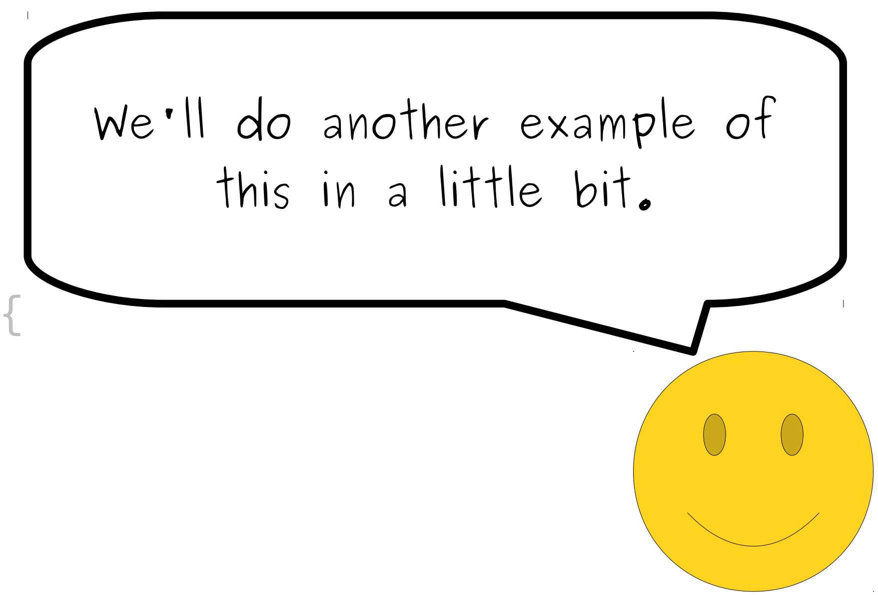

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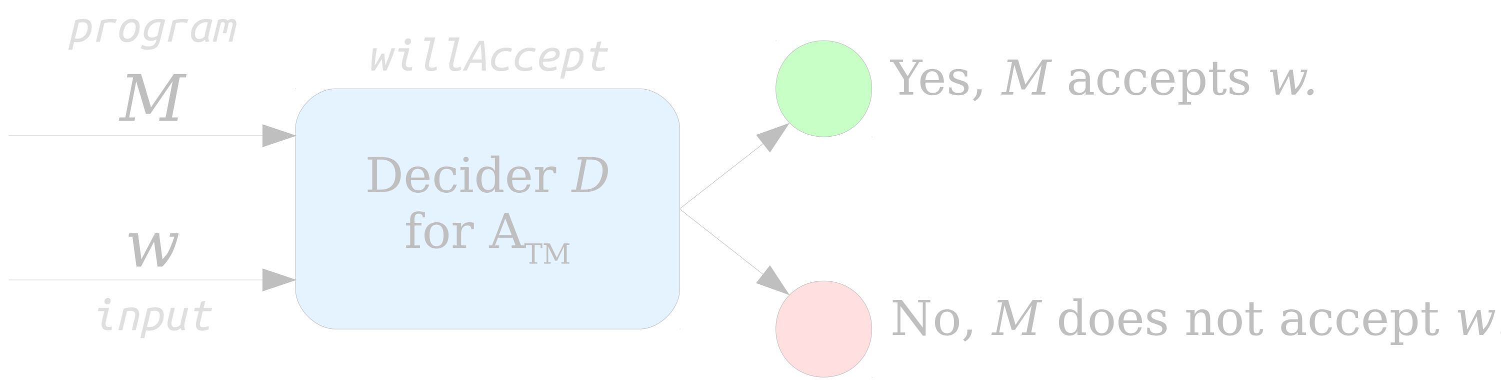


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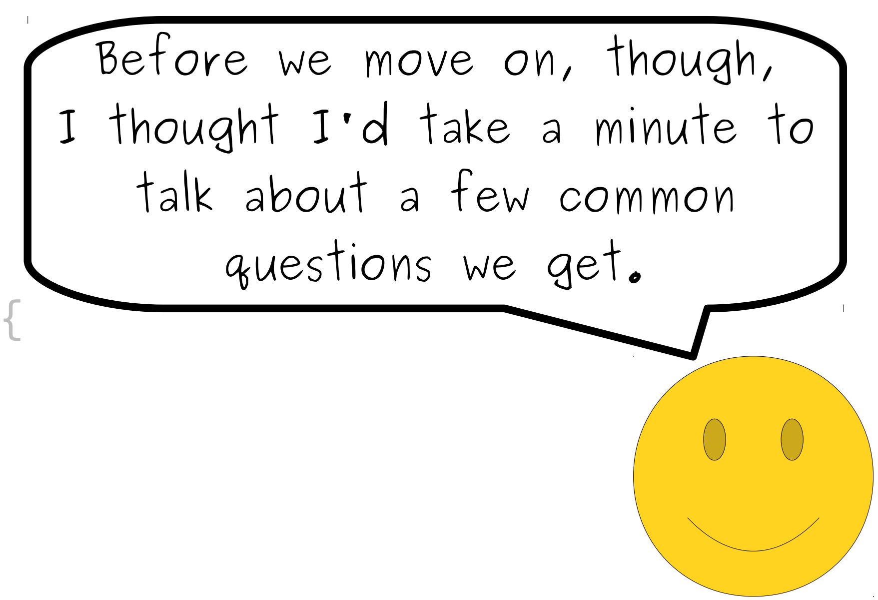

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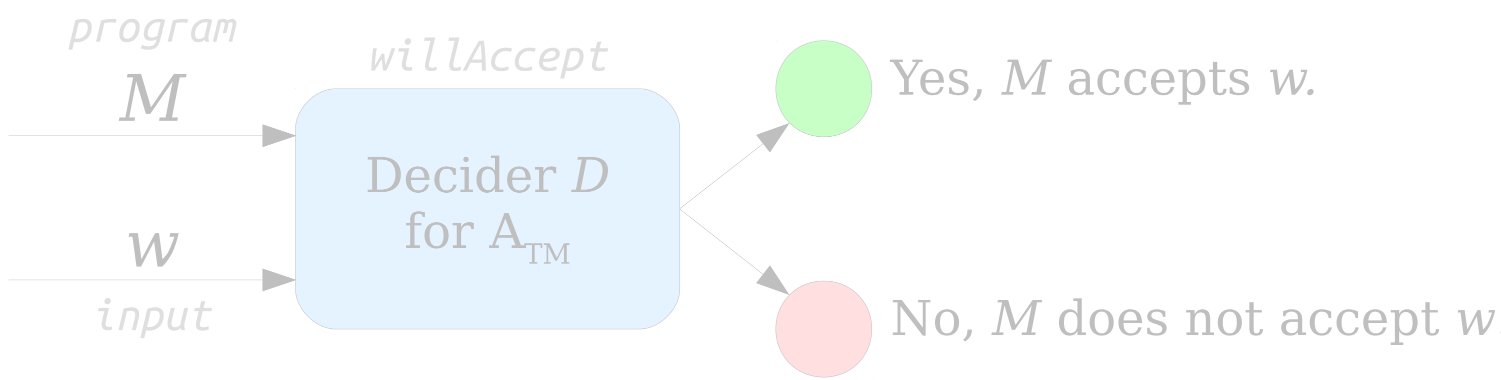


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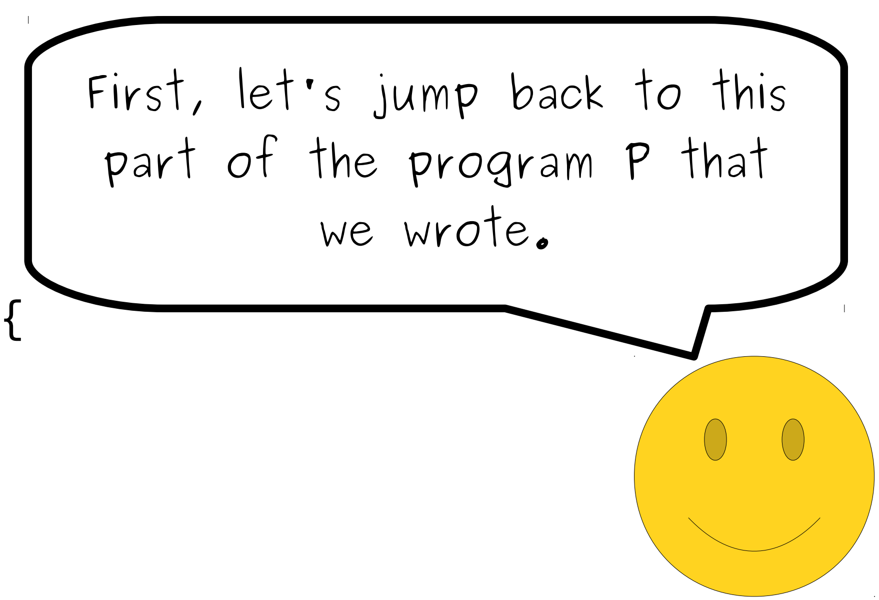

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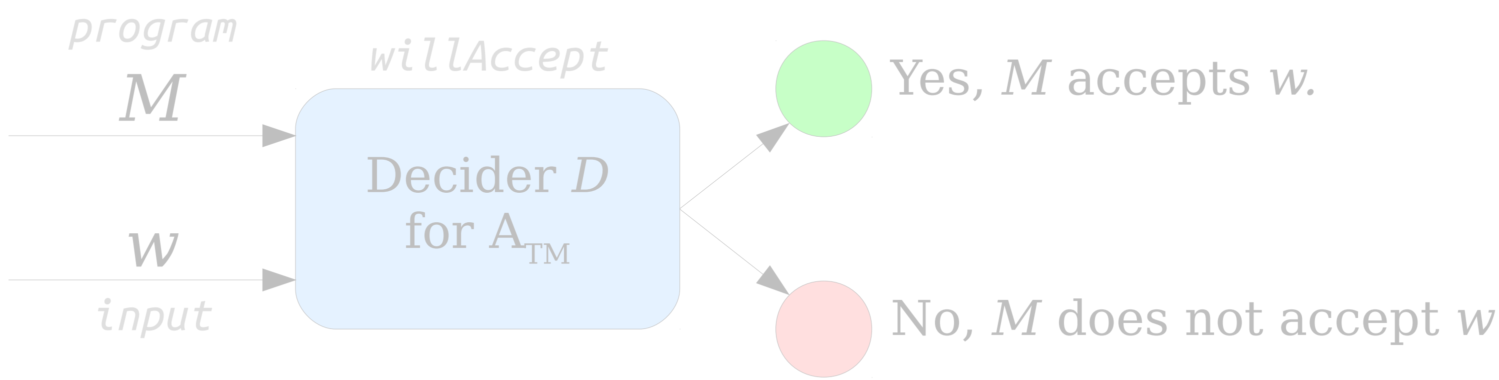


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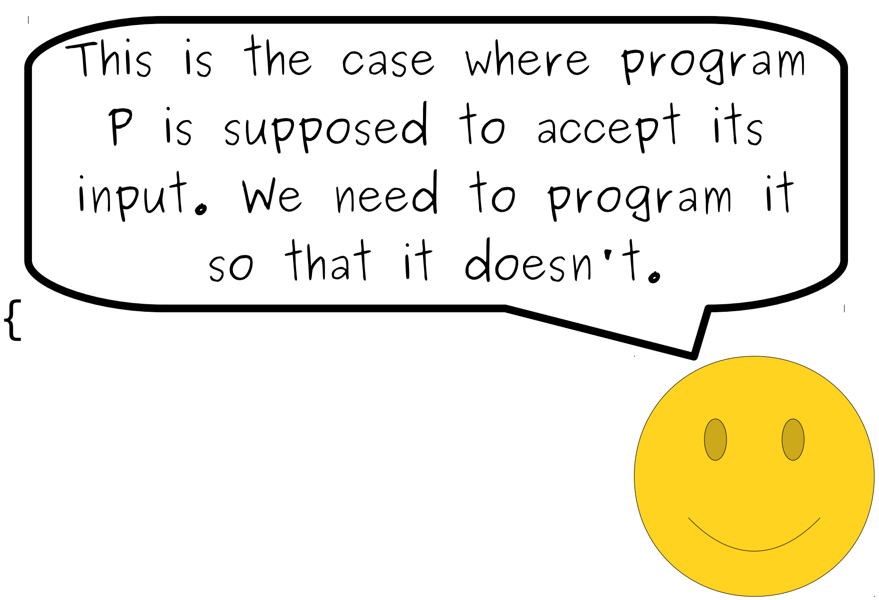

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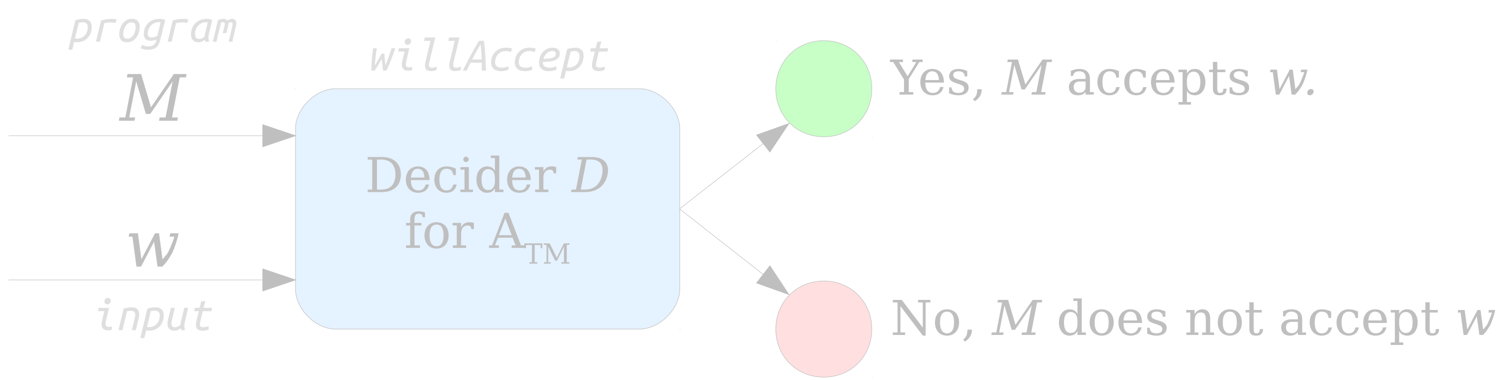


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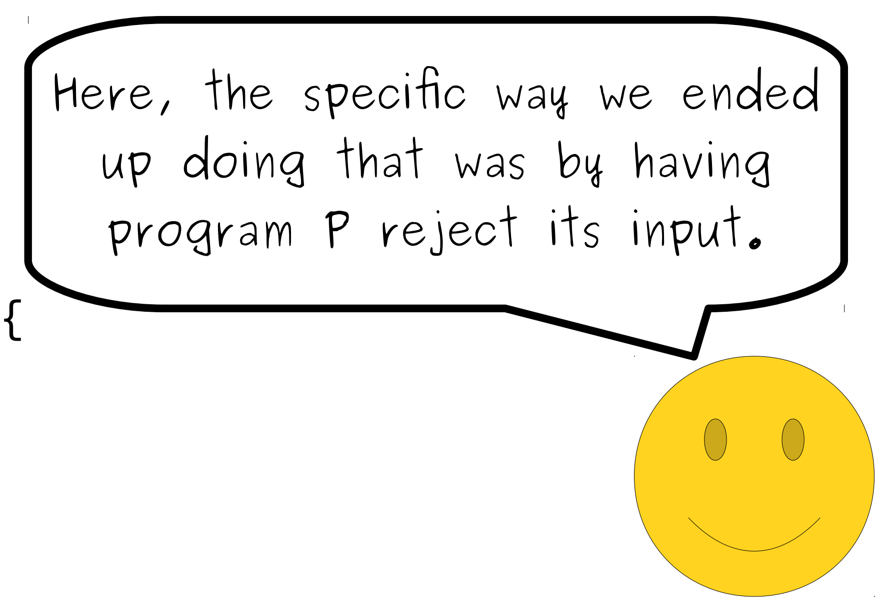

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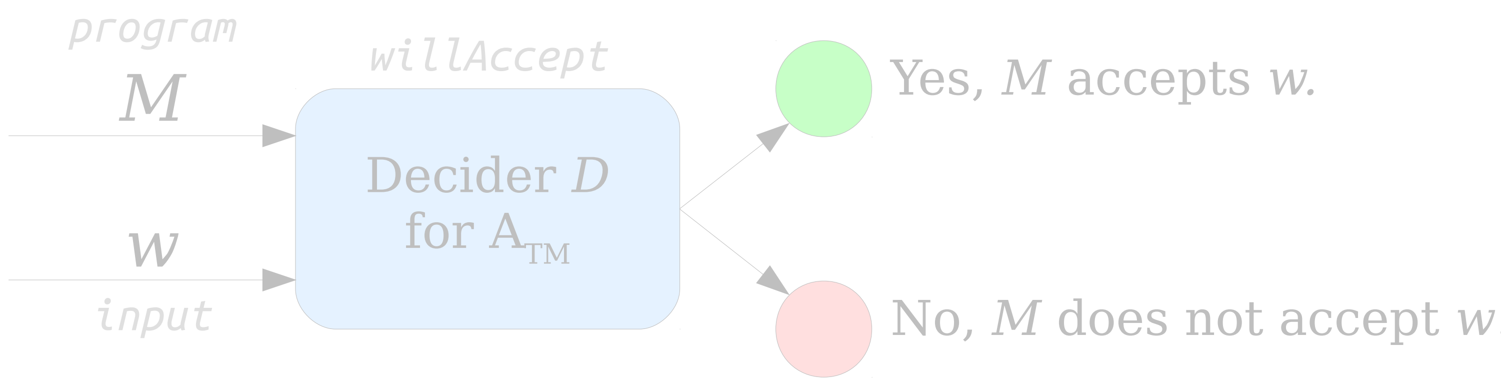


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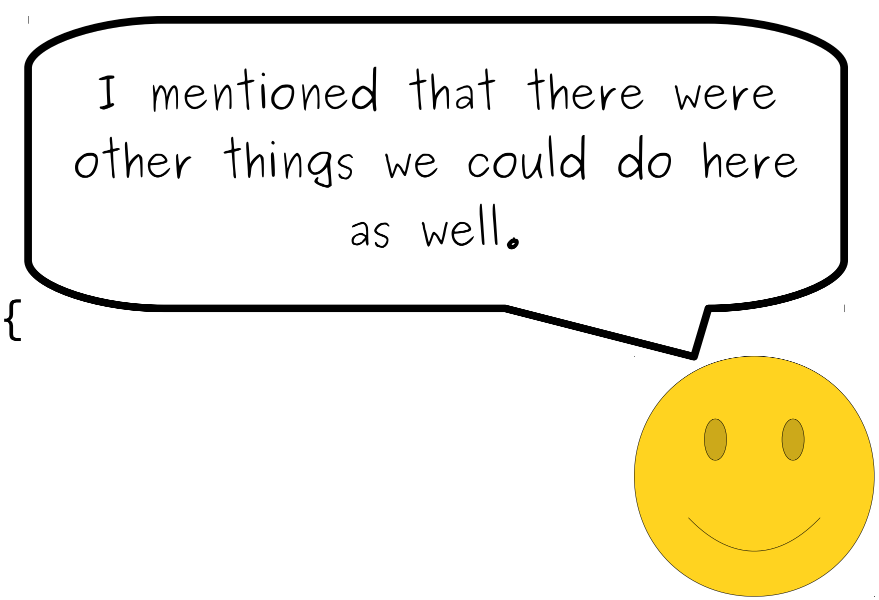

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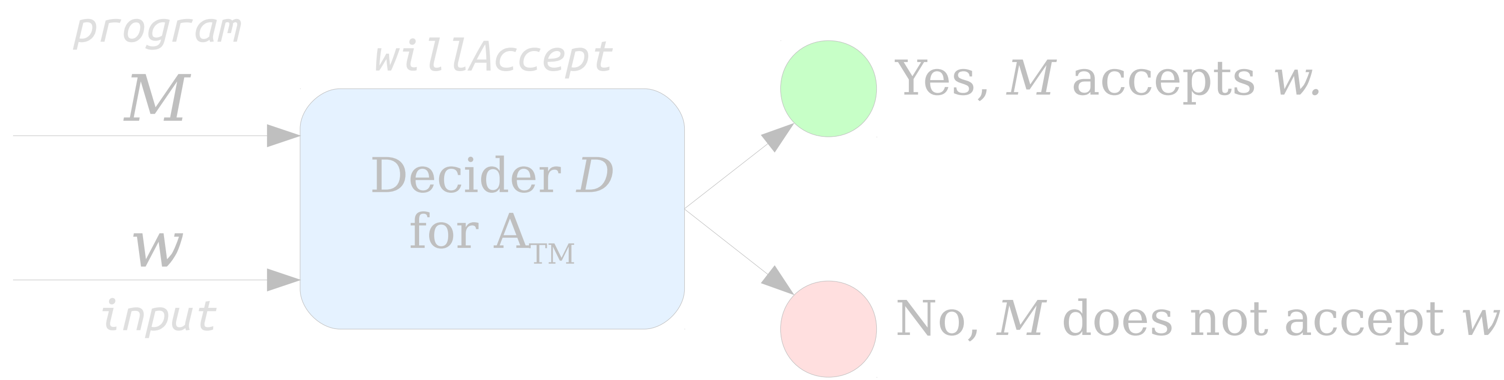


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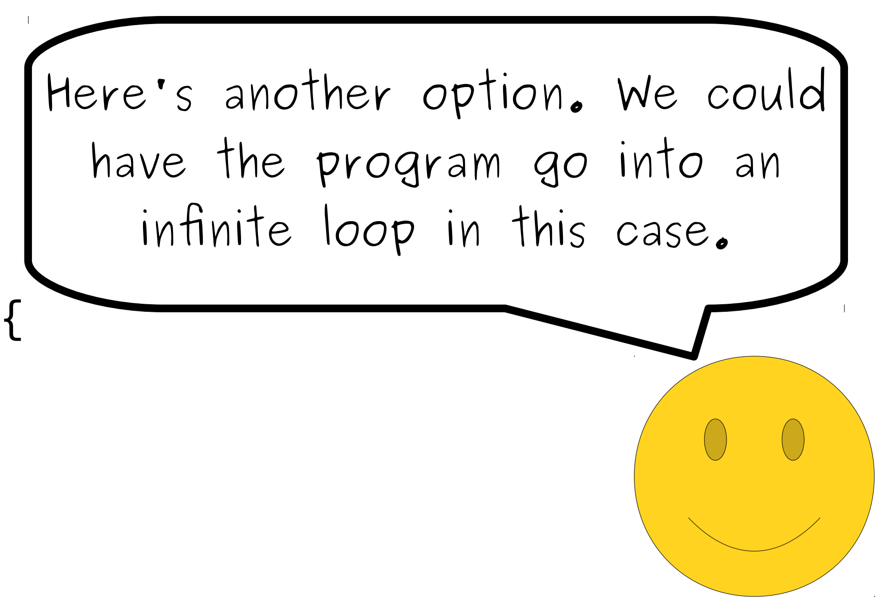

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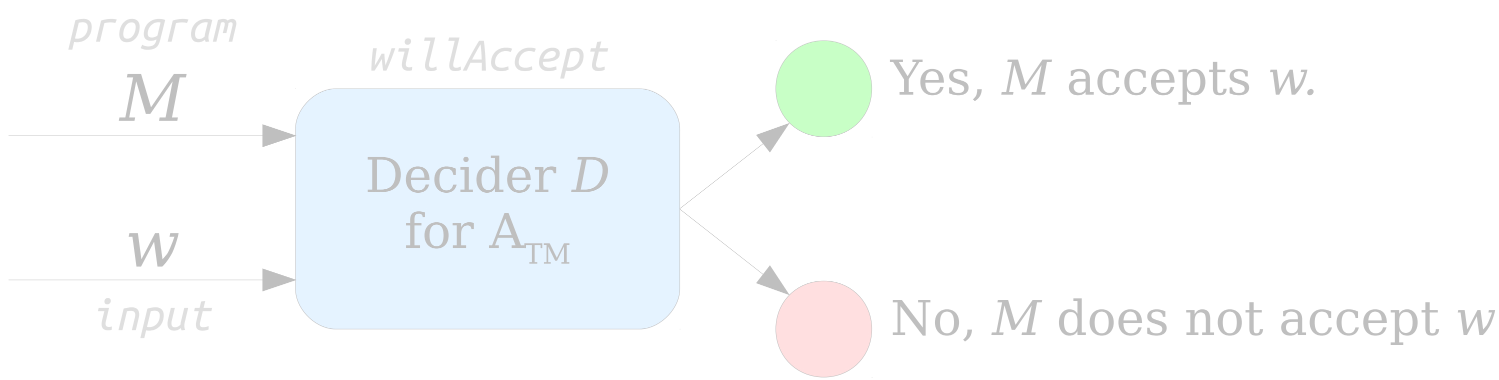


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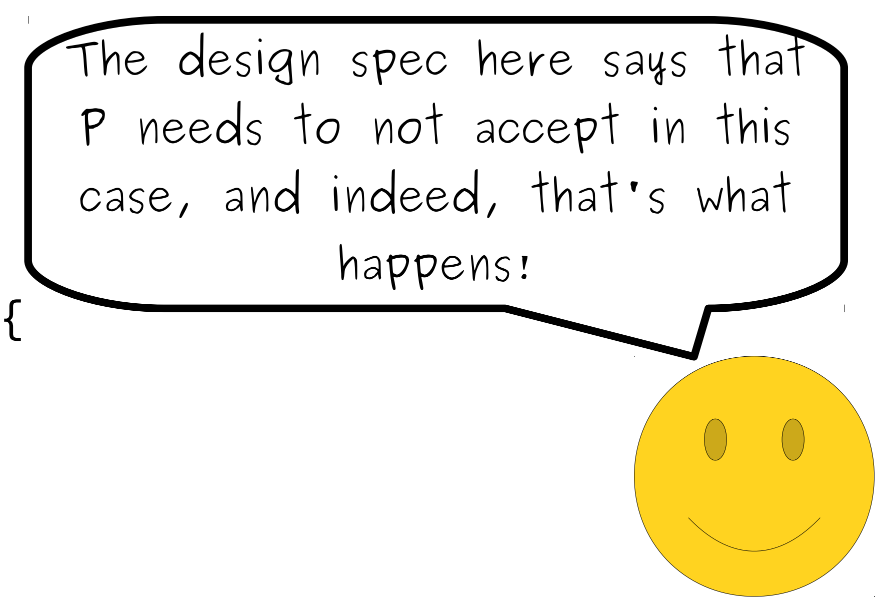

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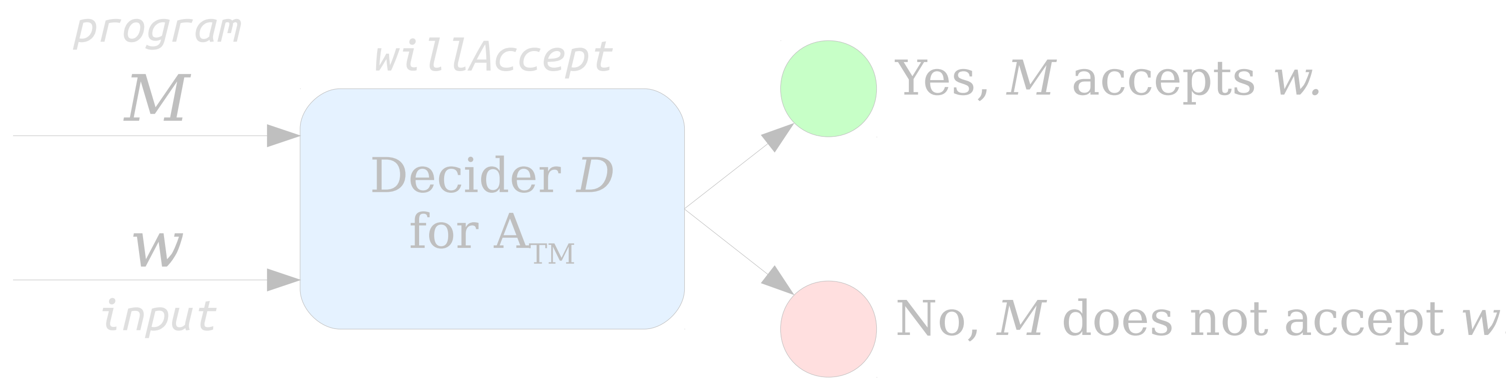


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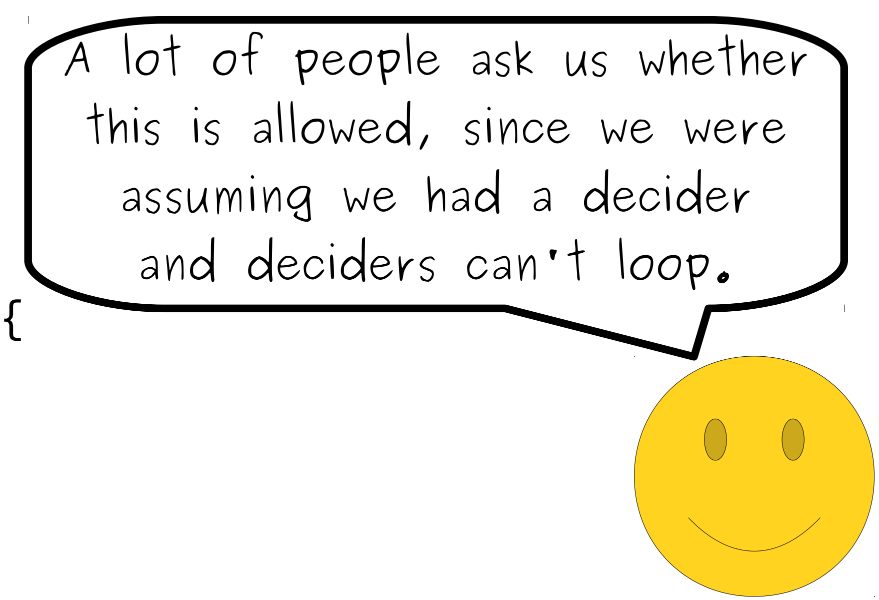

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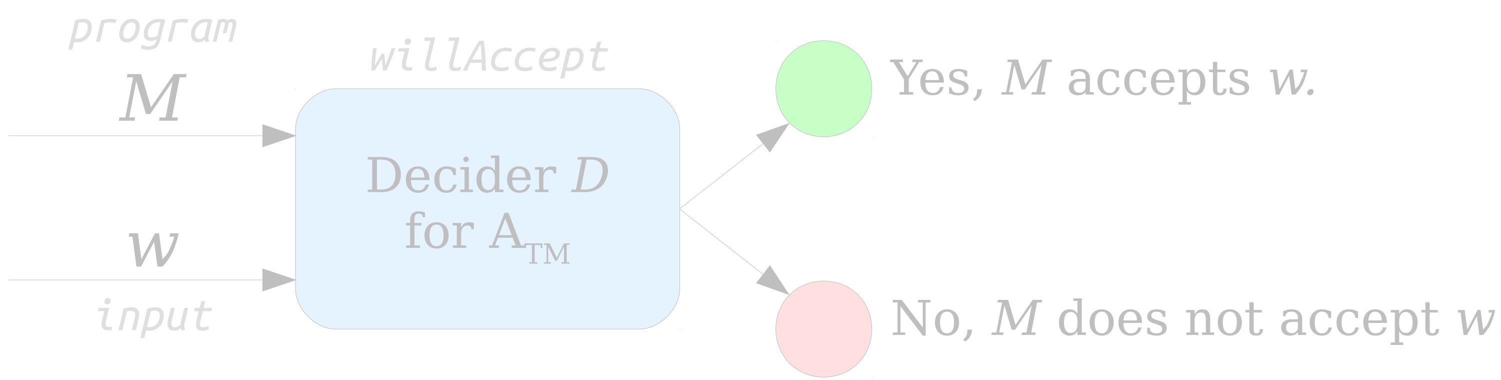


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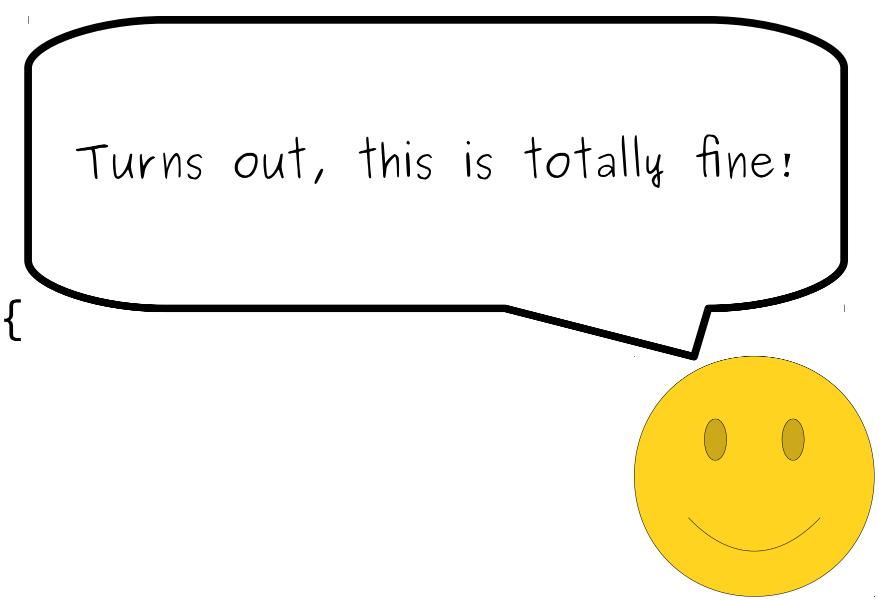

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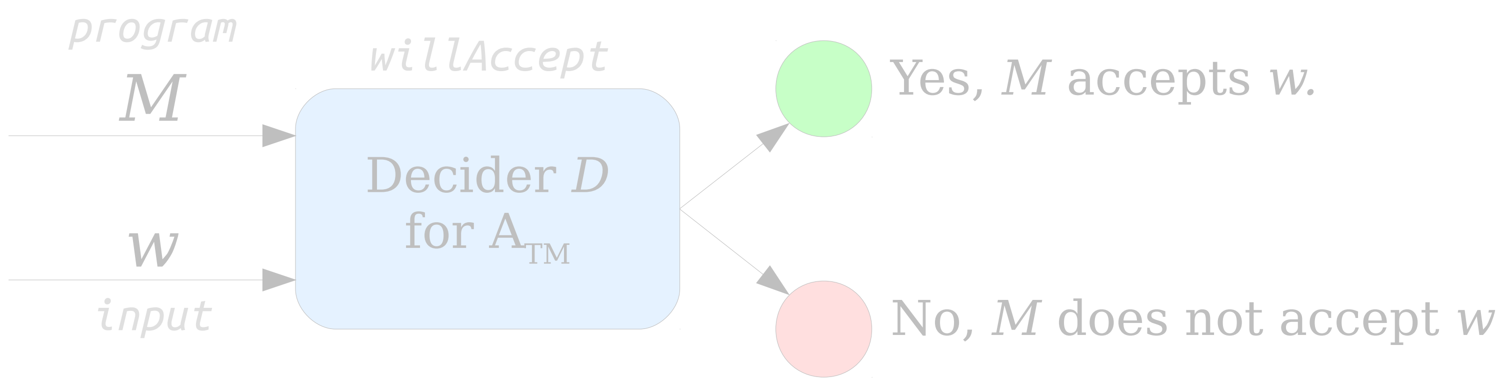


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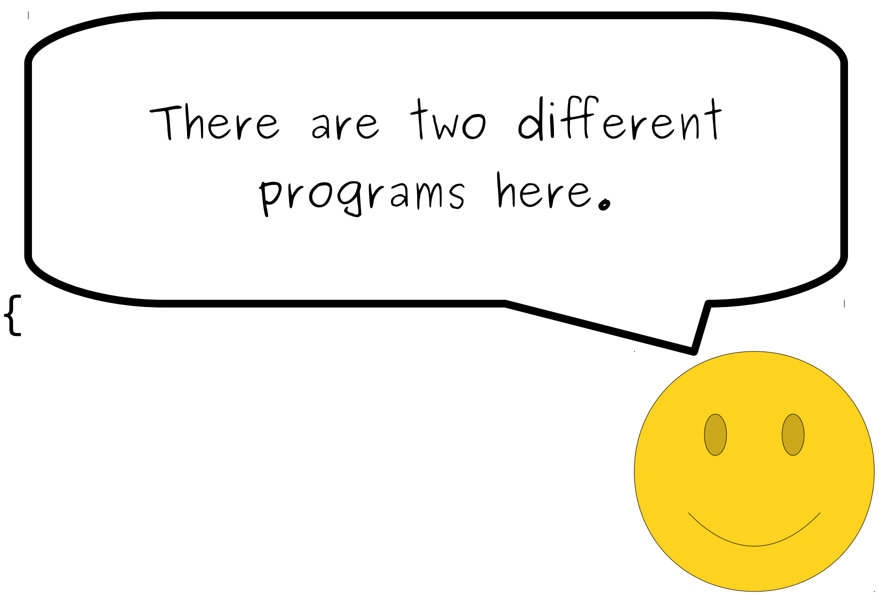

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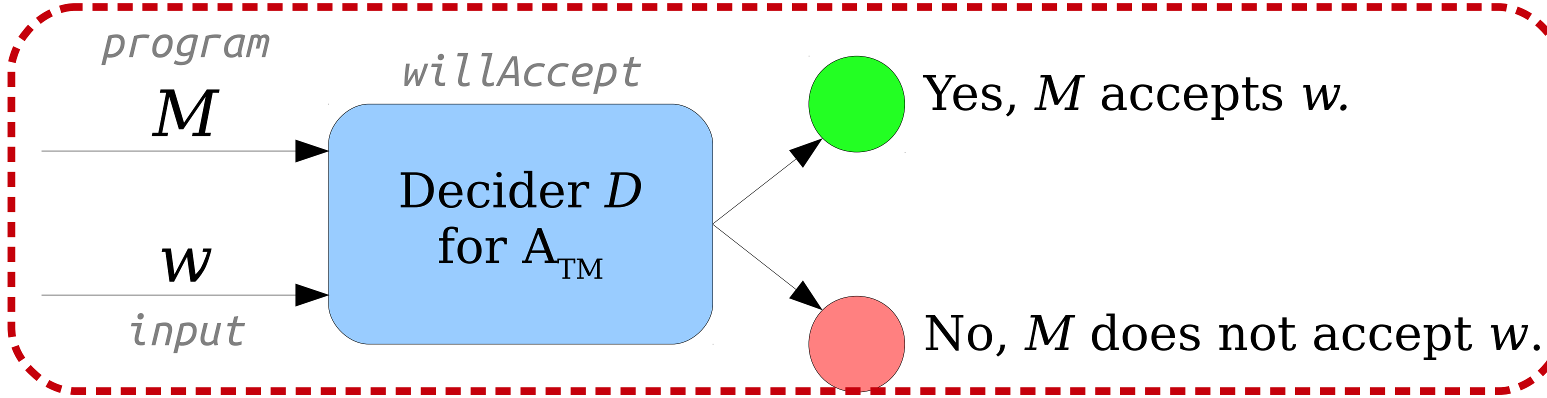


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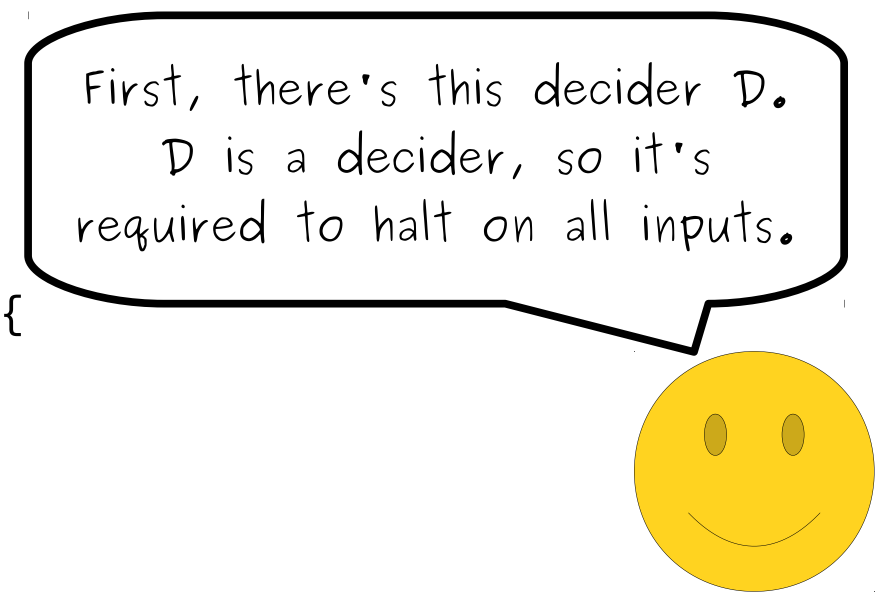

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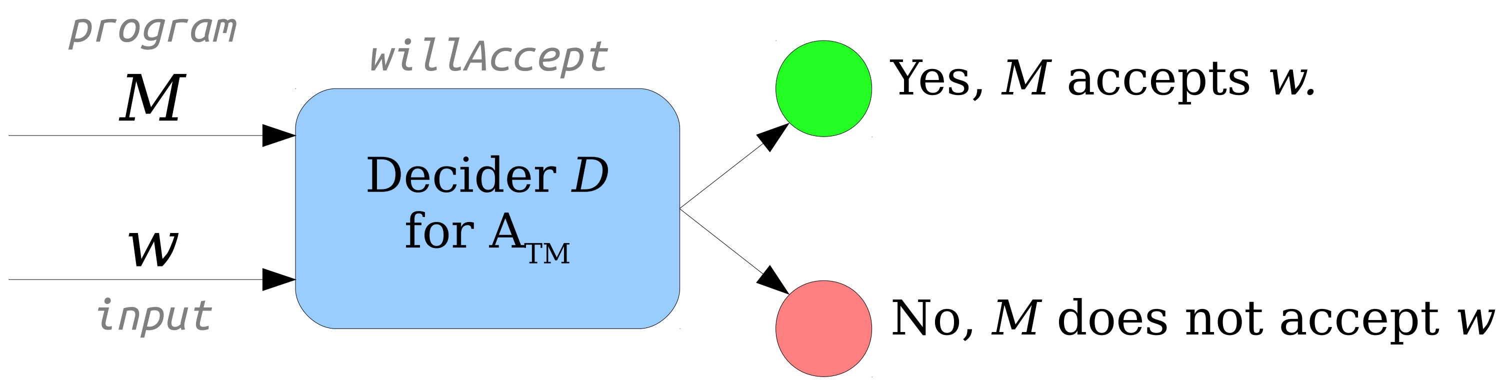


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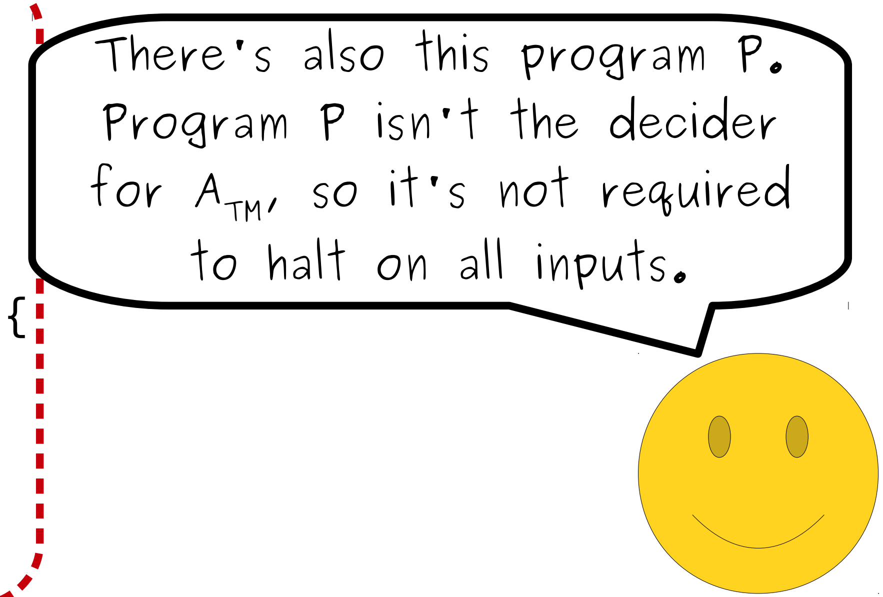

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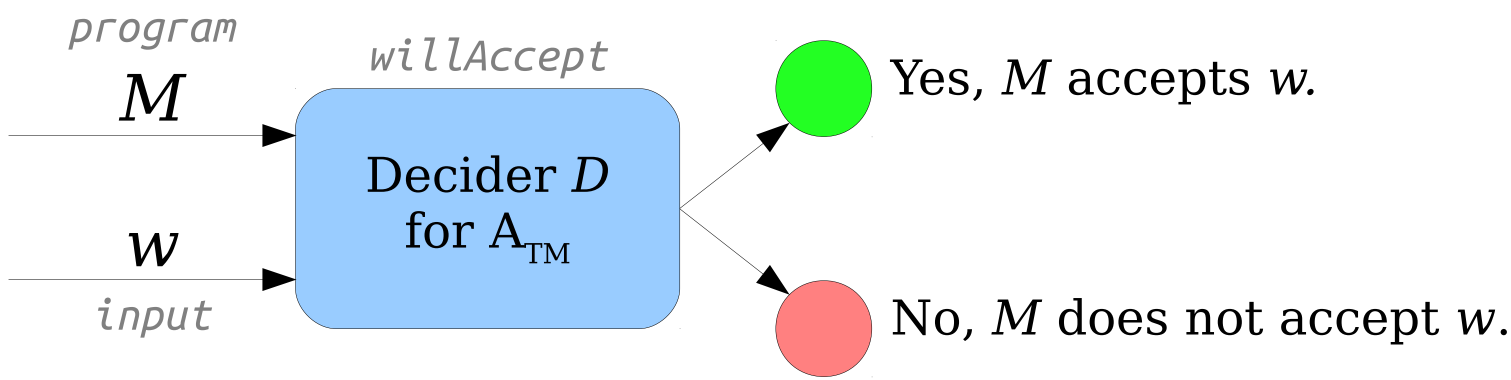


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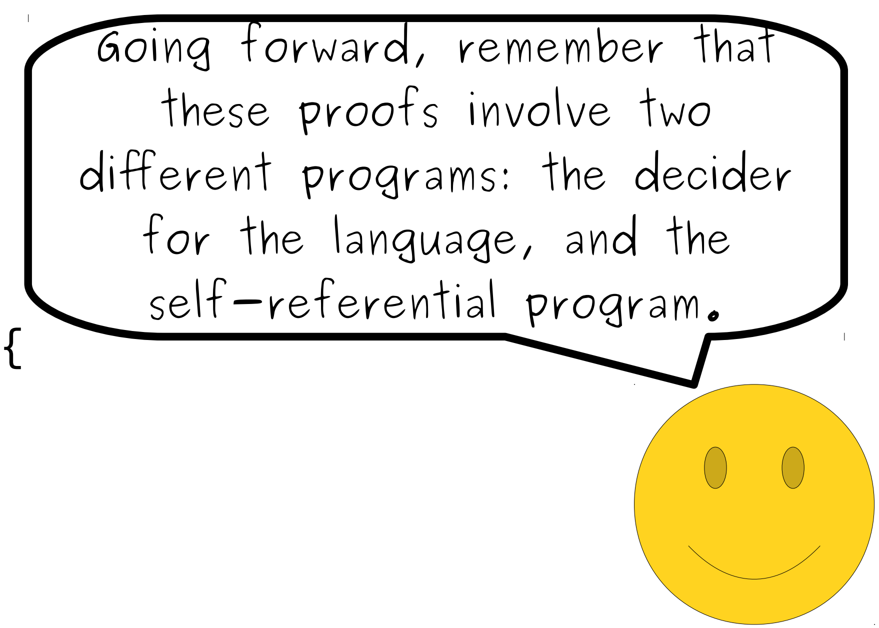

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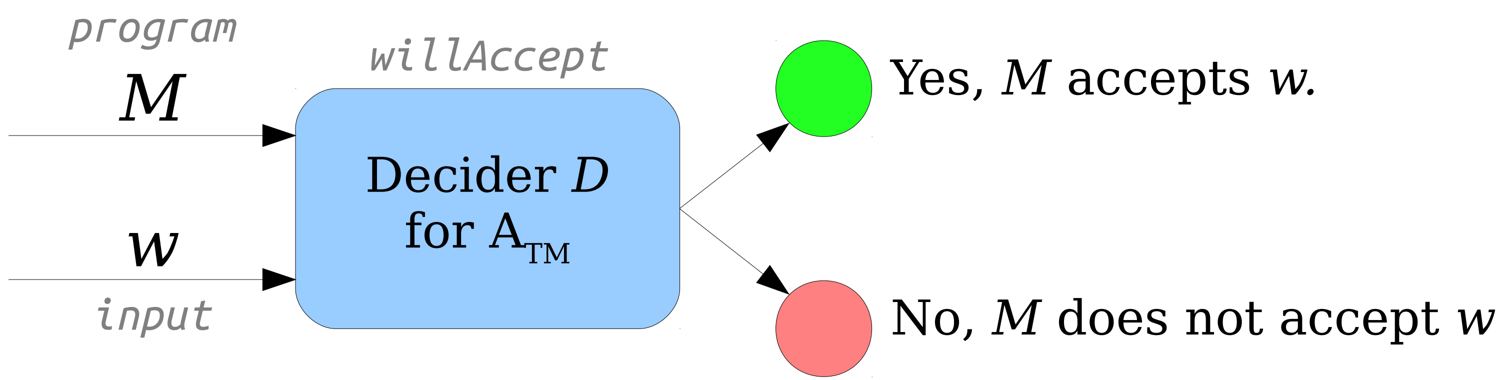


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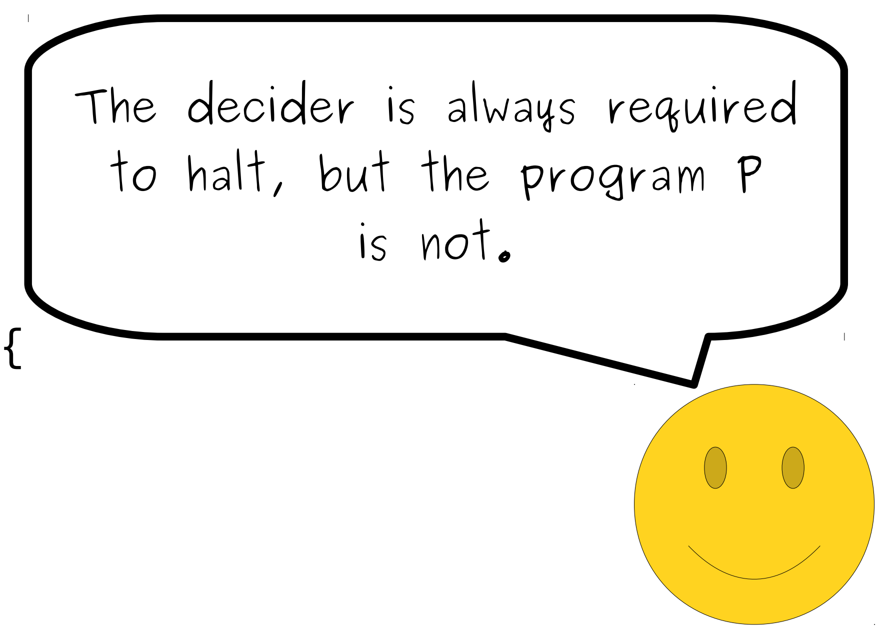

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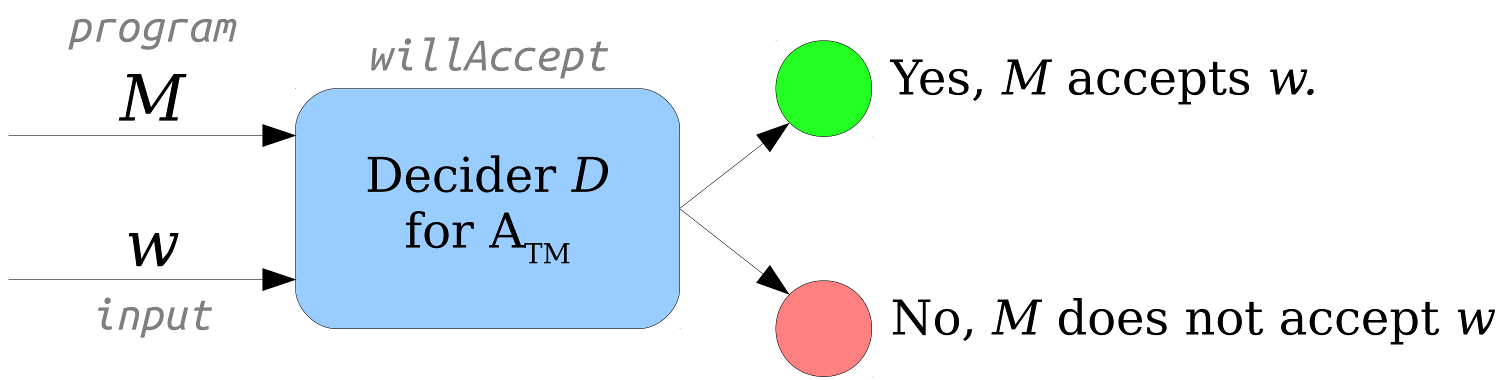


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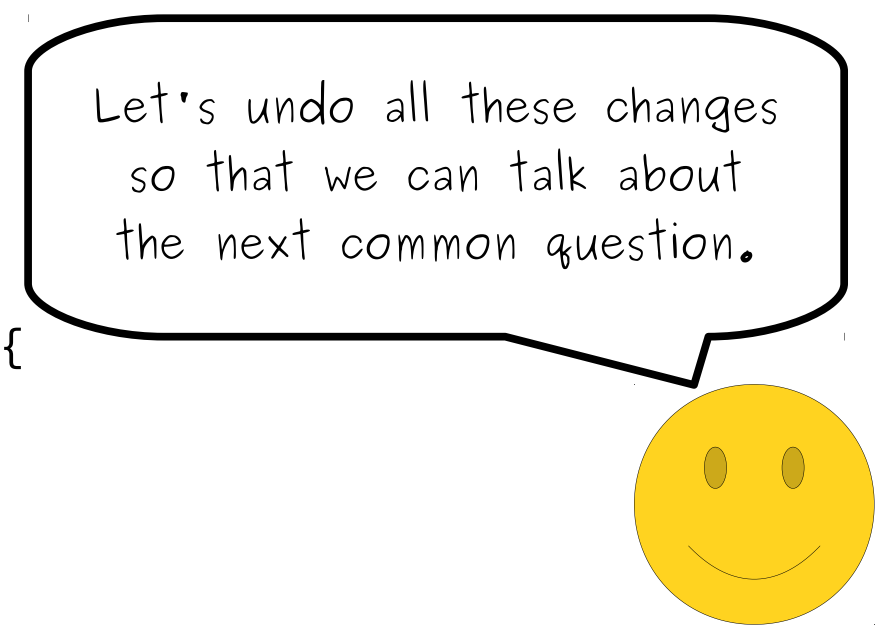

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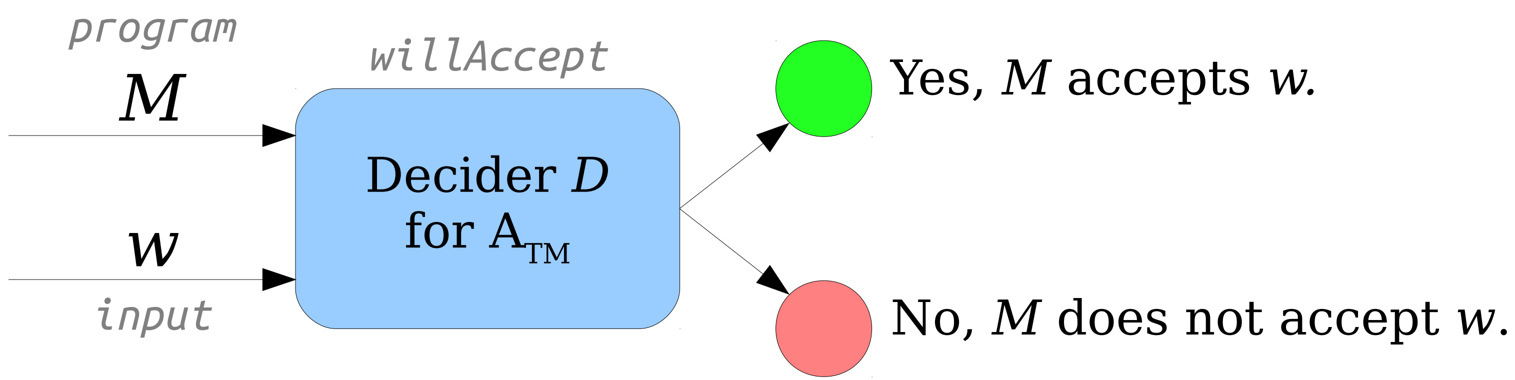


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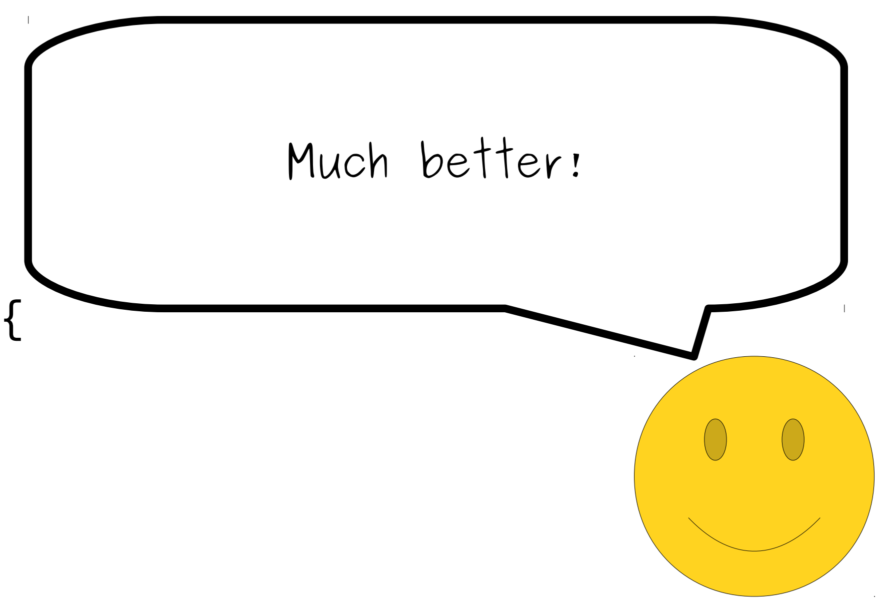

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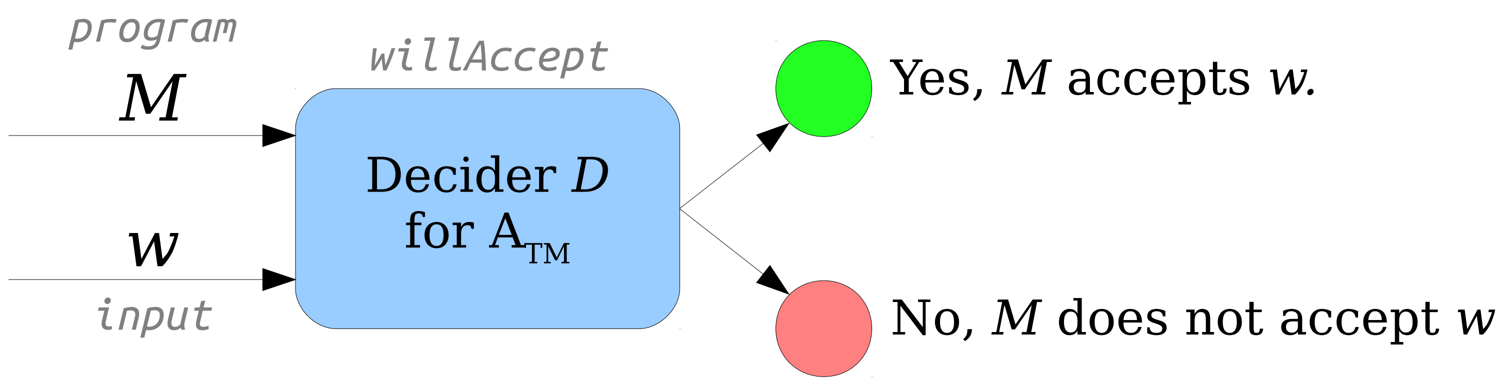


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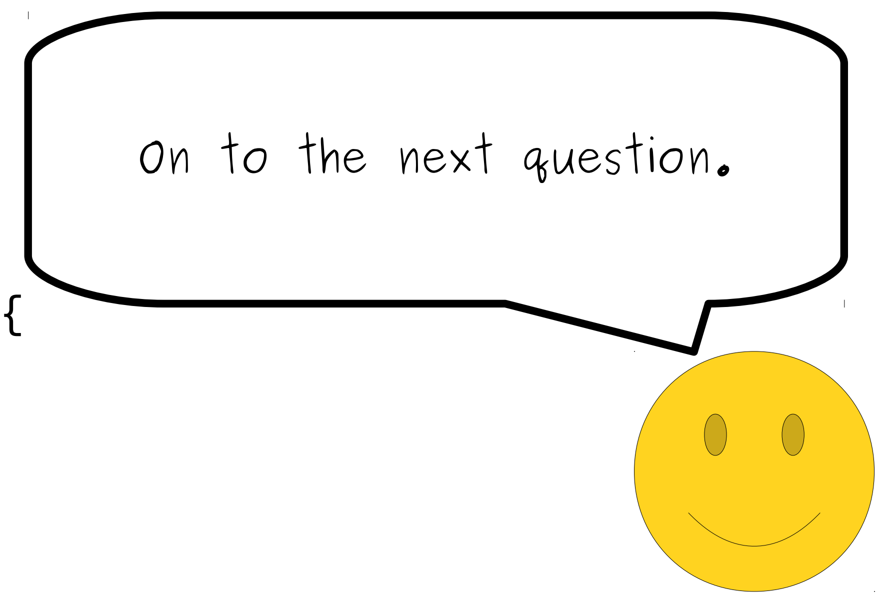

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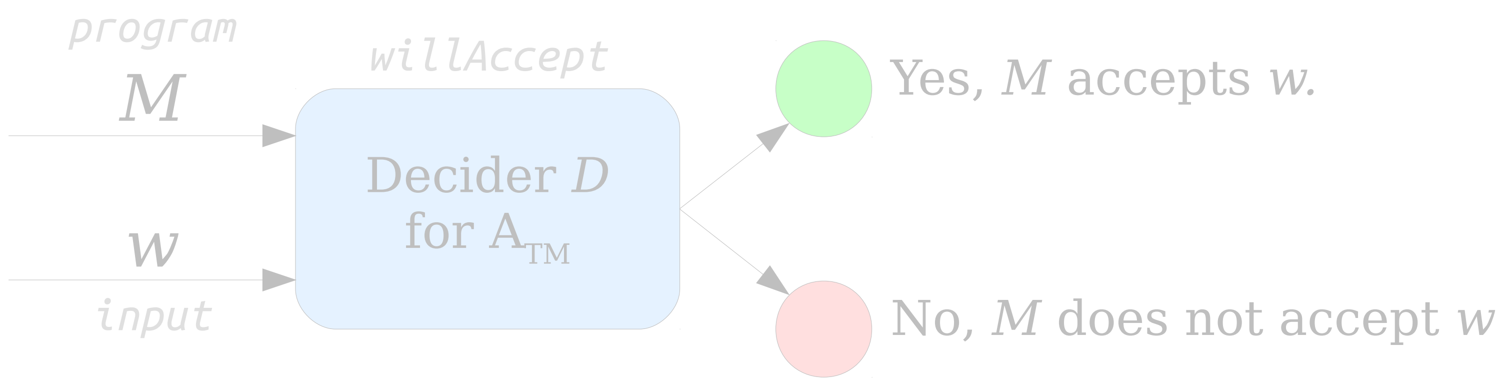


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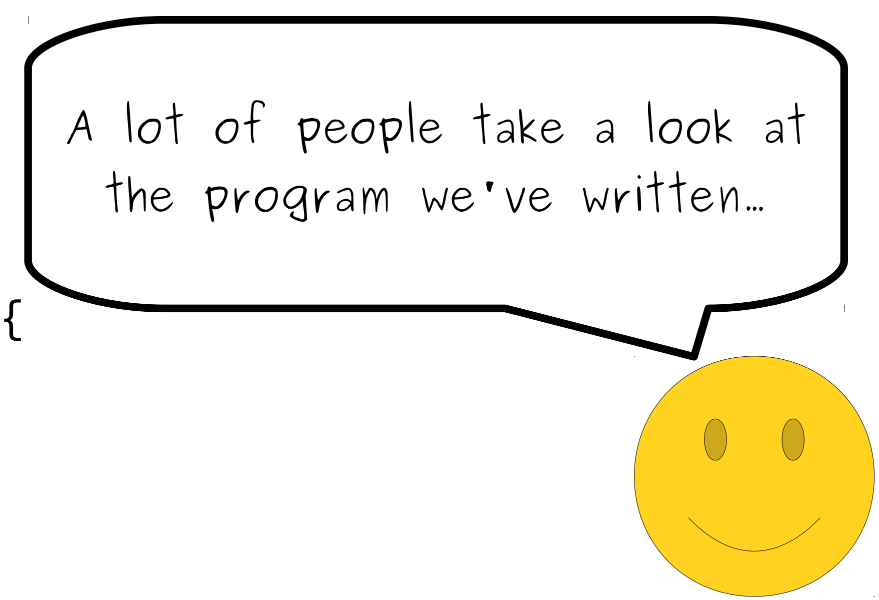

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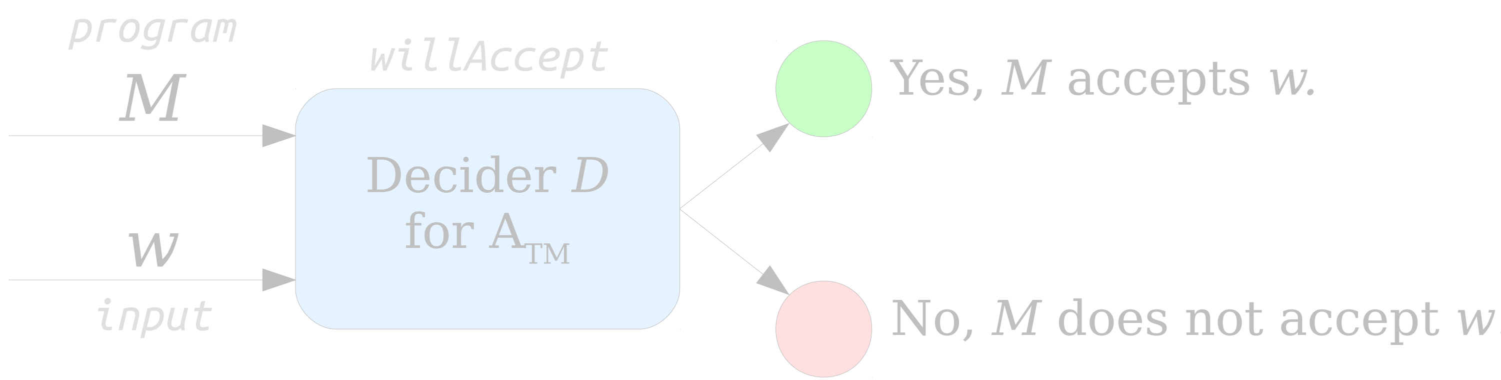


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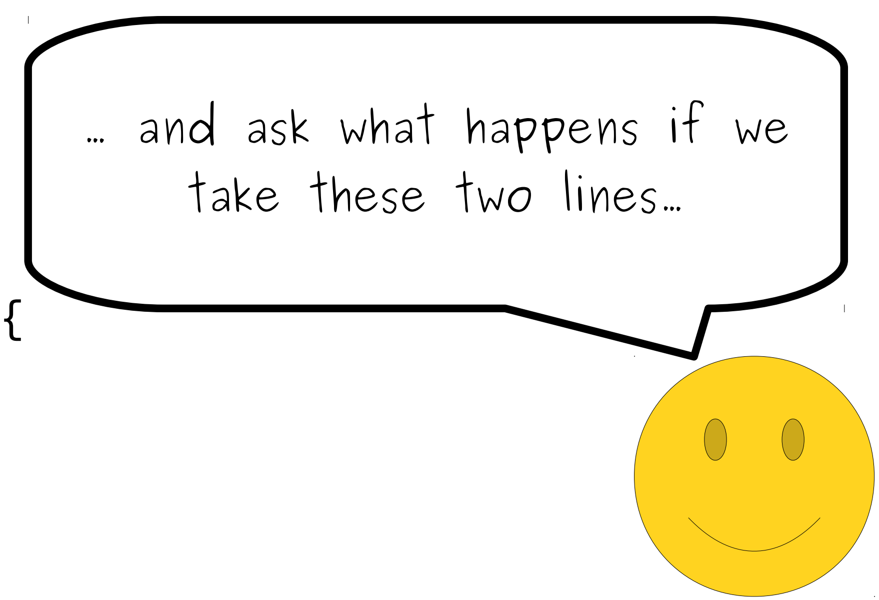

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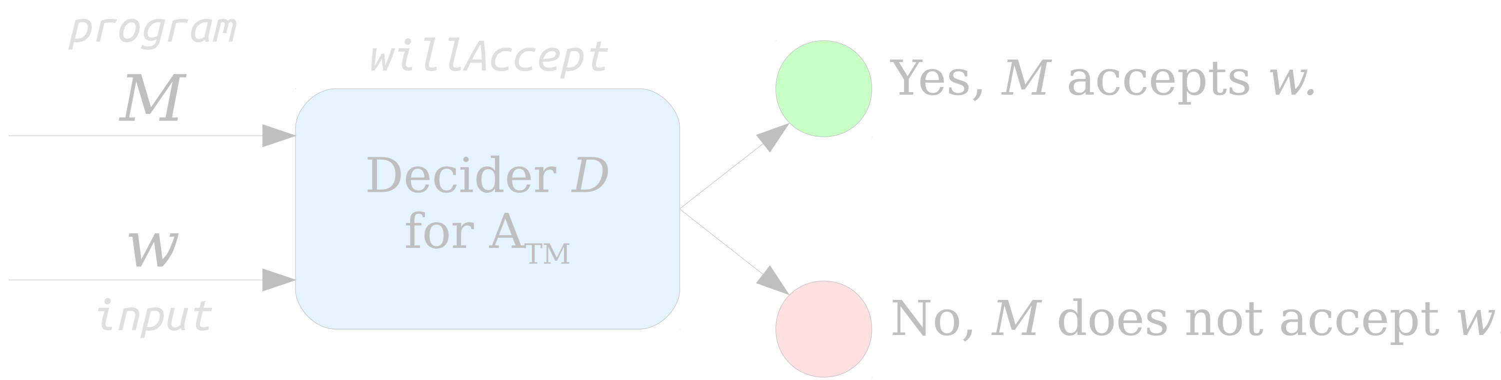


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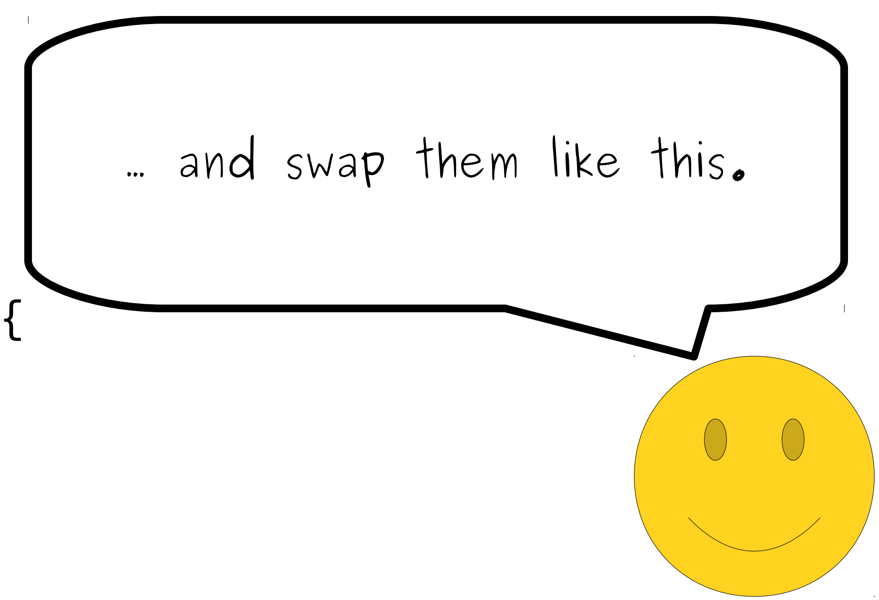

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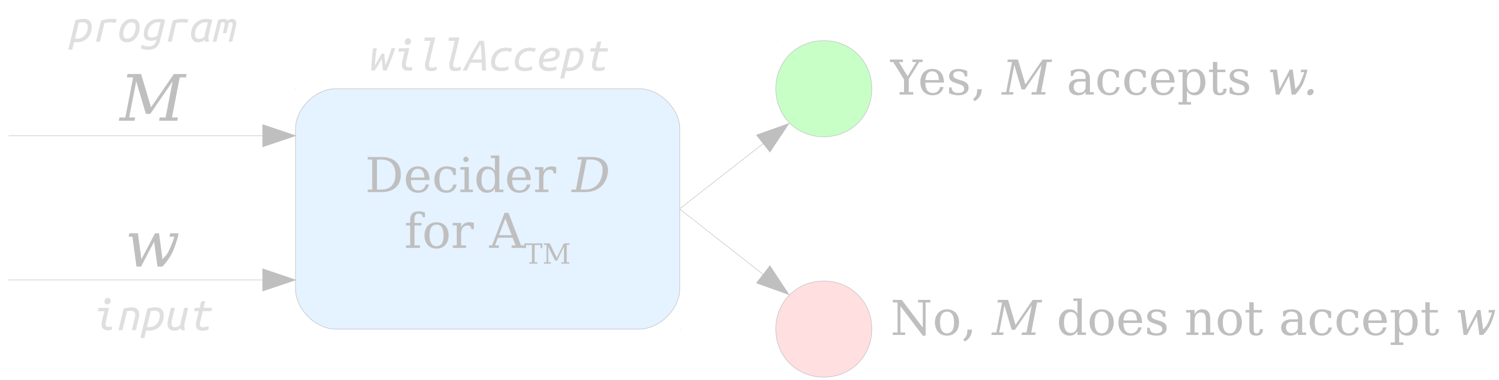


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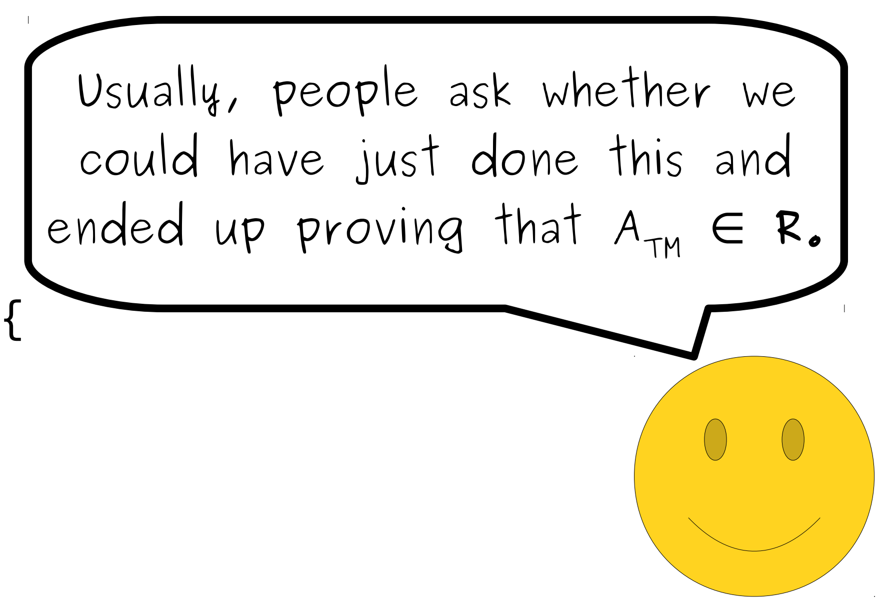

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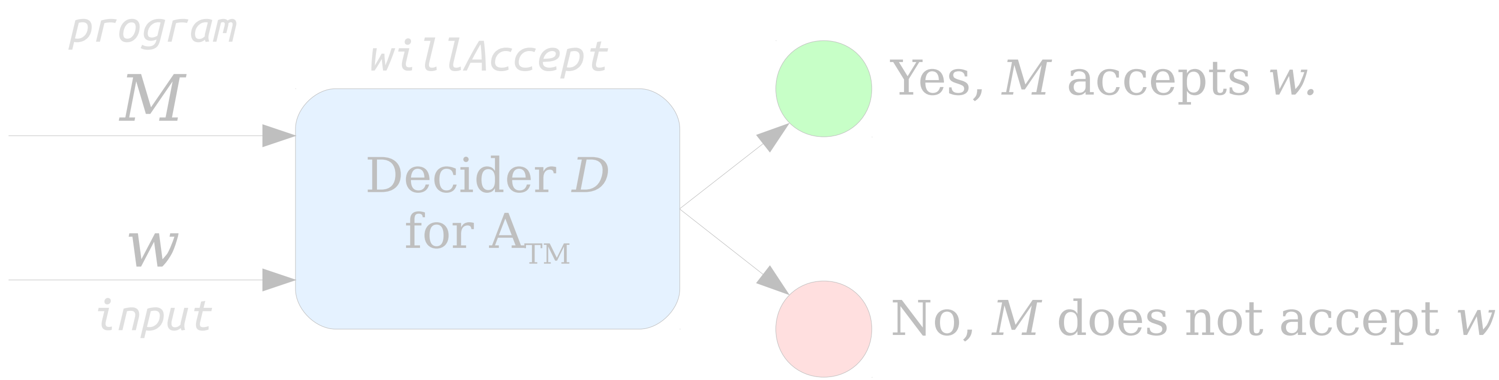
Contradiction!

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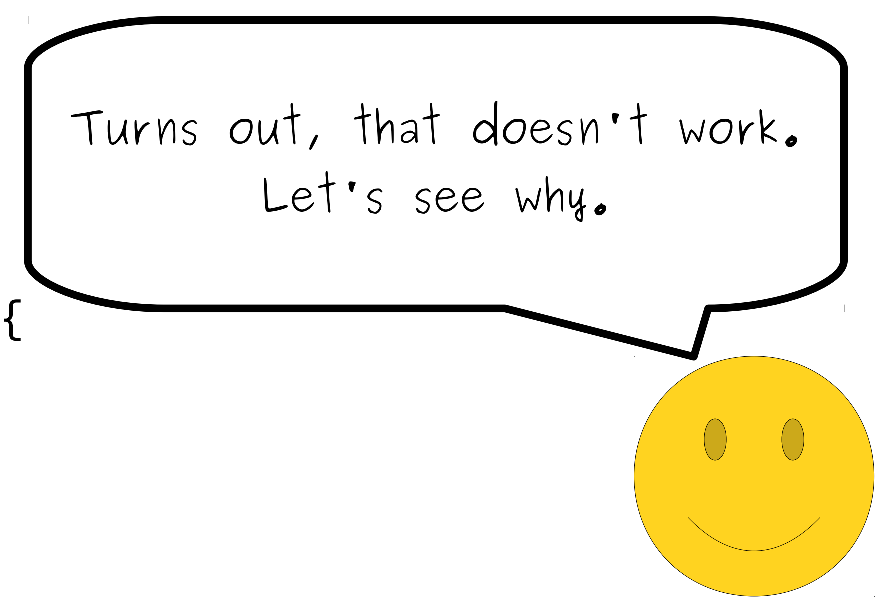

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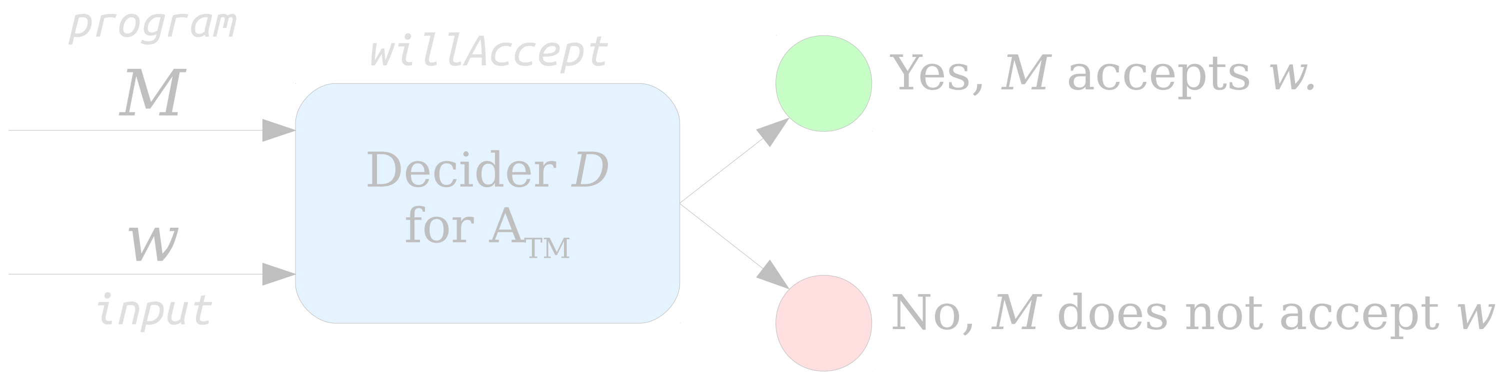


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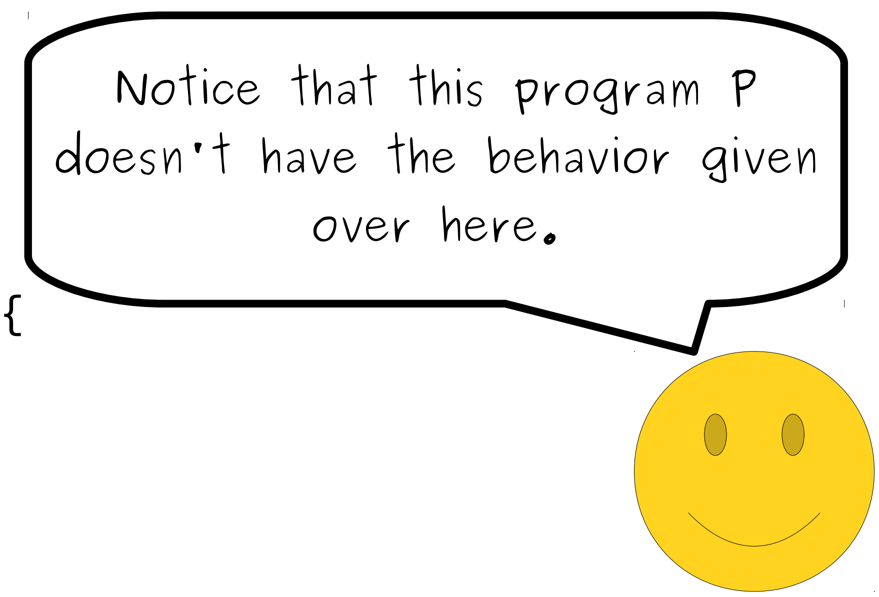

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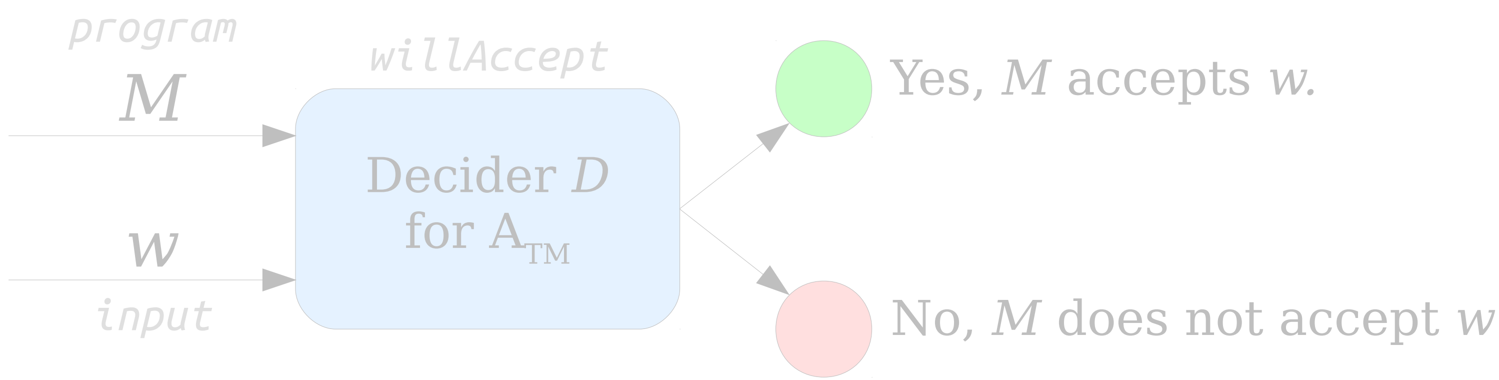


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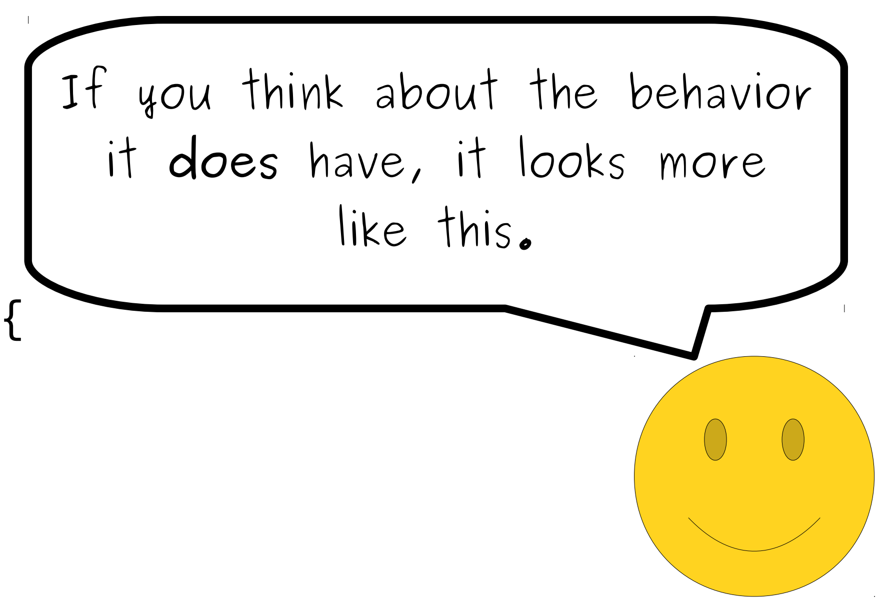

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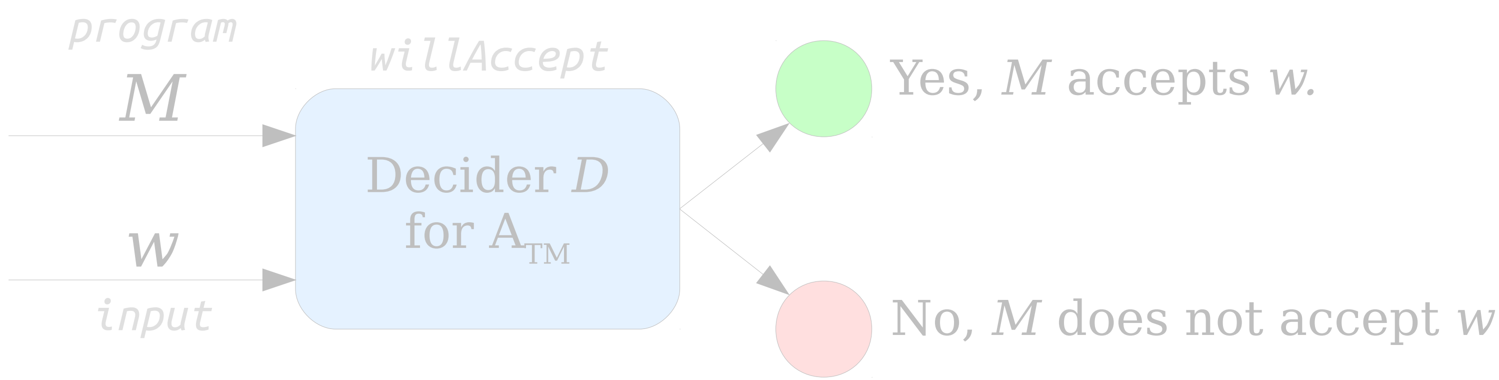


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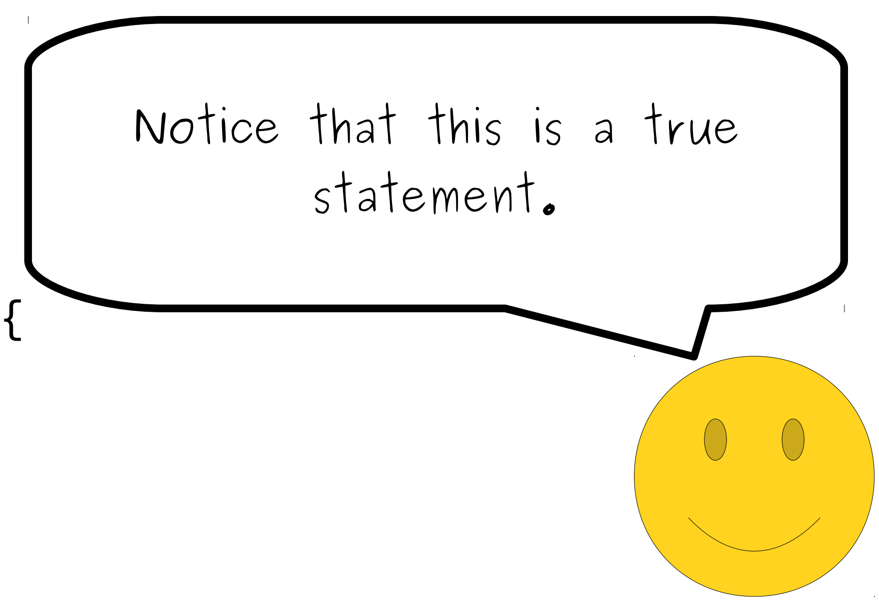

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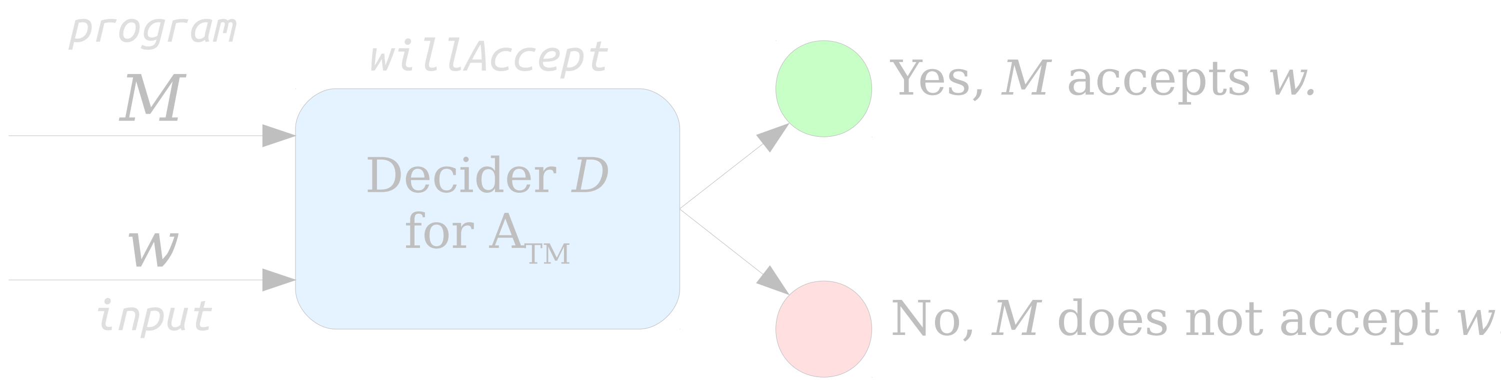


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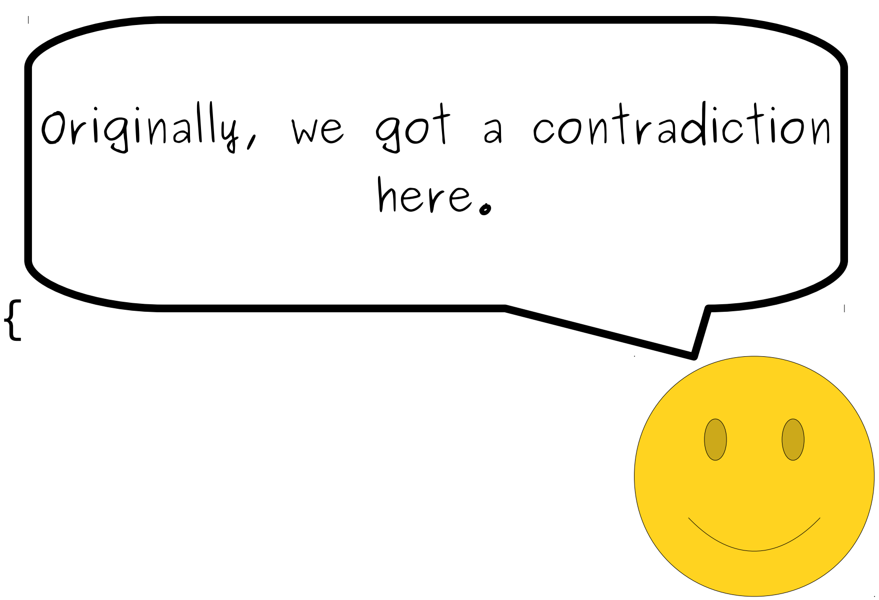

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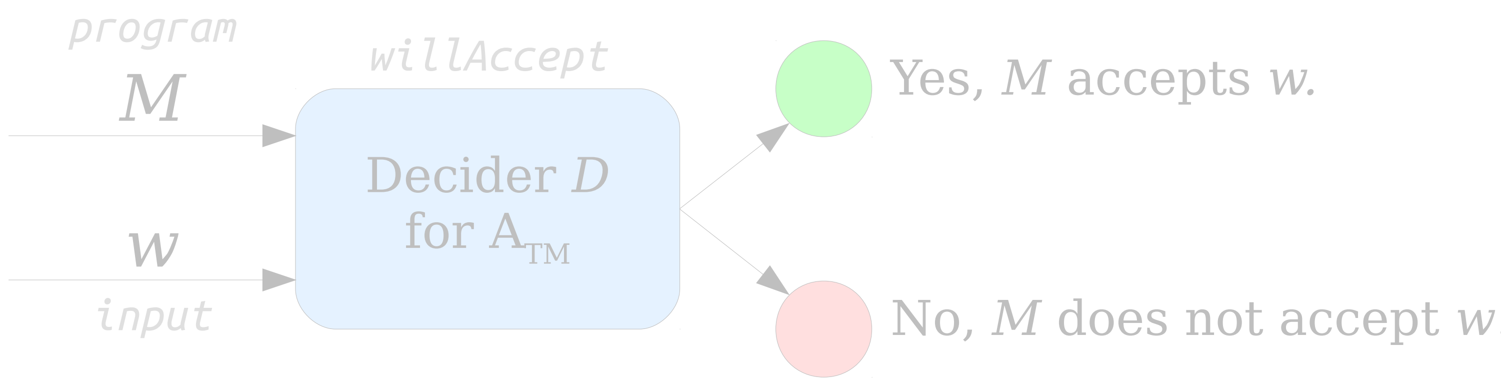


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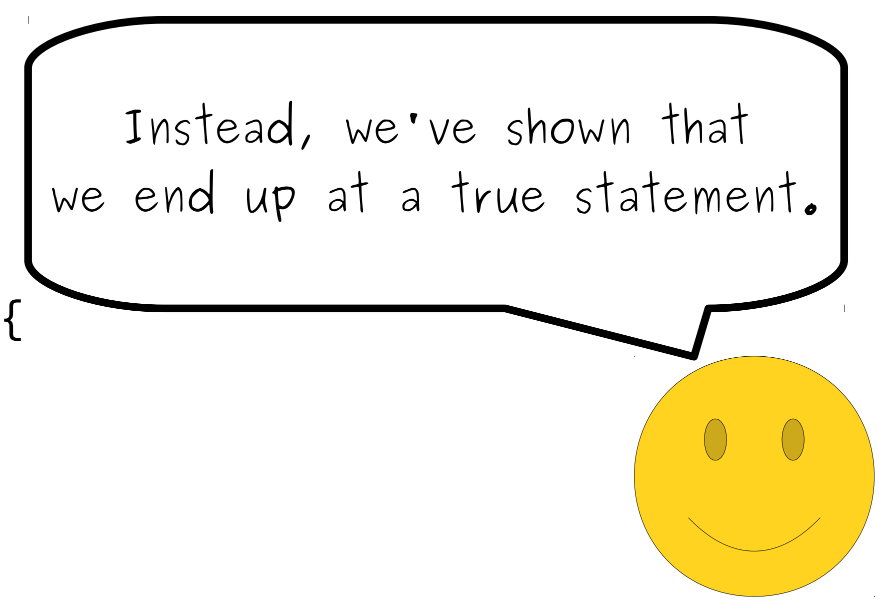

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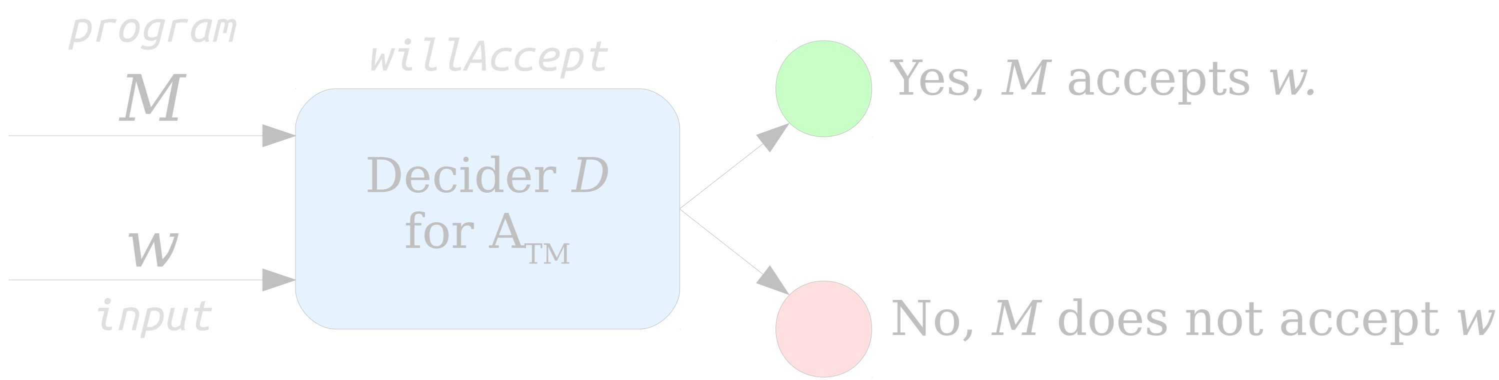


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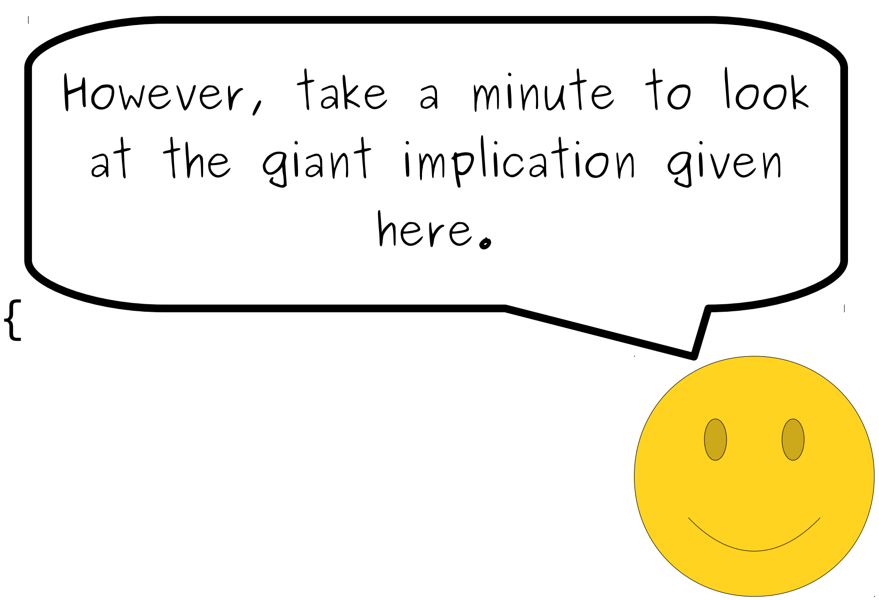

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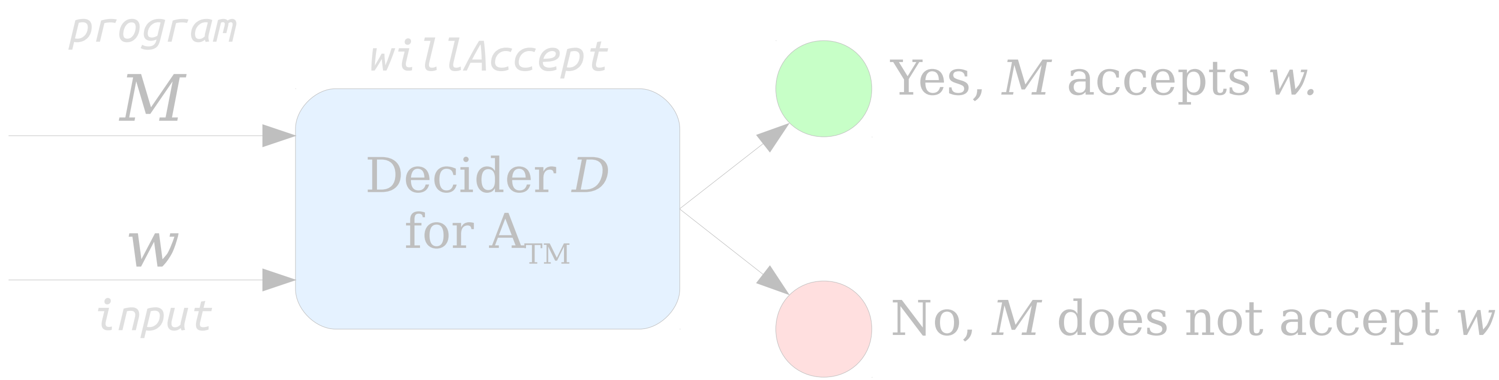


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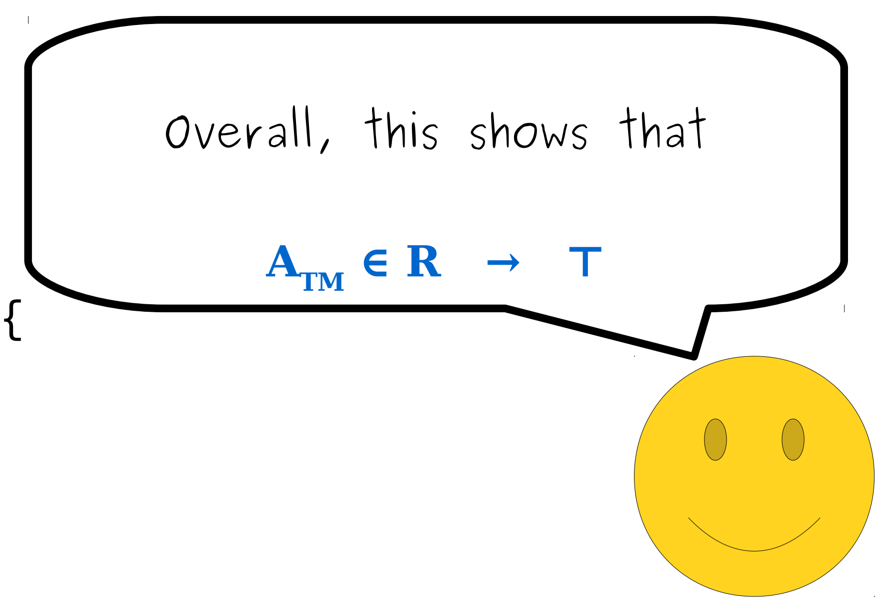

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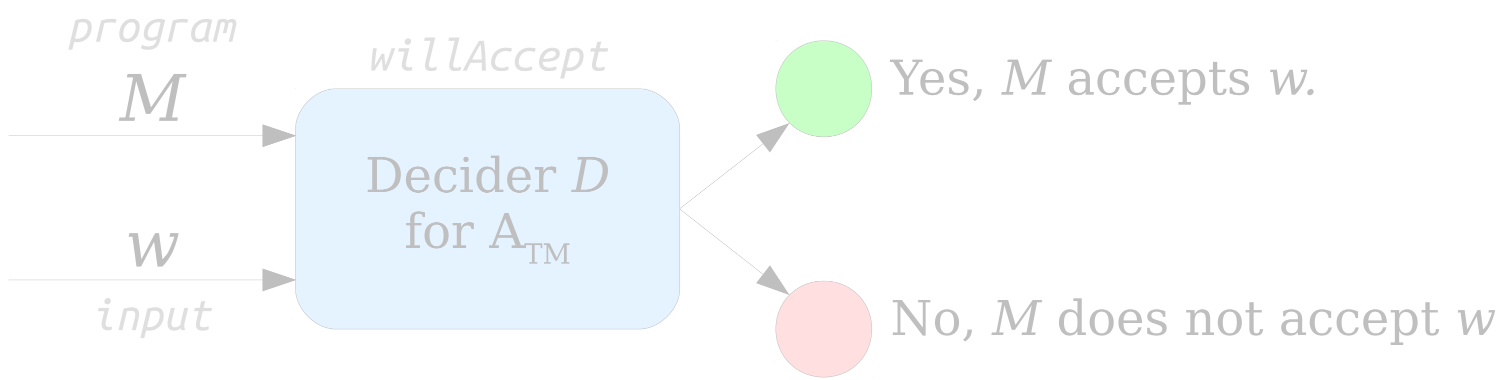


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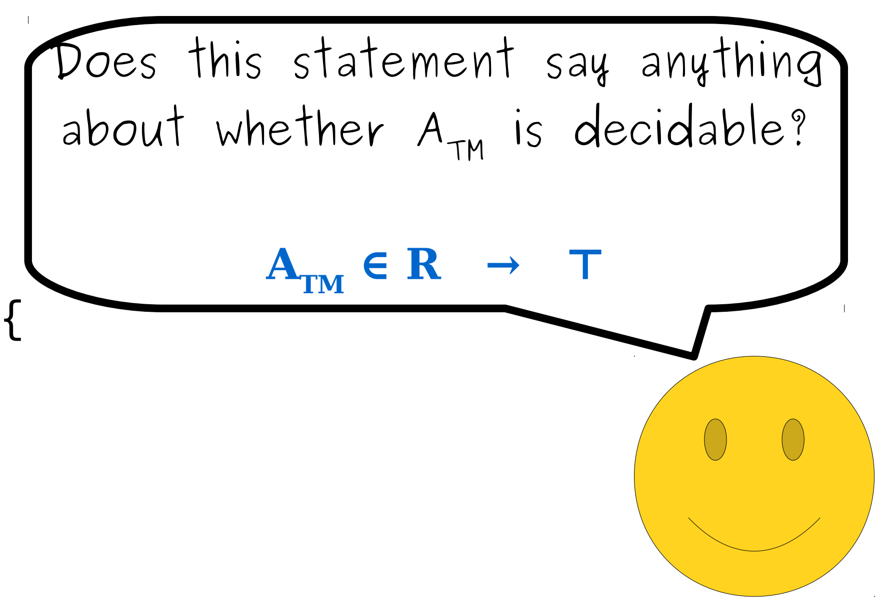

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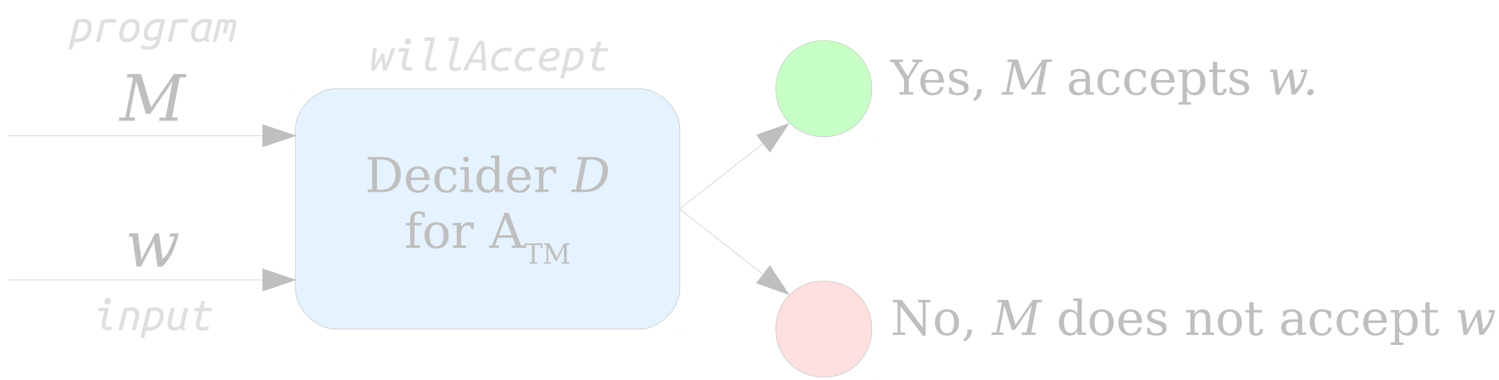


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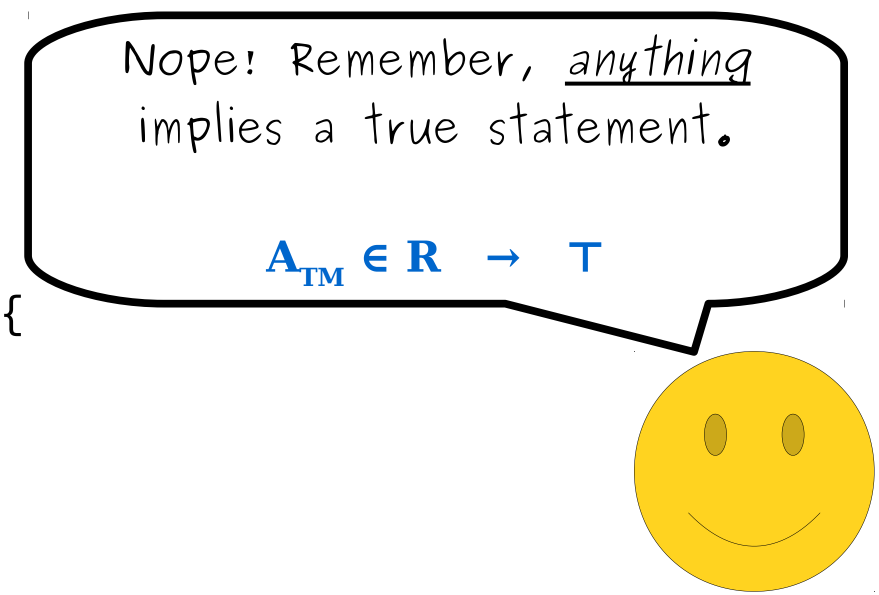

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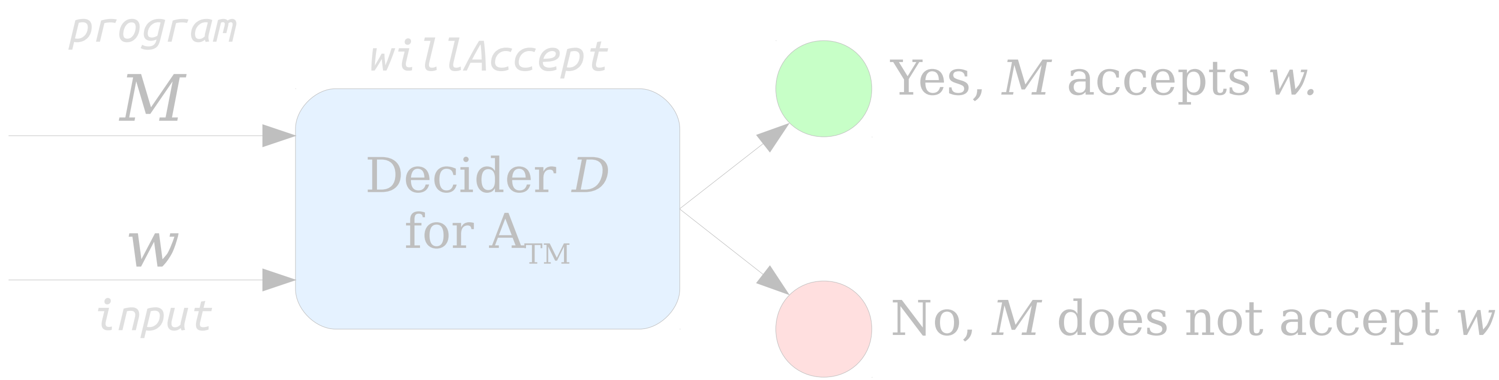


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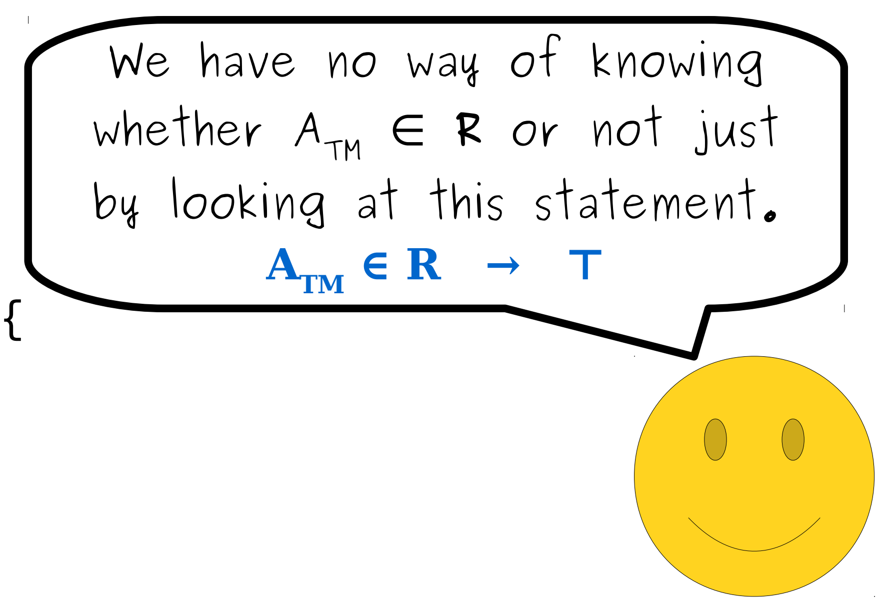

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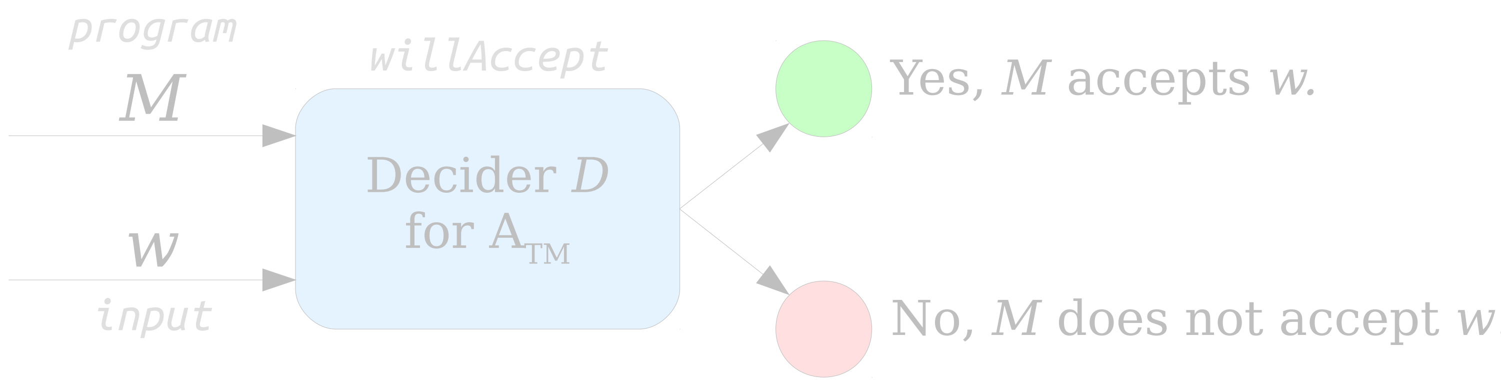


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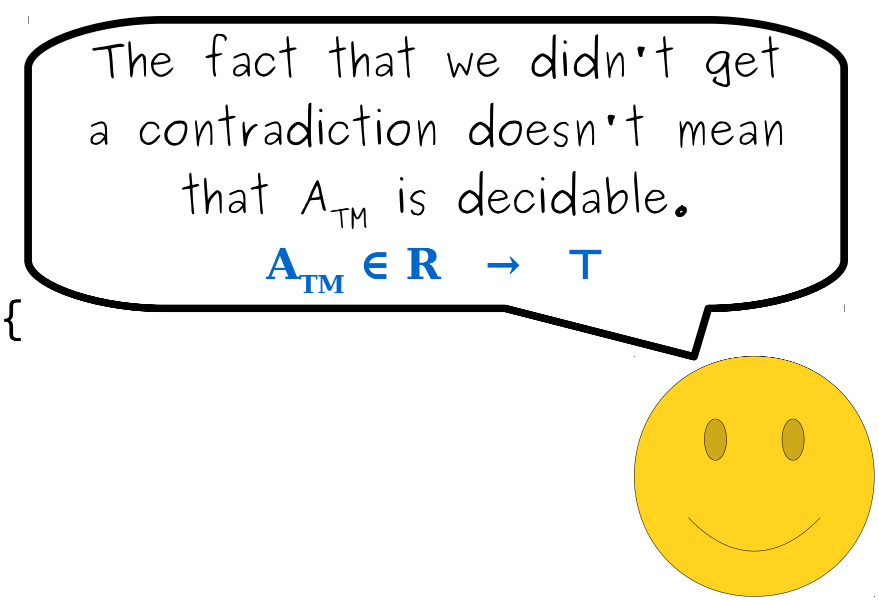

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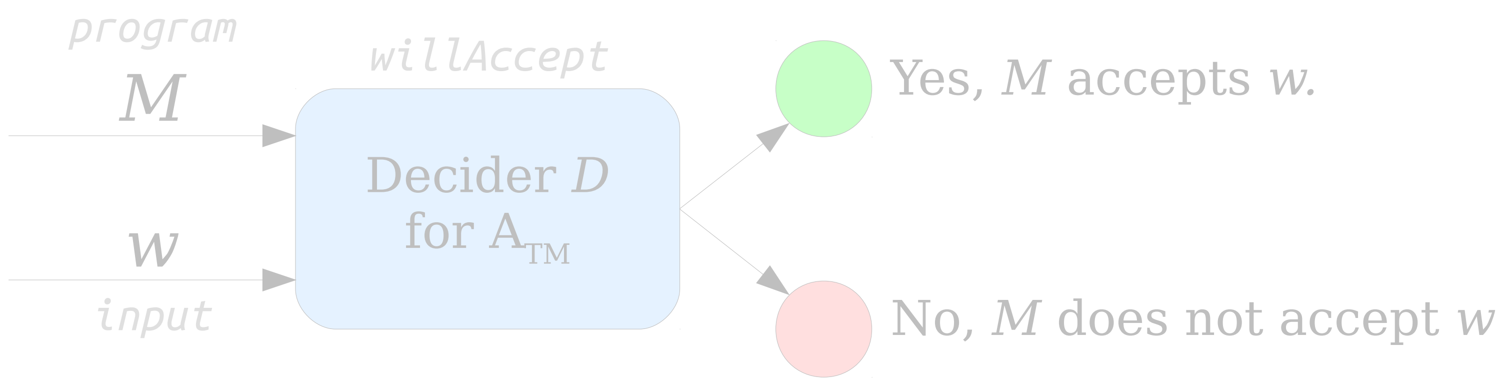


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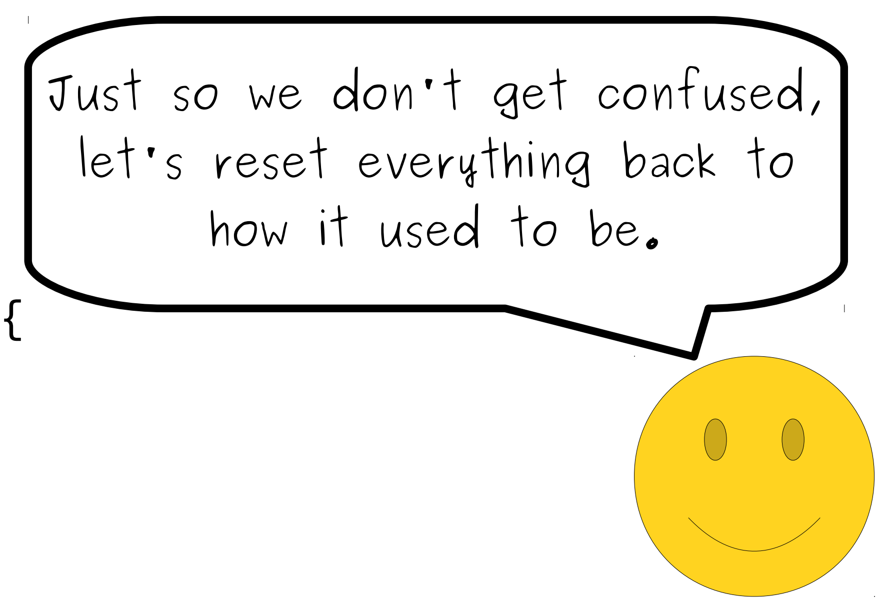

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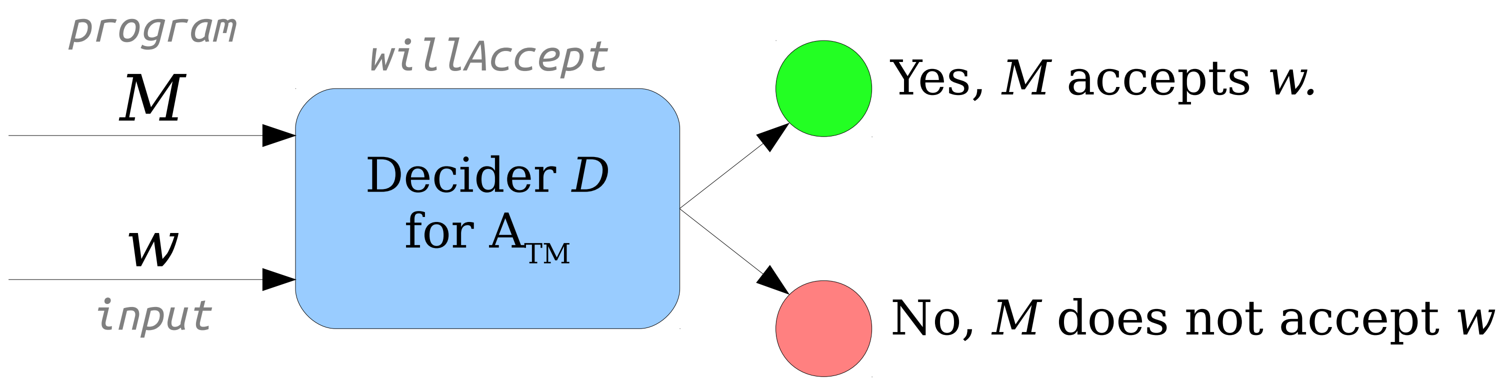


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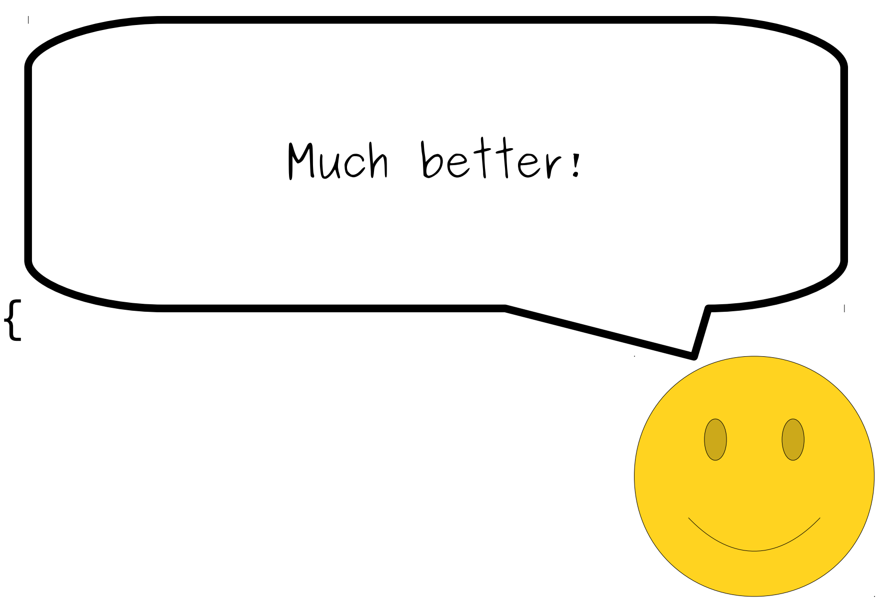

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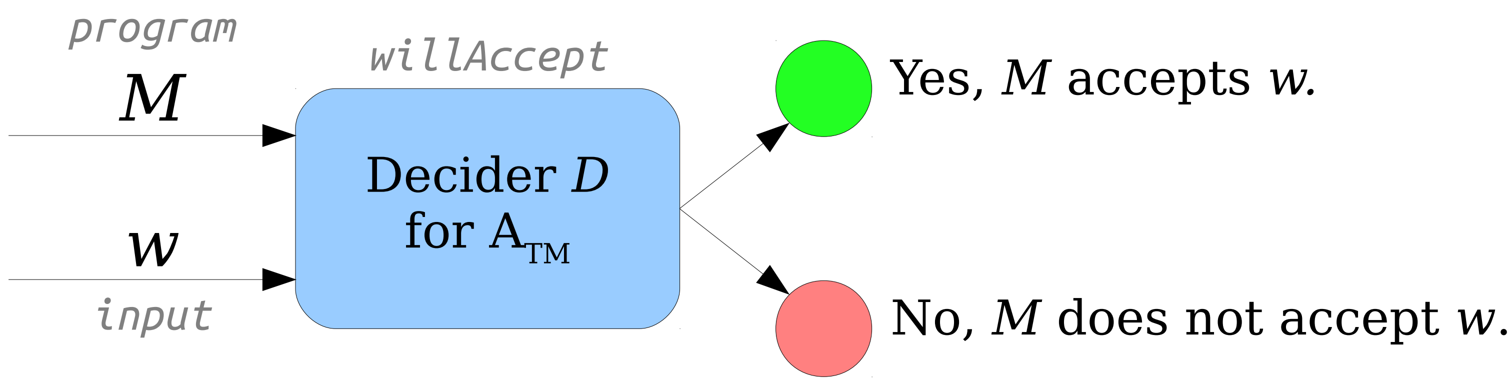


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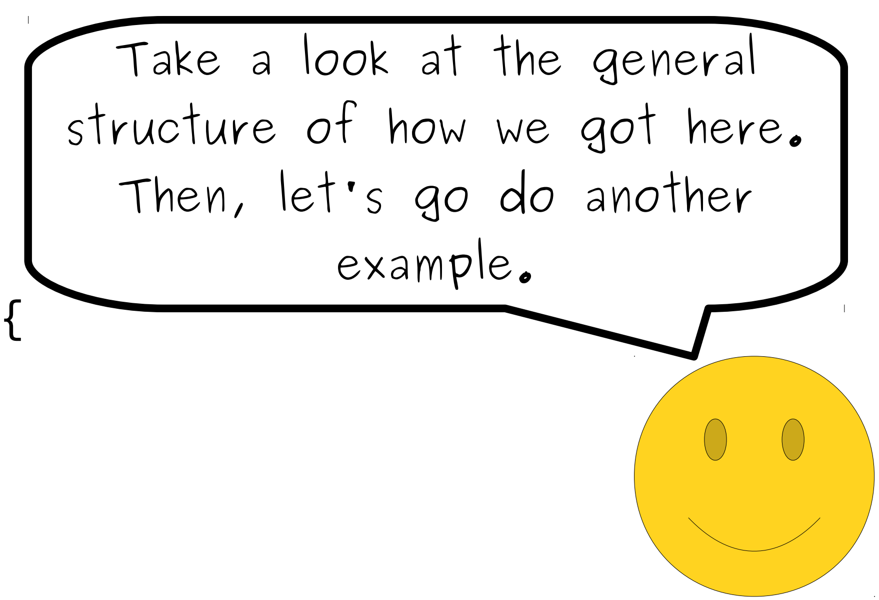
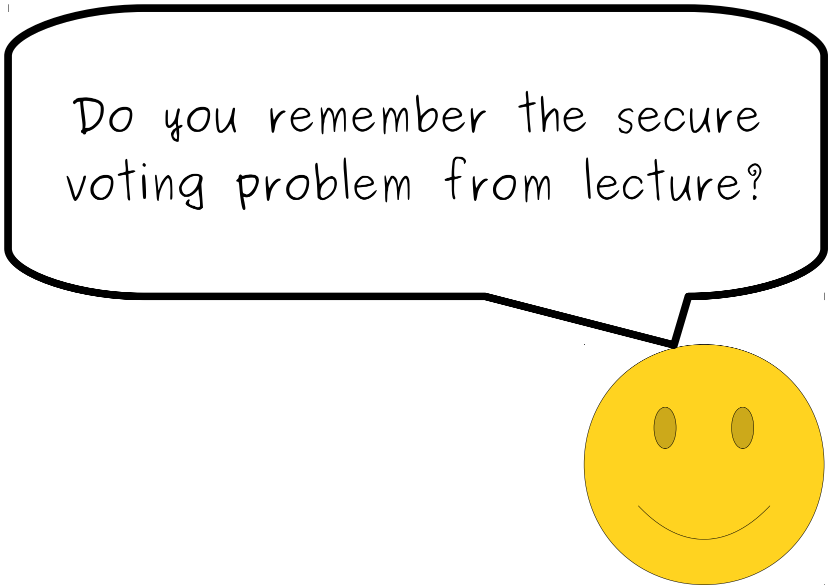
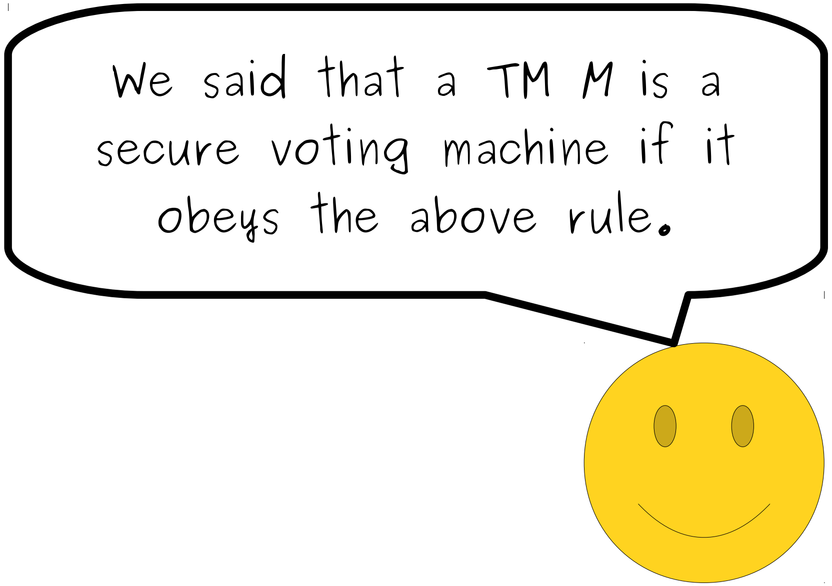



Do you remember the secure voting problem from lecture?

M is a secure voting machine

if and only if

$\mathscr{L}(M) = \{ w \in \{\texttt{r}, \texttt{d}\}^* \mid w$ has more $\texttt{r}$'s than $\texttt{d}$'s $\}$



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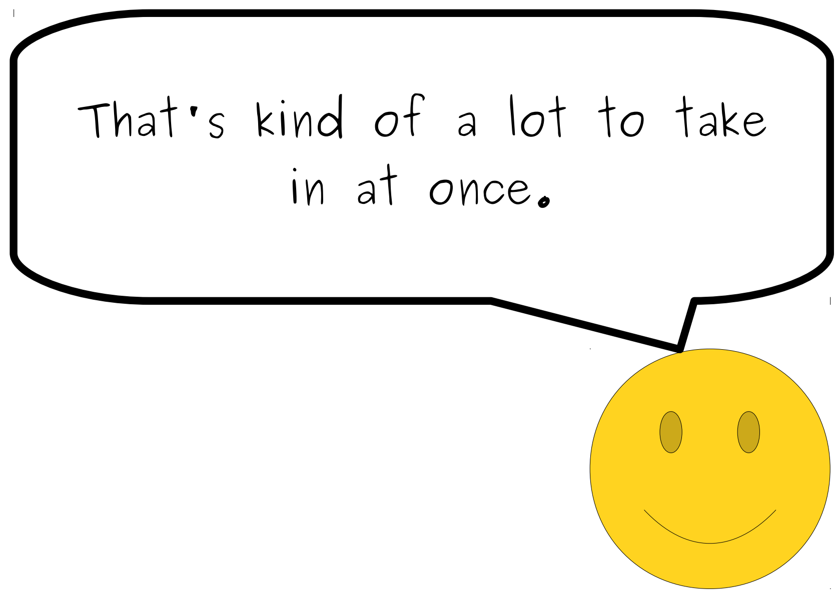

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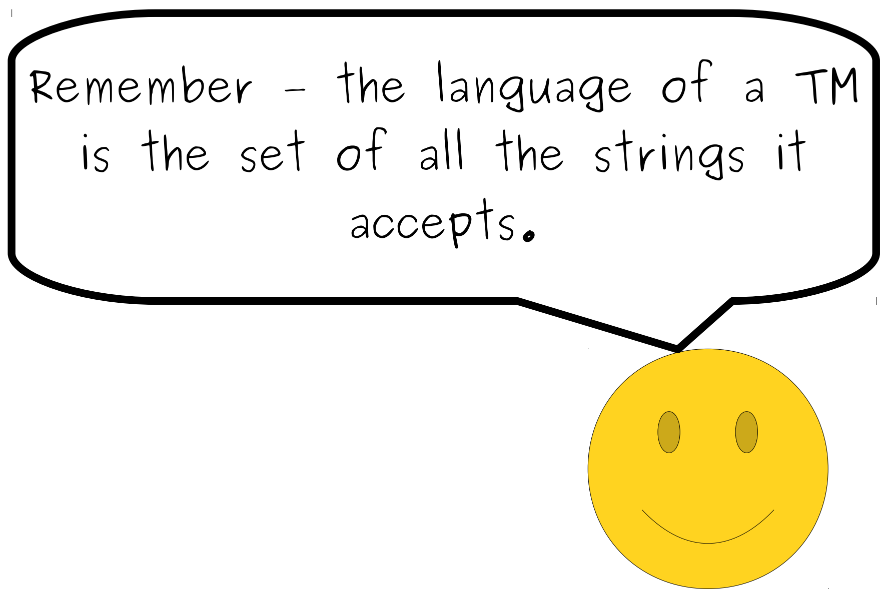

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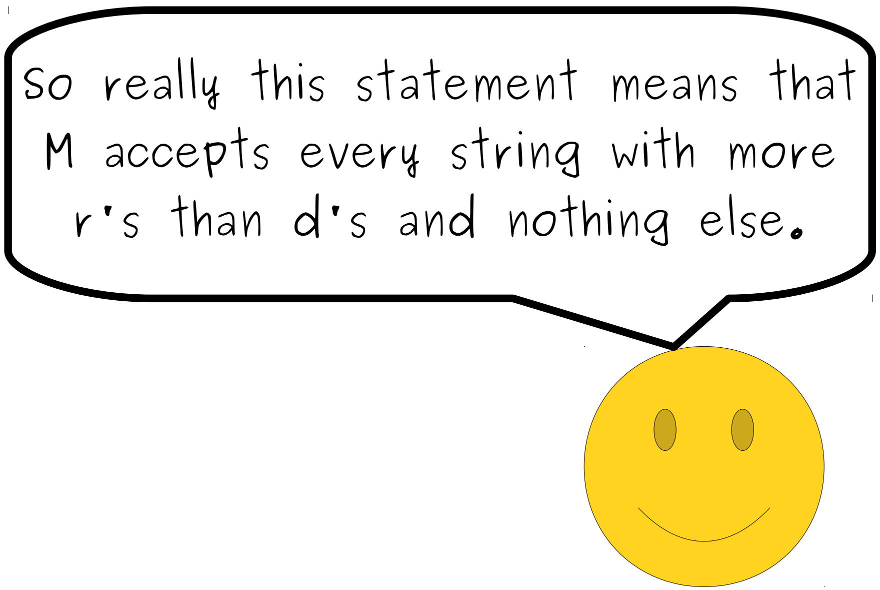

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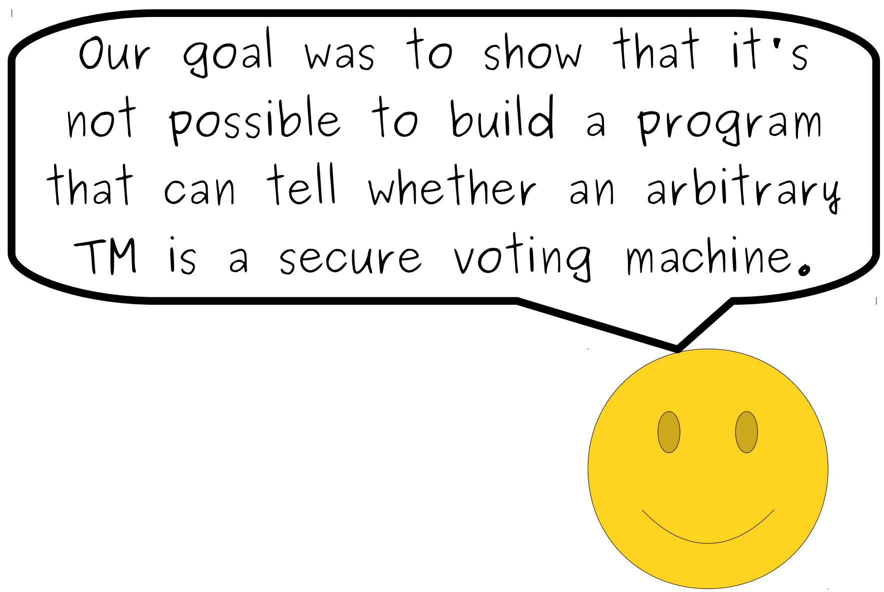

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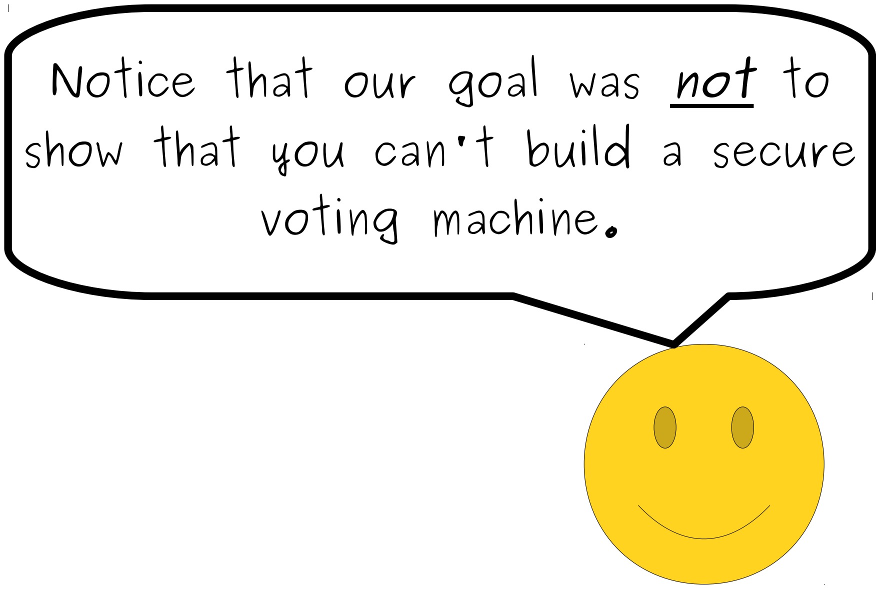

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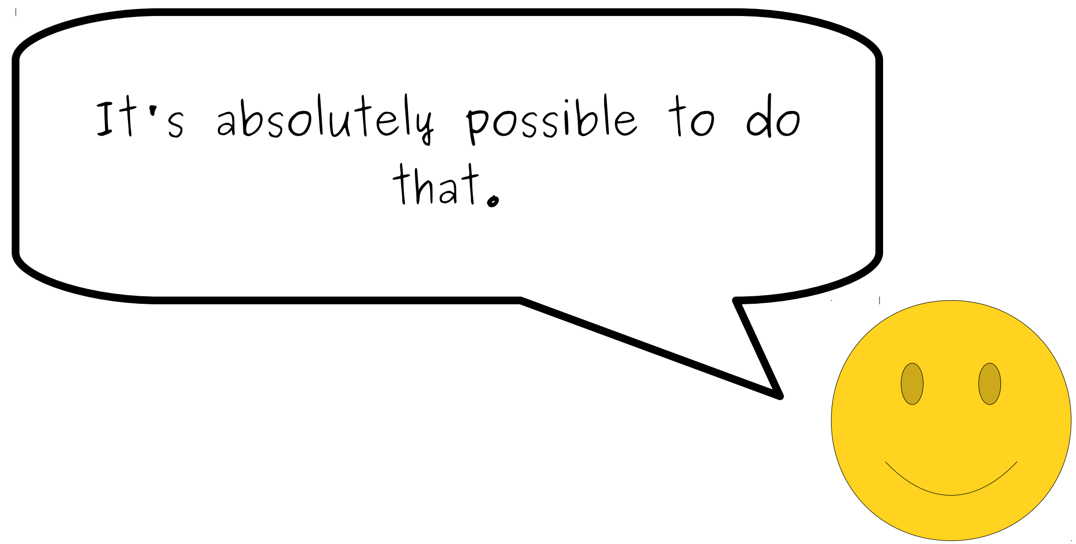


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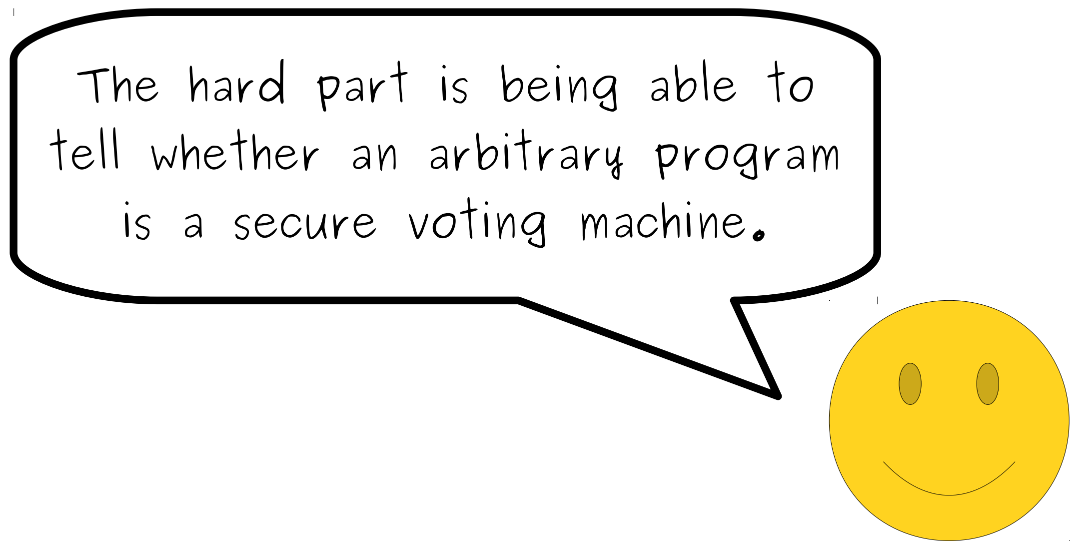


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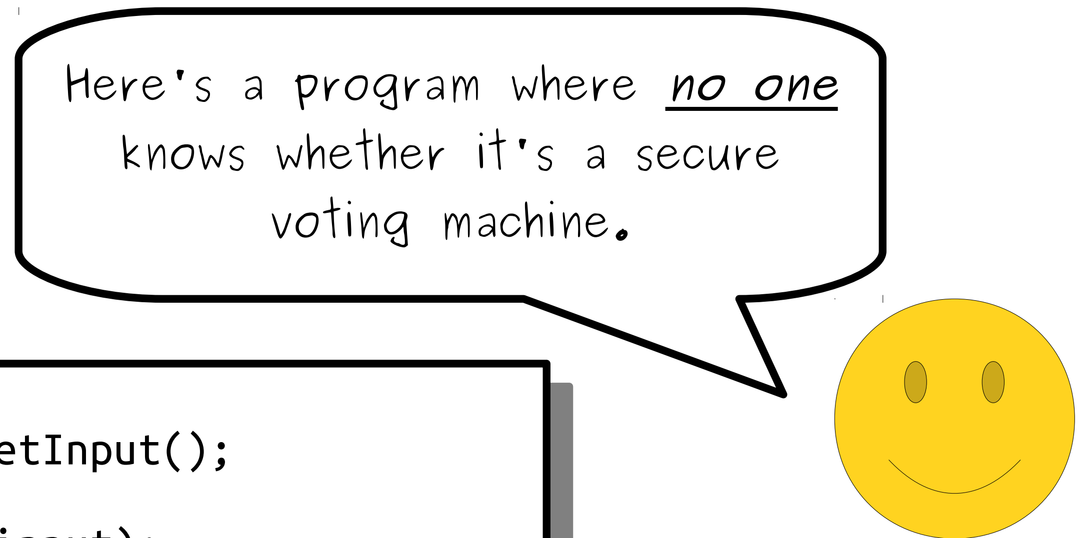
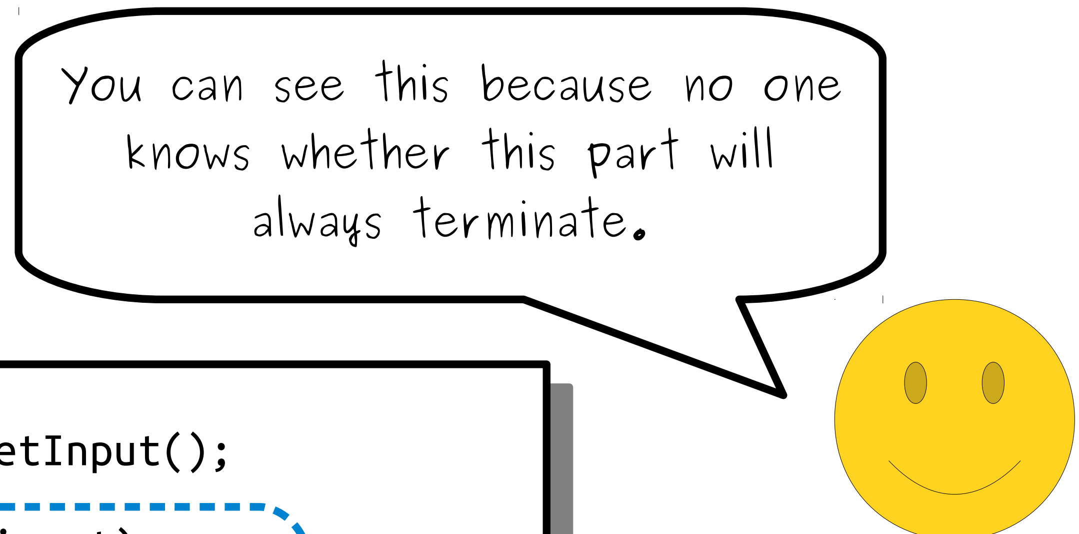



```
int main() {
  string input = getInput();

  int n = countRs(input);
  while (n > 1) {
    if (n % 2 == 0) n = n / 2;
    else n = 3*n + 1;
  }

  if (countRs(input) > countDs(input)) {
    accept();
  } else {
    reject();
  }
}
```

M is a secure voting machine

if and only if

$\mathscr{L}(M) = \{ w \in \{\texttt{r}, \texttt{d}\}^* \mid w \text{ has more } \texttt{r}\text{'s than } \texttt{d}\text{'s} \}$




```
int main() {
  string input = getInput();

  int n = countRs(input);
  while (n > 1) {
    if (n % 2 == 0) n = n / 2;
    else n = 3*n + 1;
  }

  if (countRs(input) > countDs(input)) {
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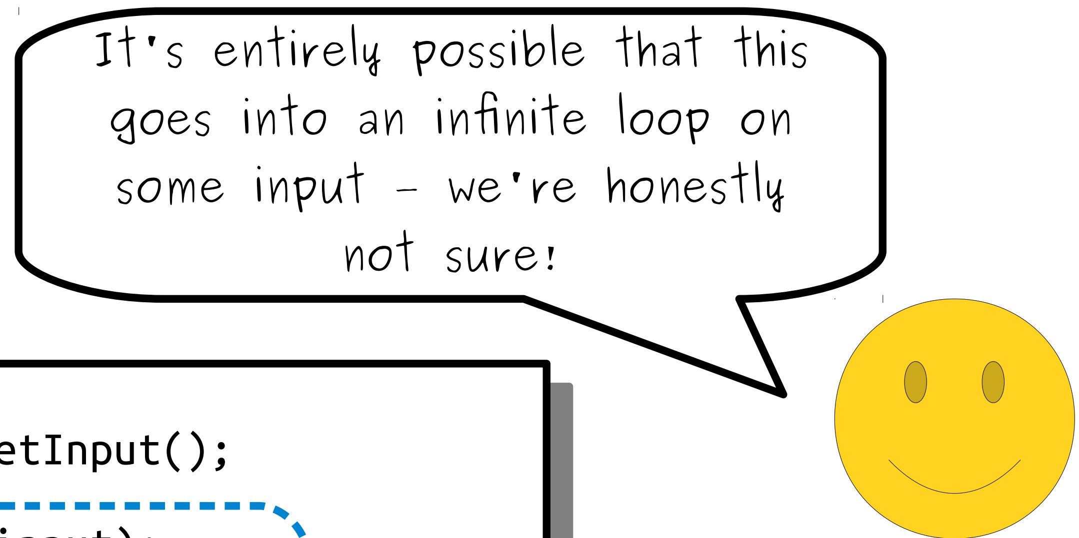


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M is a secure voting machine

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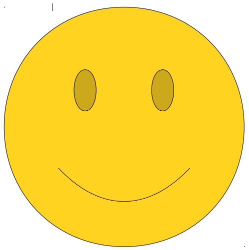
$\mathscr{L}(M) = \{\ w \in \{$`r`, `d`$\}^* \mid w$ has more `r`'s than `d`'s $\}$

So, to recap:
Building a secure voting machine isn't hard.
Checking whether an arbitrary program is a secure voting machine is really hard.



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  string input = getInput();

  int n = countRs(input);
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}
```
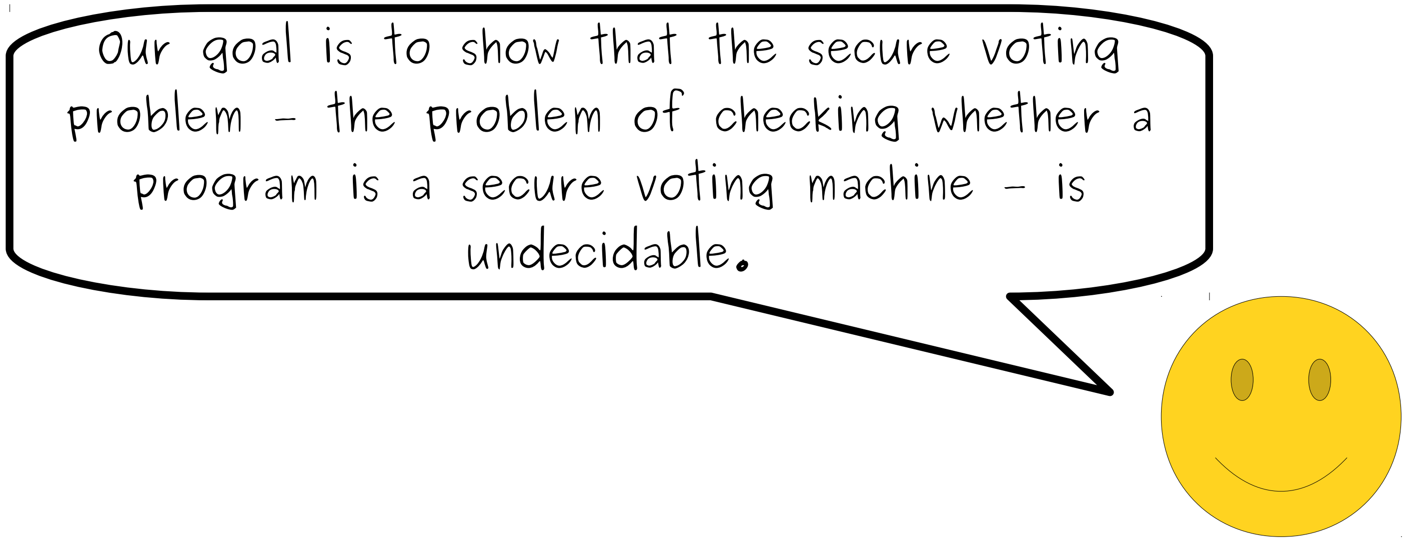
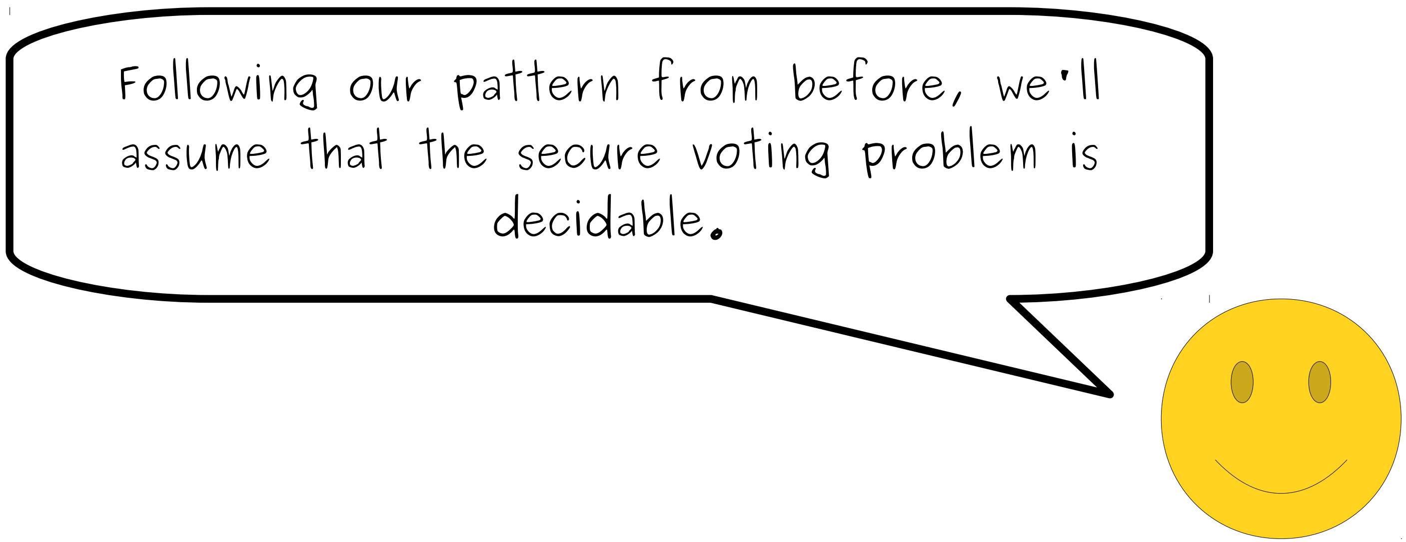


Our goal is to show that the secure voting problem – the problem of checking whether a program is a secure voting machine – is undecidable.

The secure voting problem is decidable.

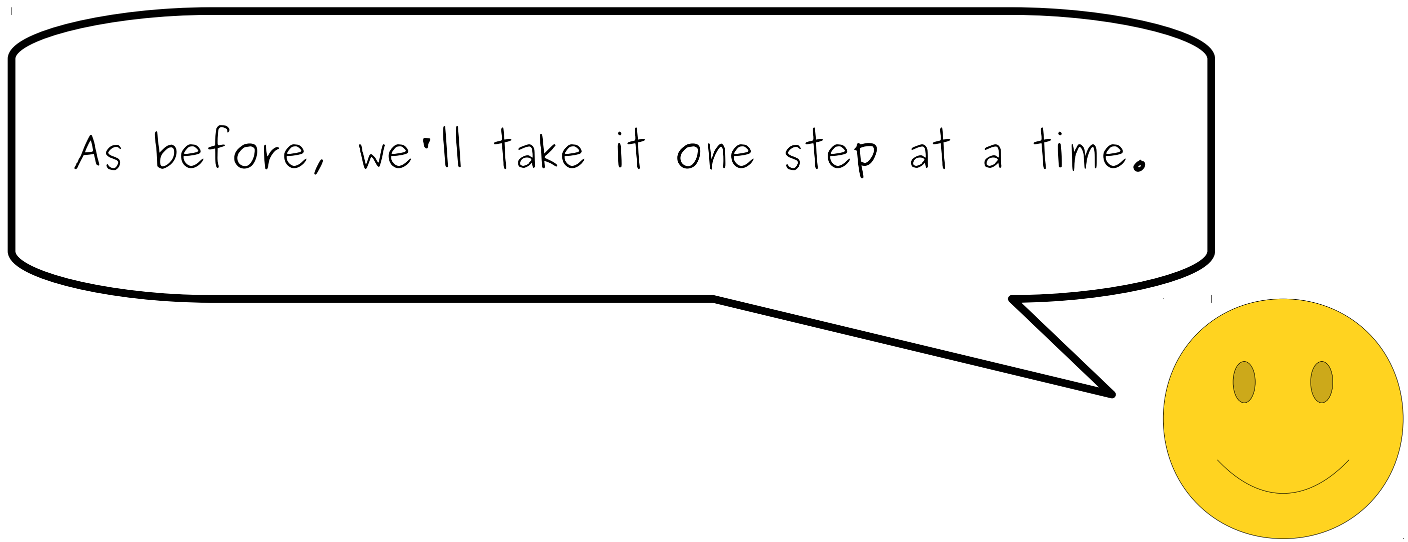


The secure voting problem is decidable.

Contradiction!

The secure voting problem is decidable.

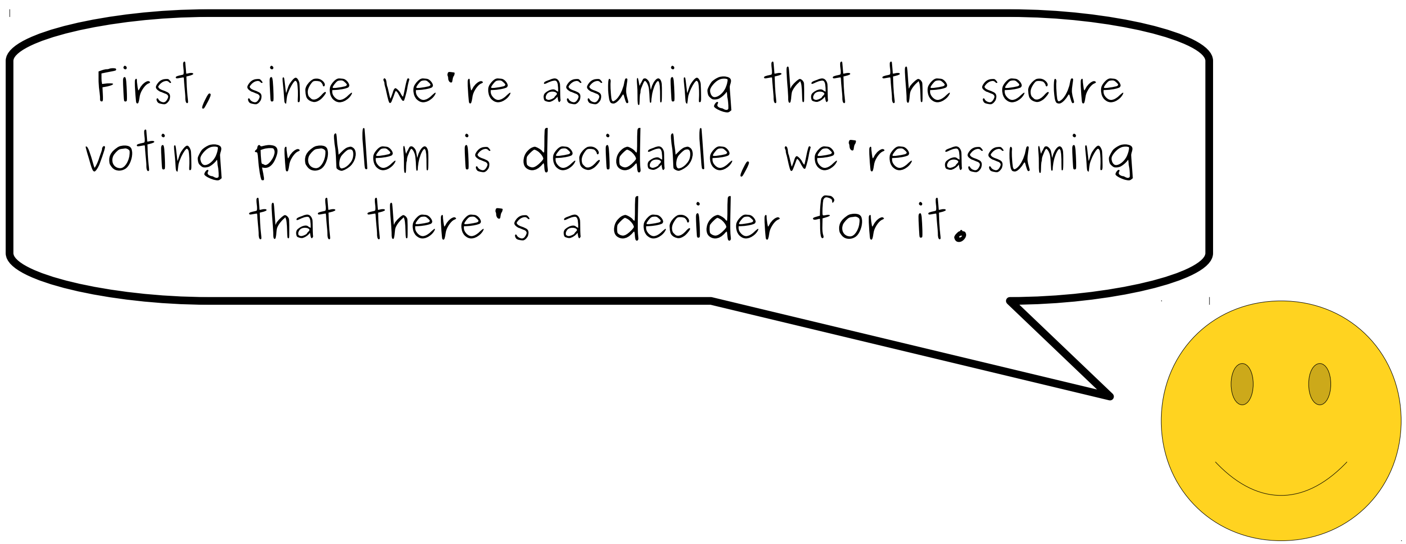
Contradiction!



The secure voting problem is decidable.

↓

There is a decider *D* for the secure voting problem



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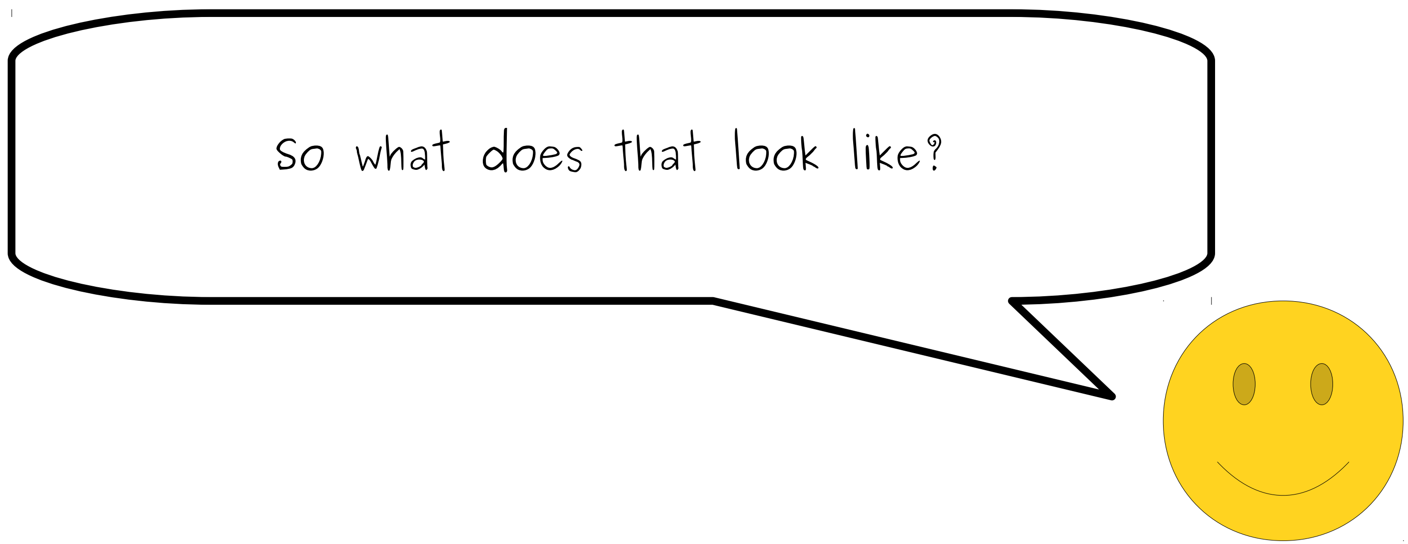
Contradiction!

Decider D for the secure voting problem



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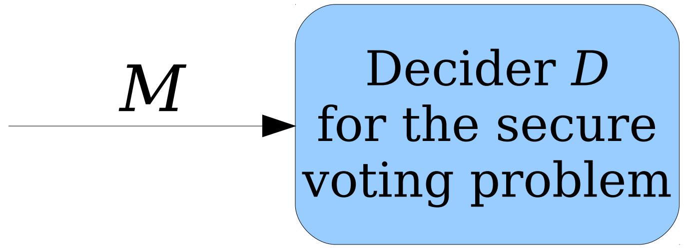

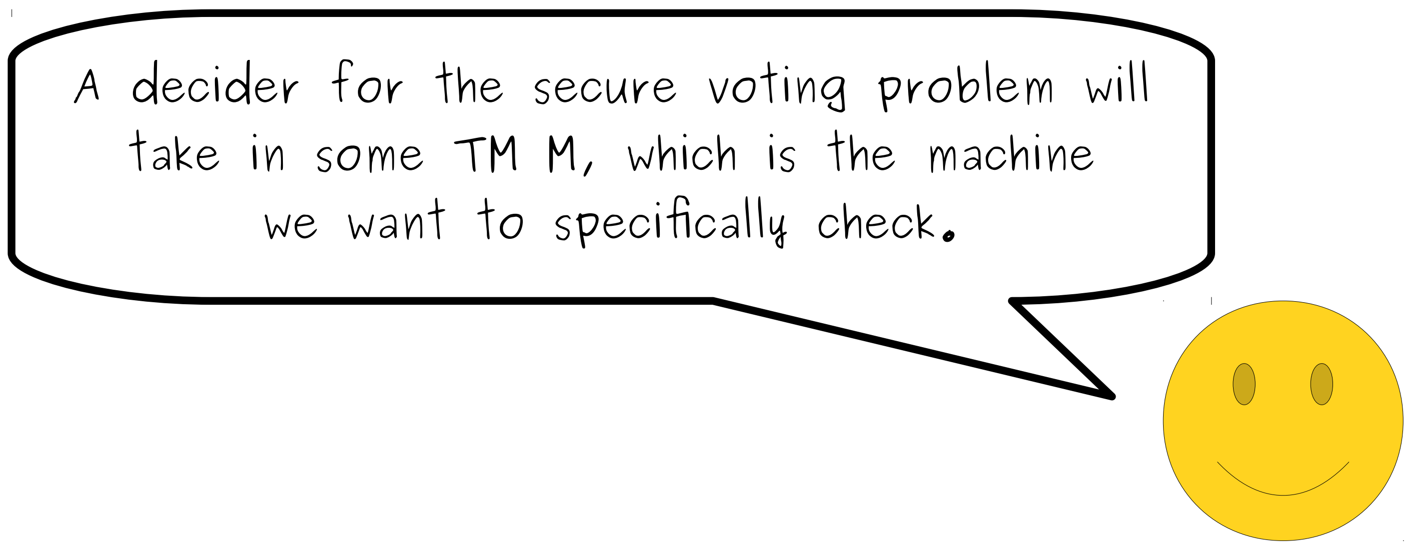

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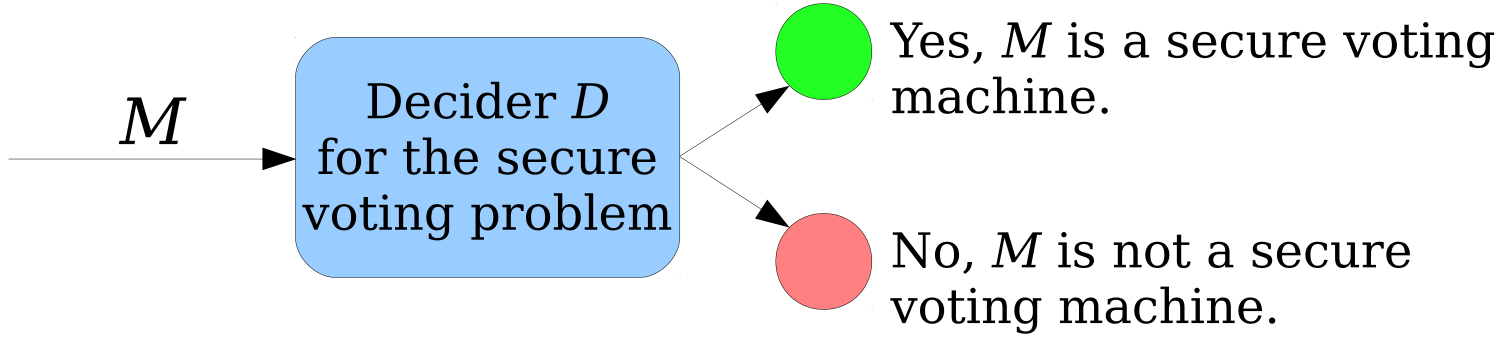
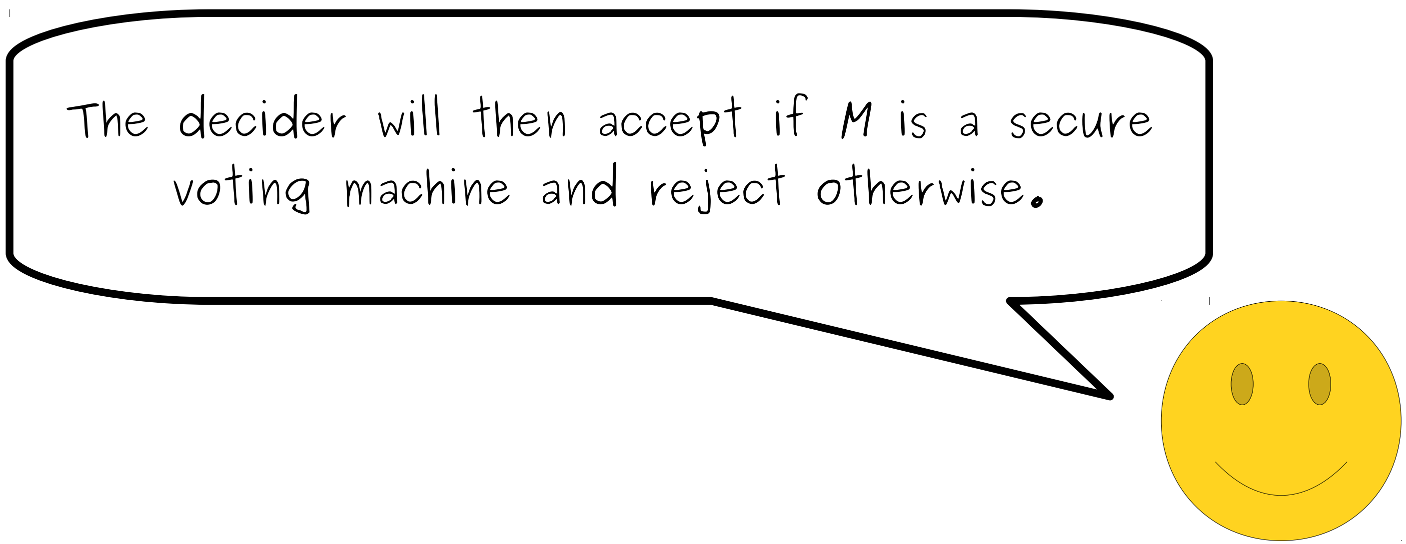




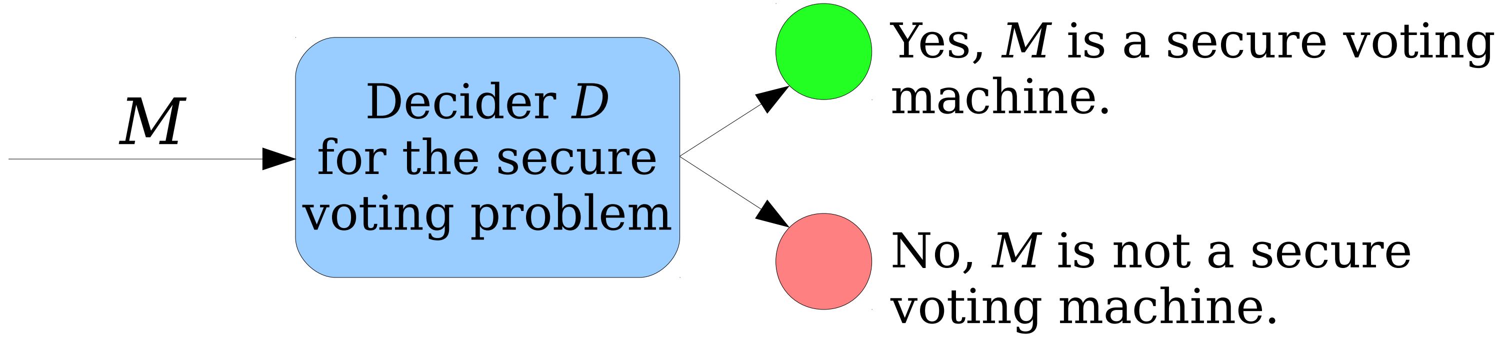
Contradiction!

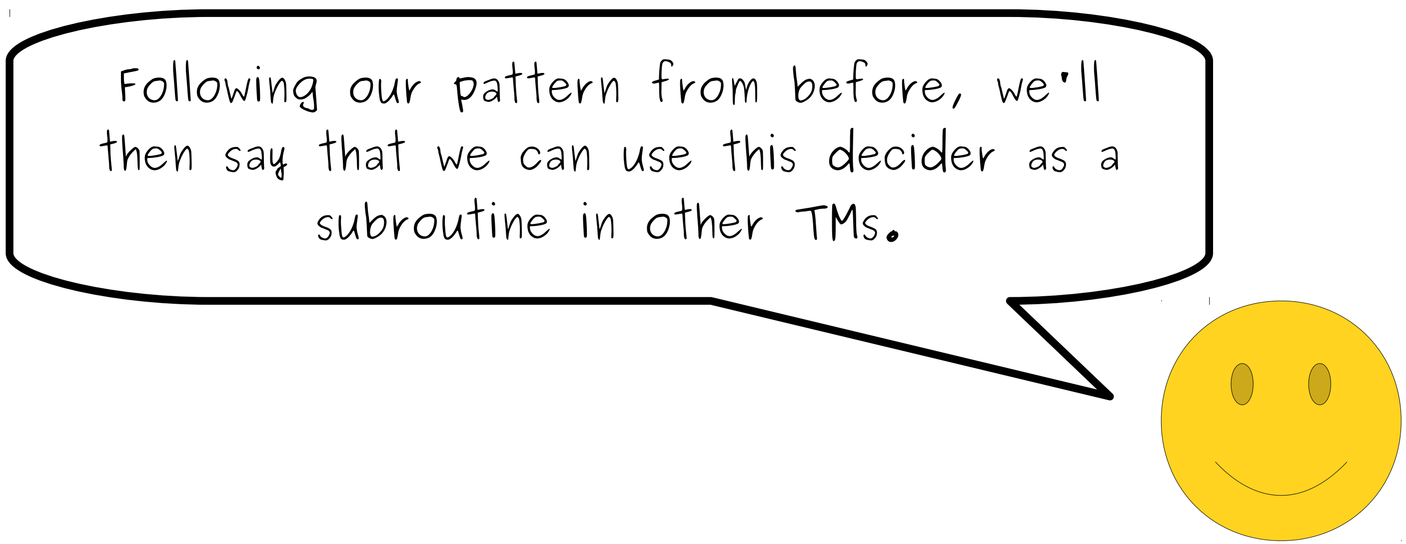
The secure voting problem is decidable.

There is a decider D for the secure voting problem

We can write programs that use D as a helper method

Contradiction!





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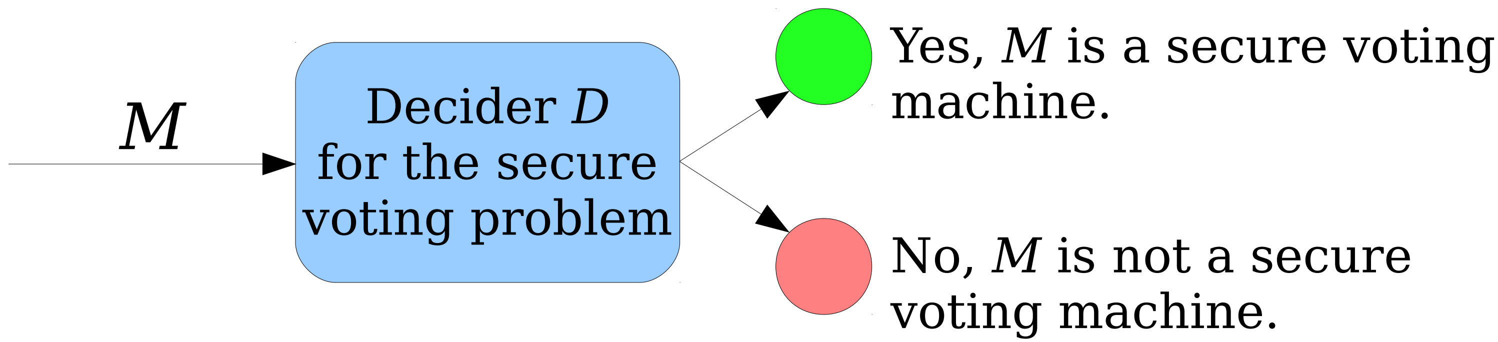
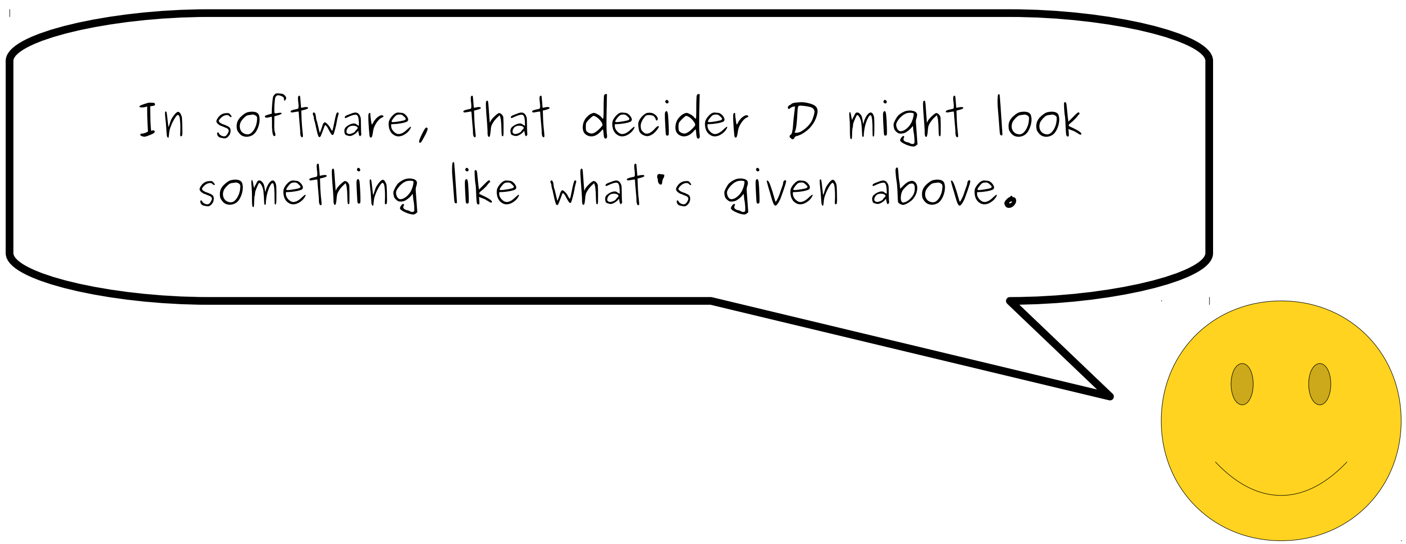



```
bool isSecure(string program)
```



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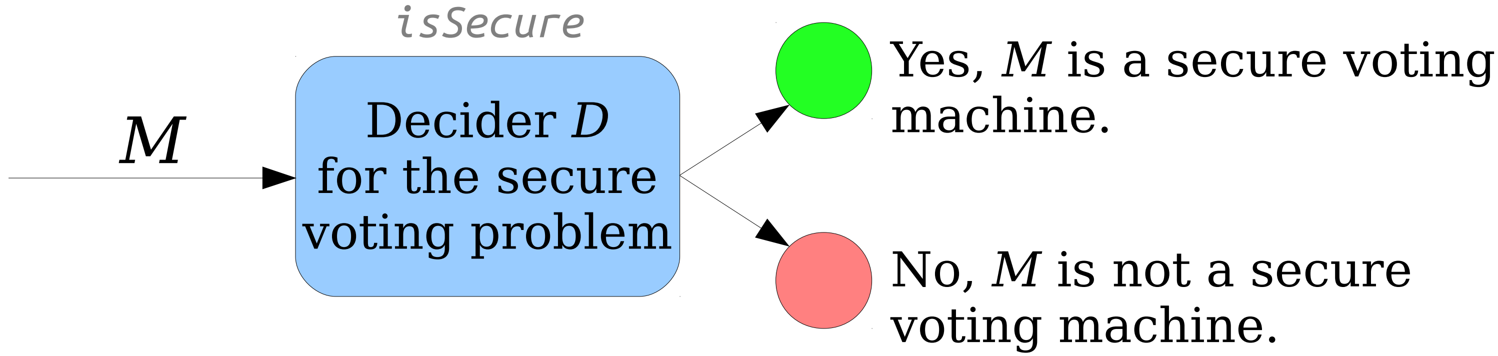


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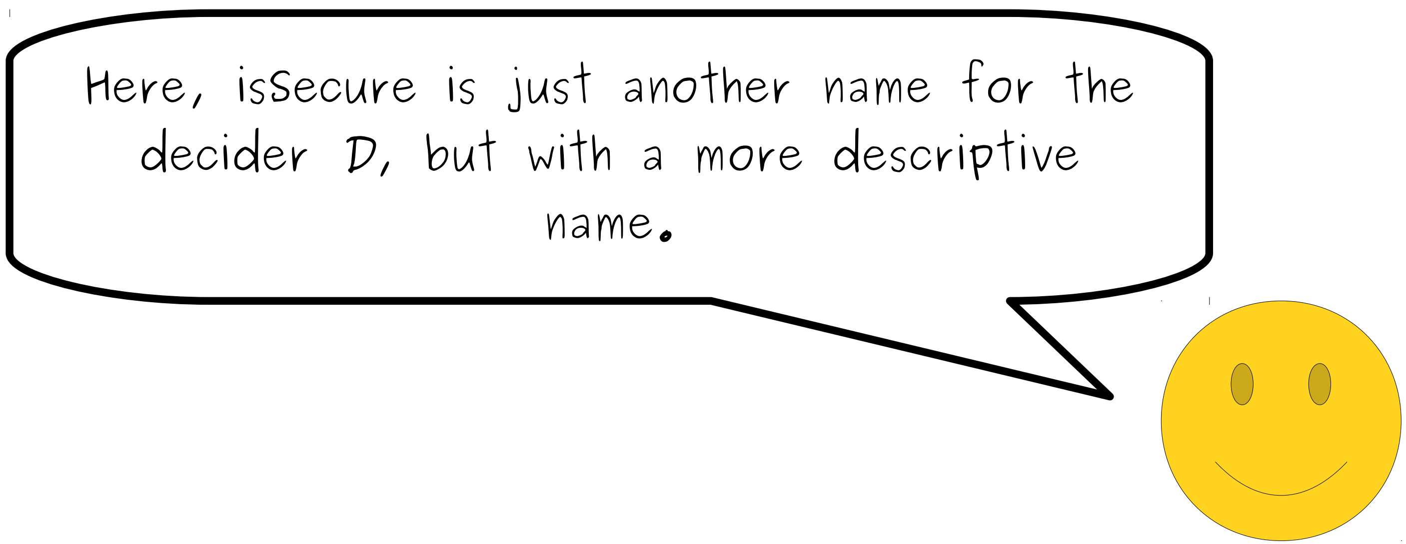

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There is a decider *D* for the secure voting problem

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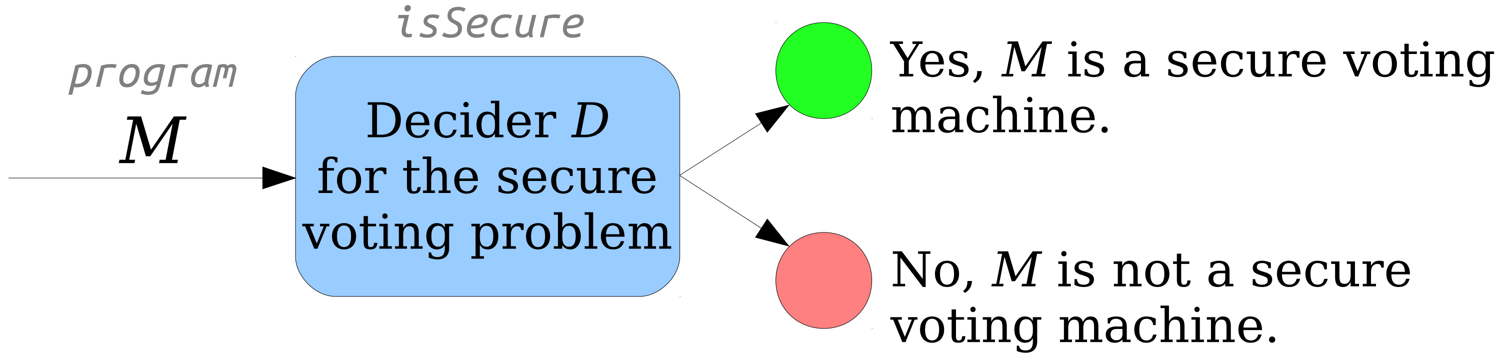


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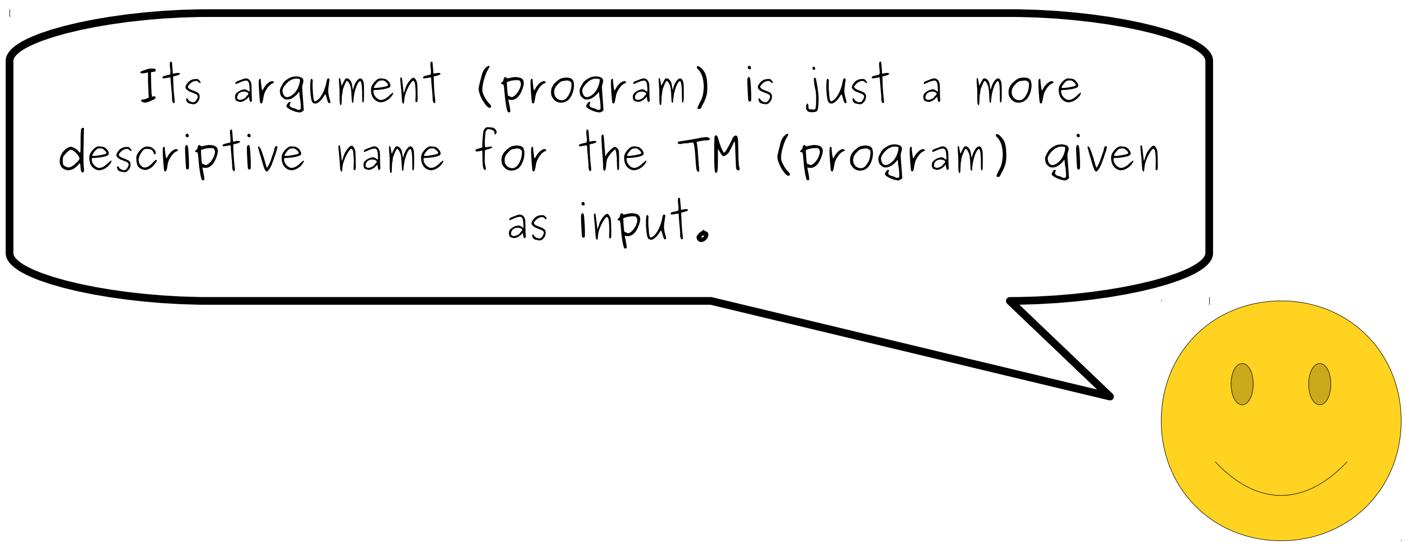

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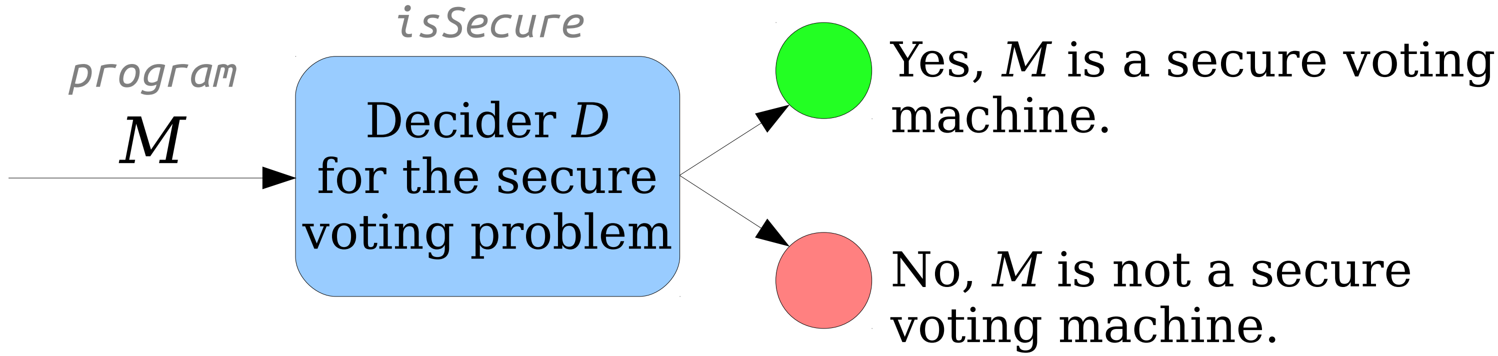


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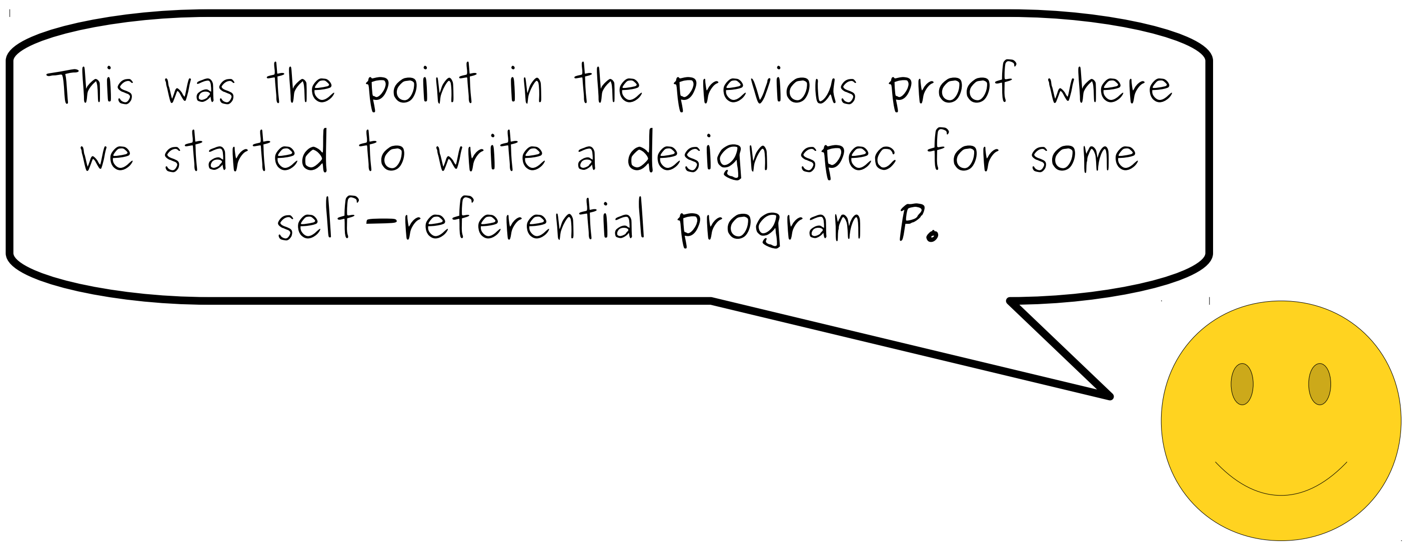

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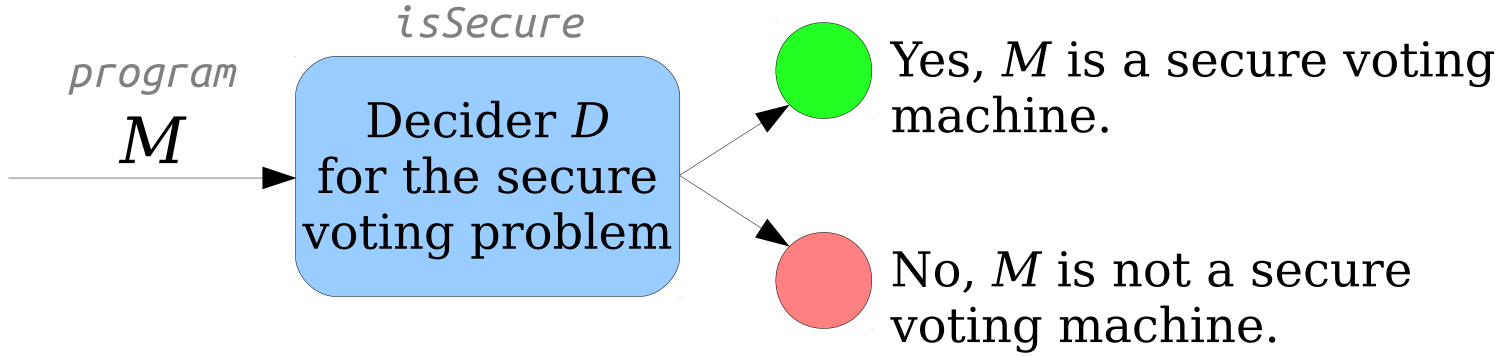


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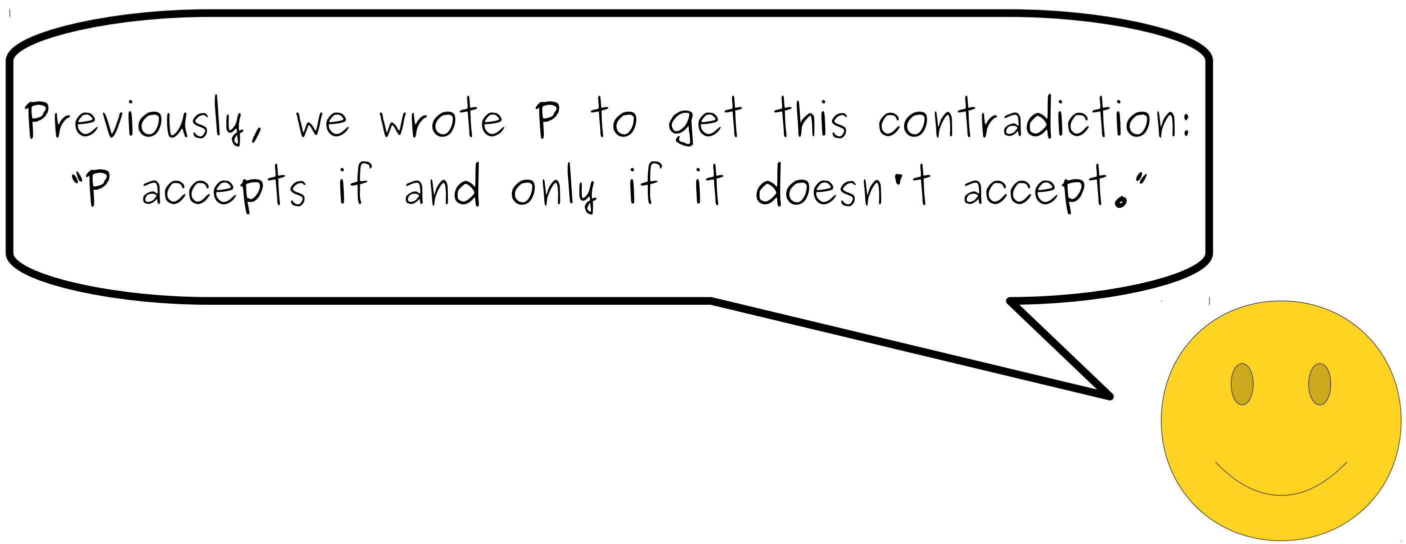

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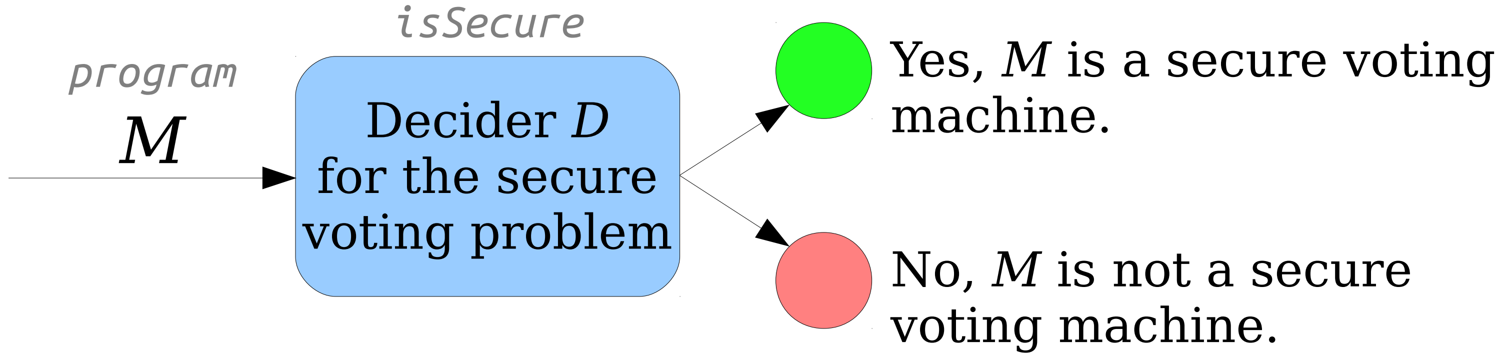


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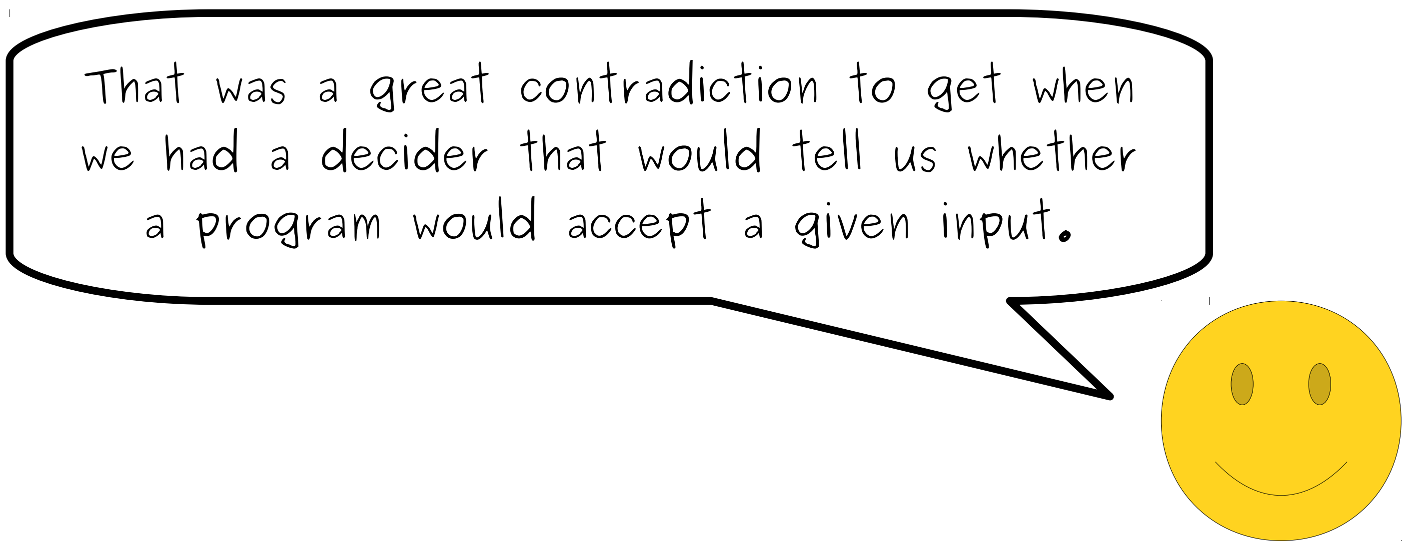

The secure voting problem is decidable.

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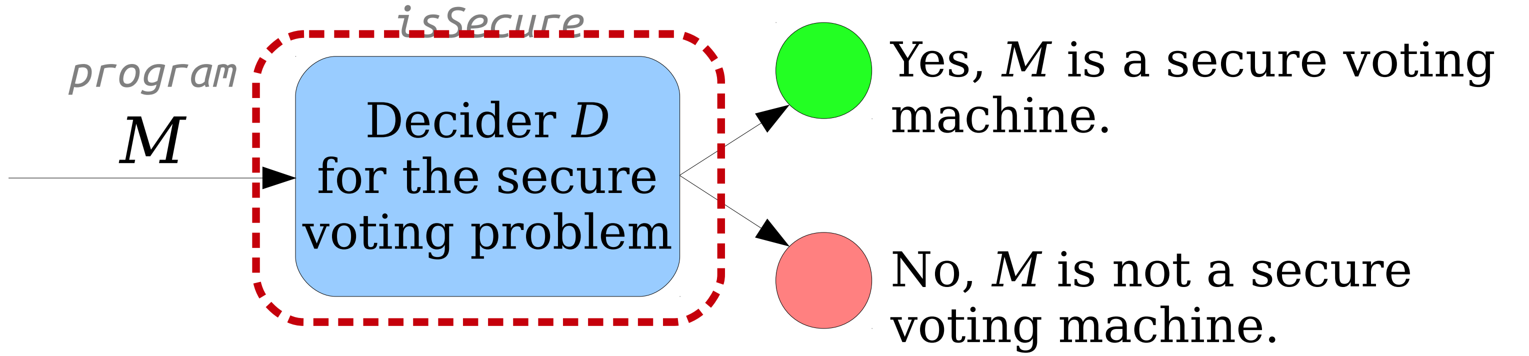
Contradiction!




```
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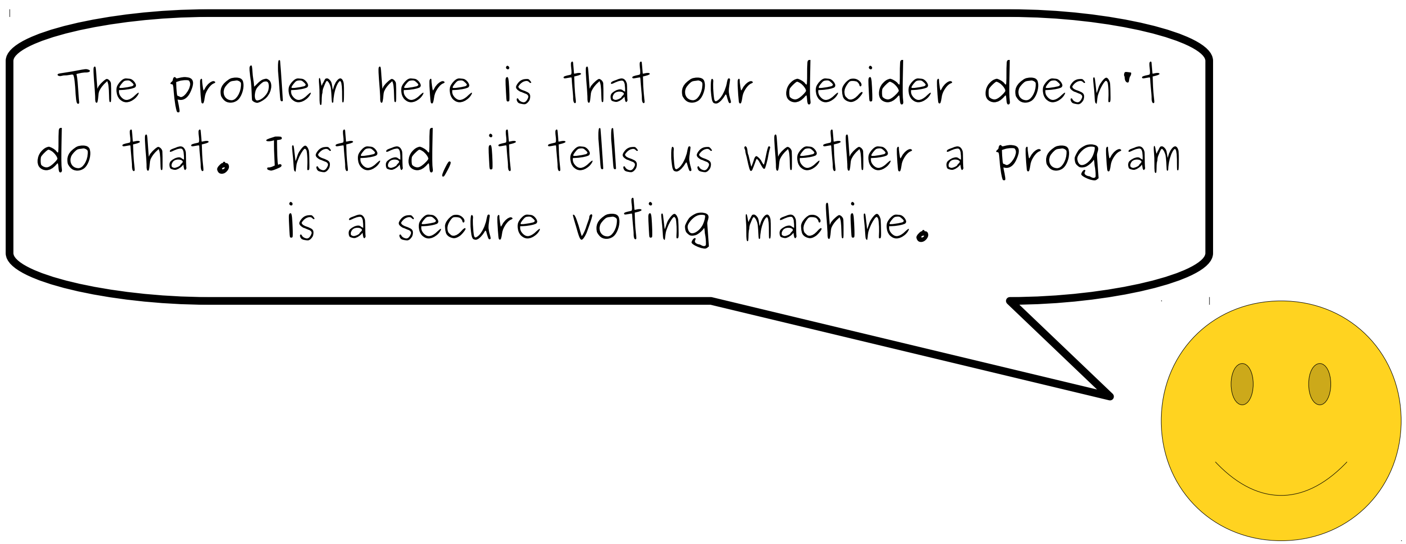

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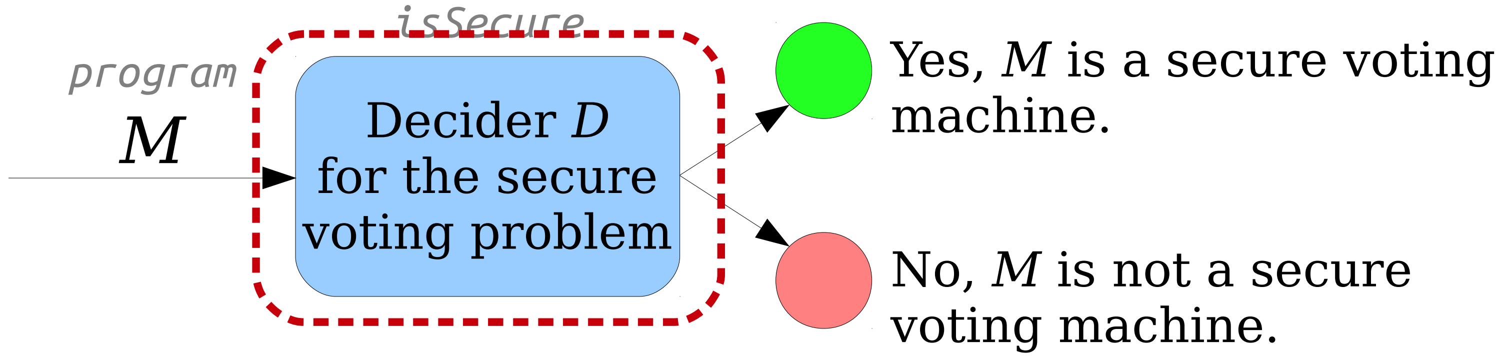


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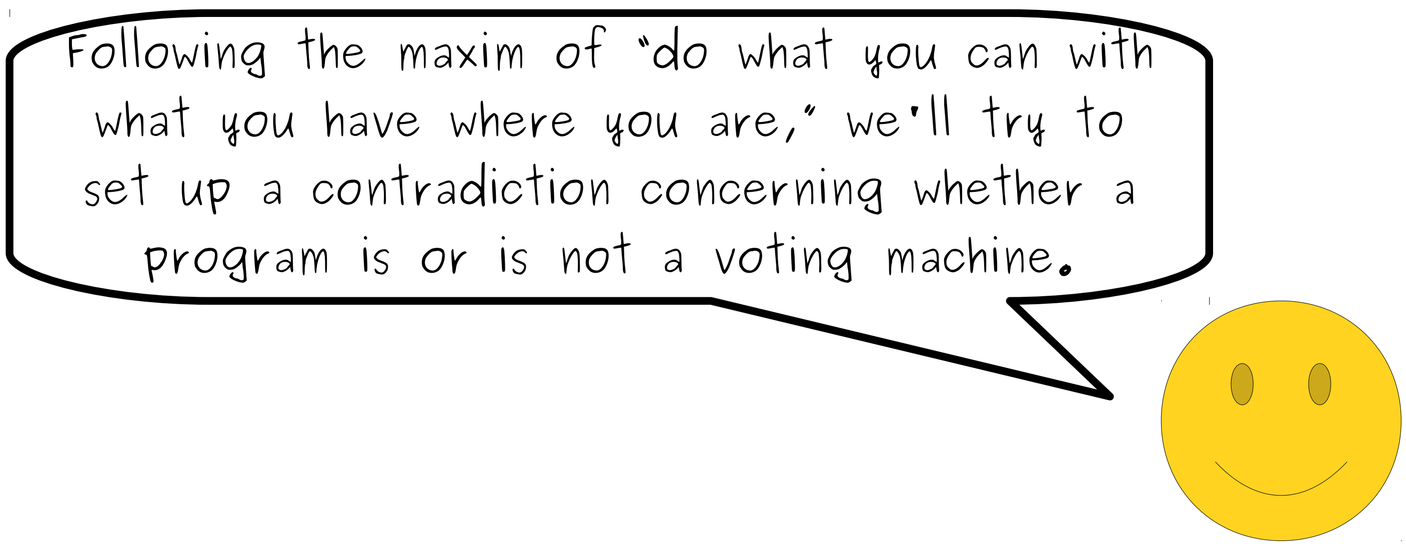

The secure voting problem is decidable.

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There is a decider *D* for the secure voting problem

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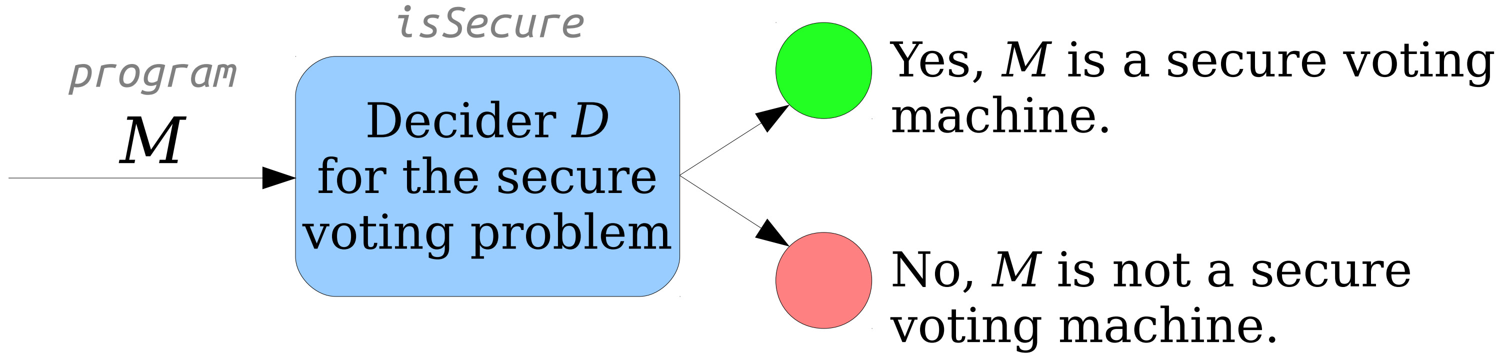
We can write programs that use *D* as a helper method

↓

Program *P* is secure if and only if program *P* is not secure.

Contradiction!




```
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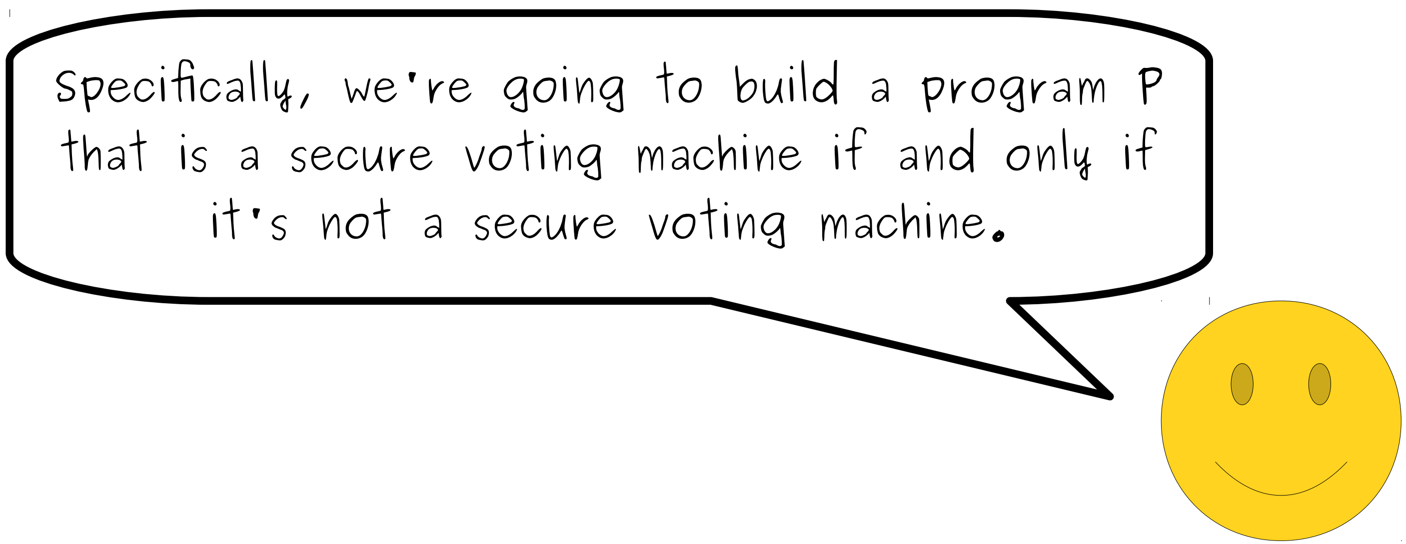

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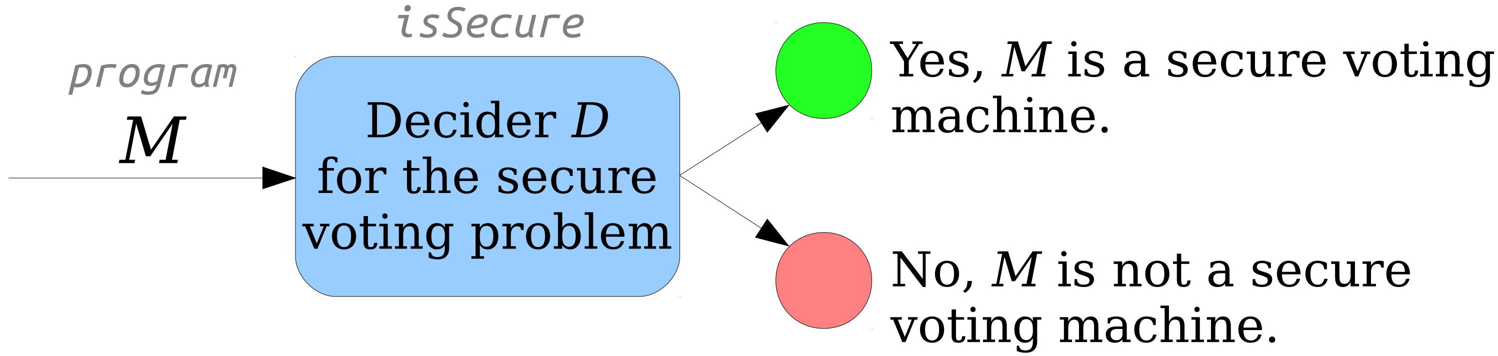


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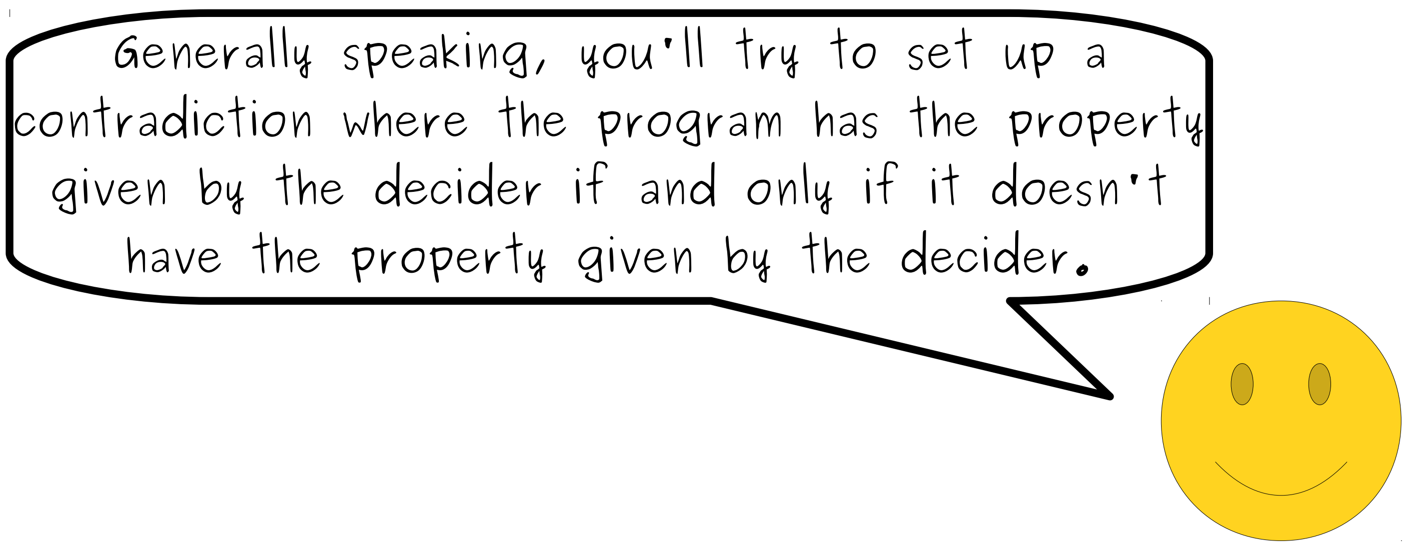

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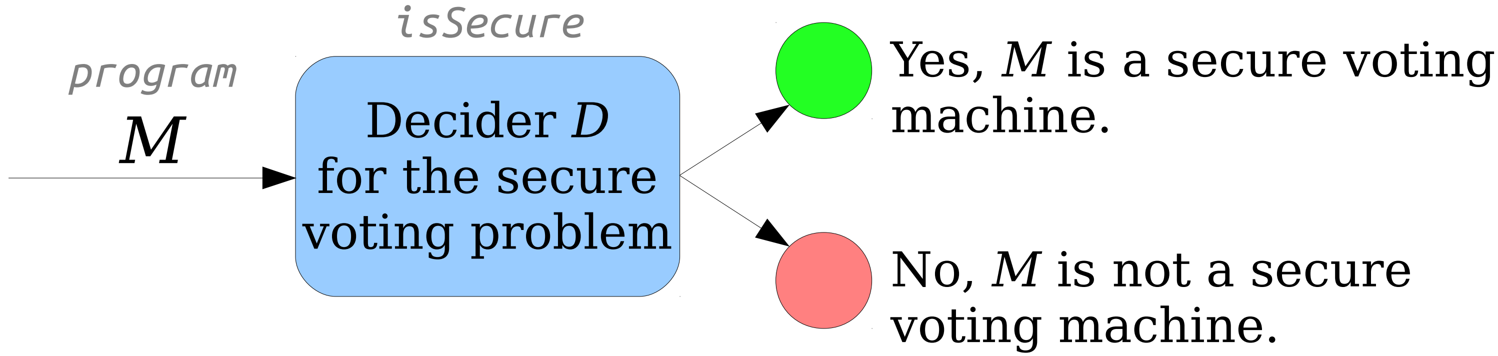


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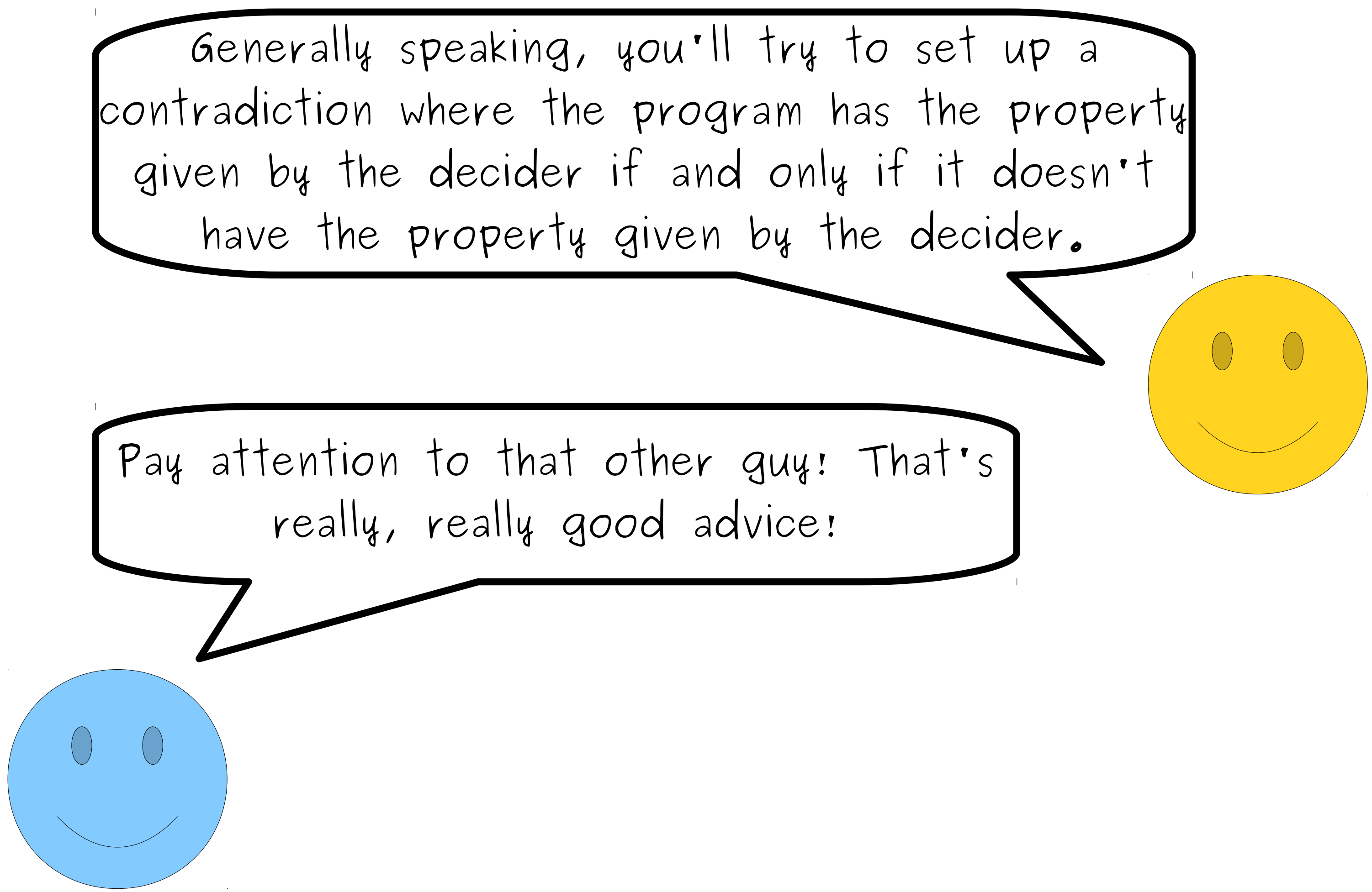

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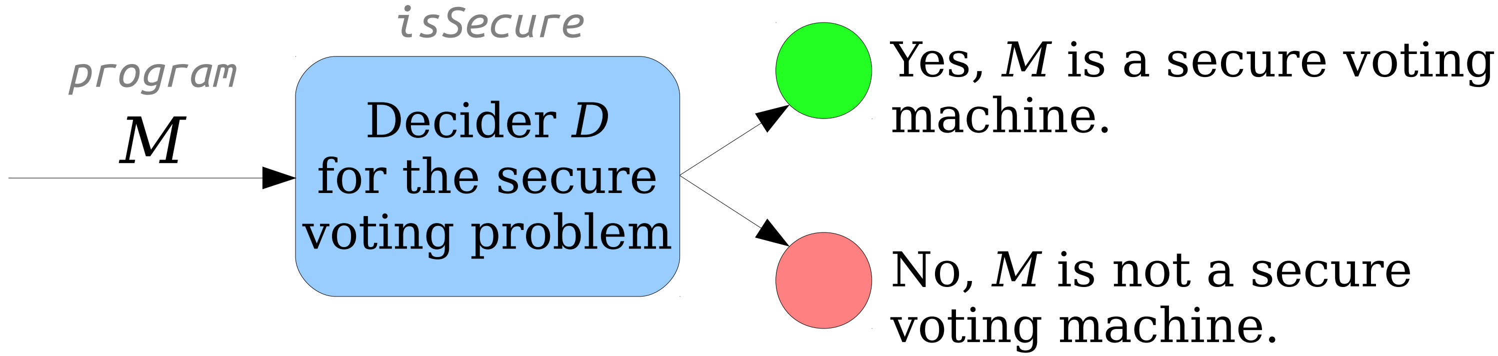


```
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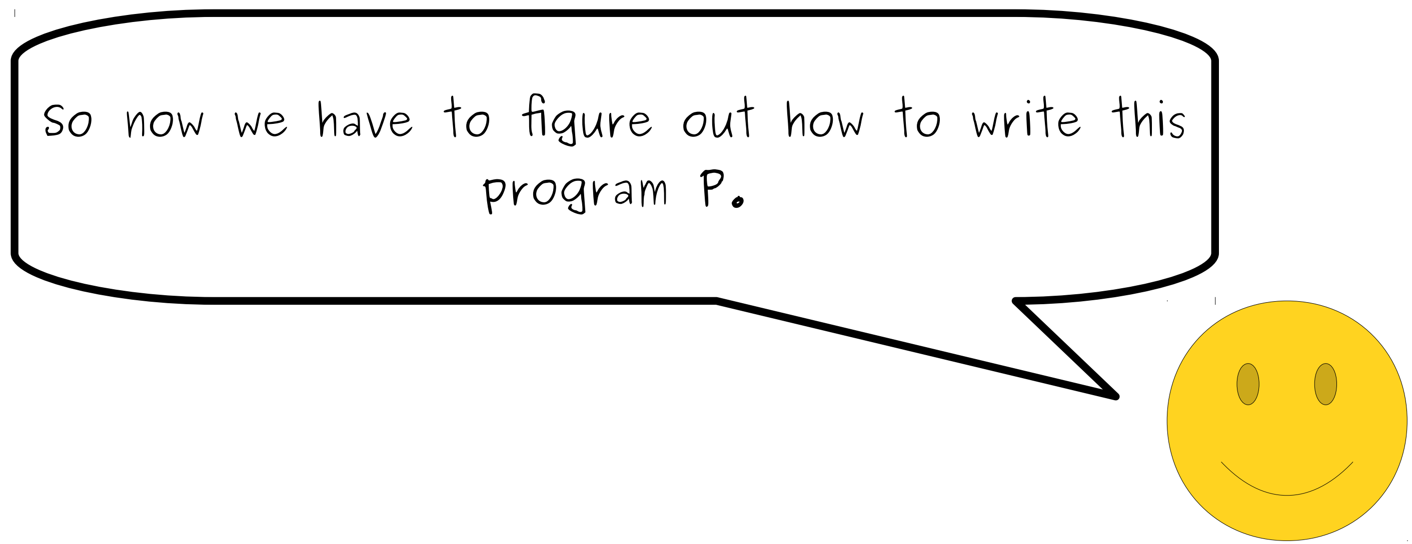

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There is a decider D for the secure voting problem

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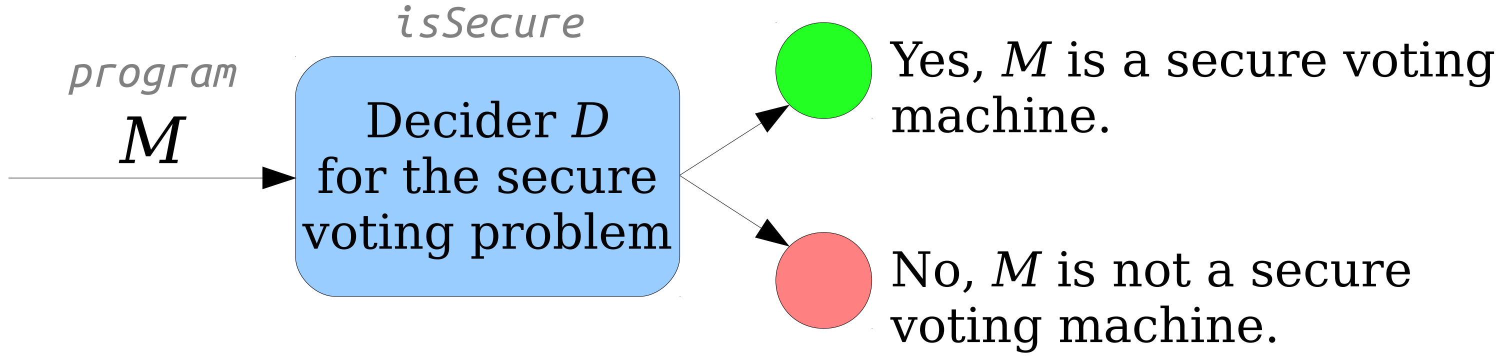
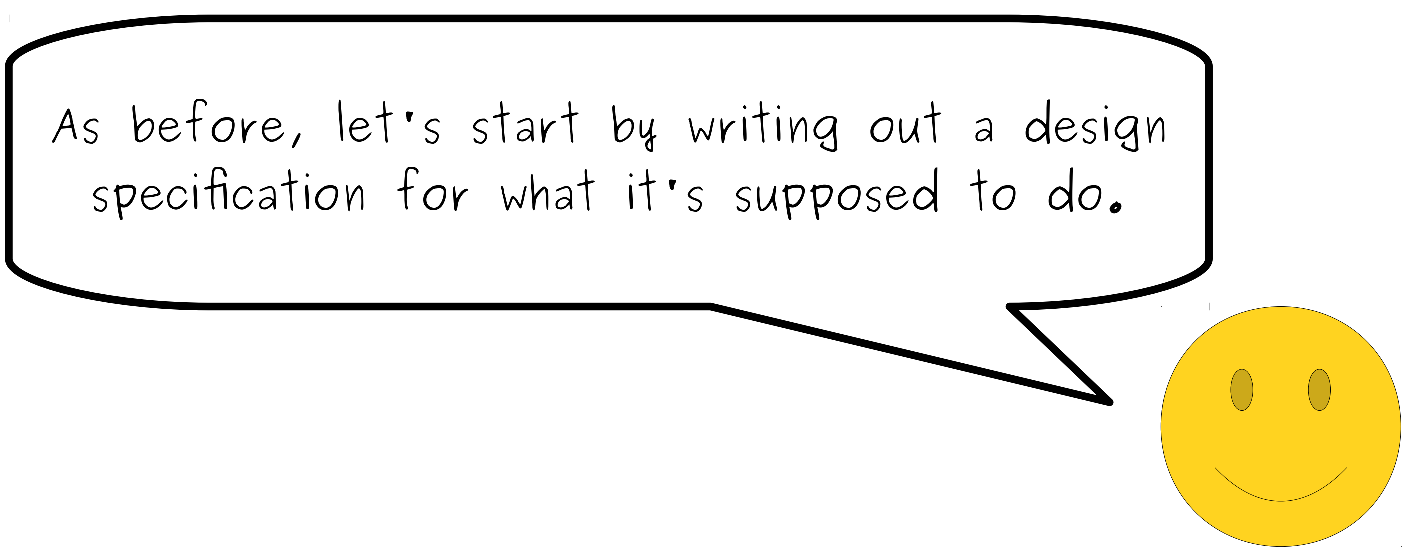
Program P is secure if and only if program P is not secure.

Contradiction!




```
bool isSecure(string program)
```

Program P design specification:



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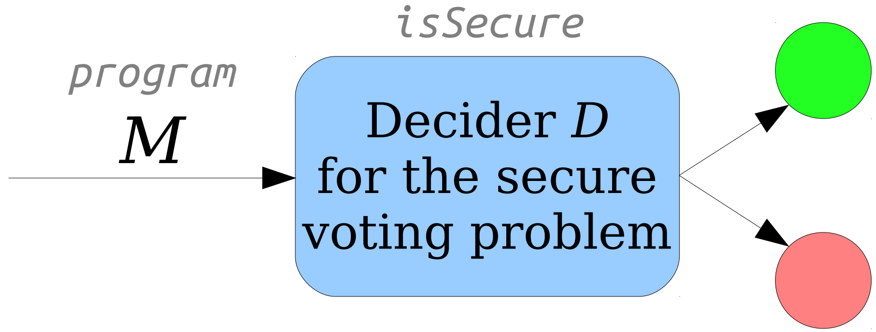


Yes, *M* is a secure voting machine.

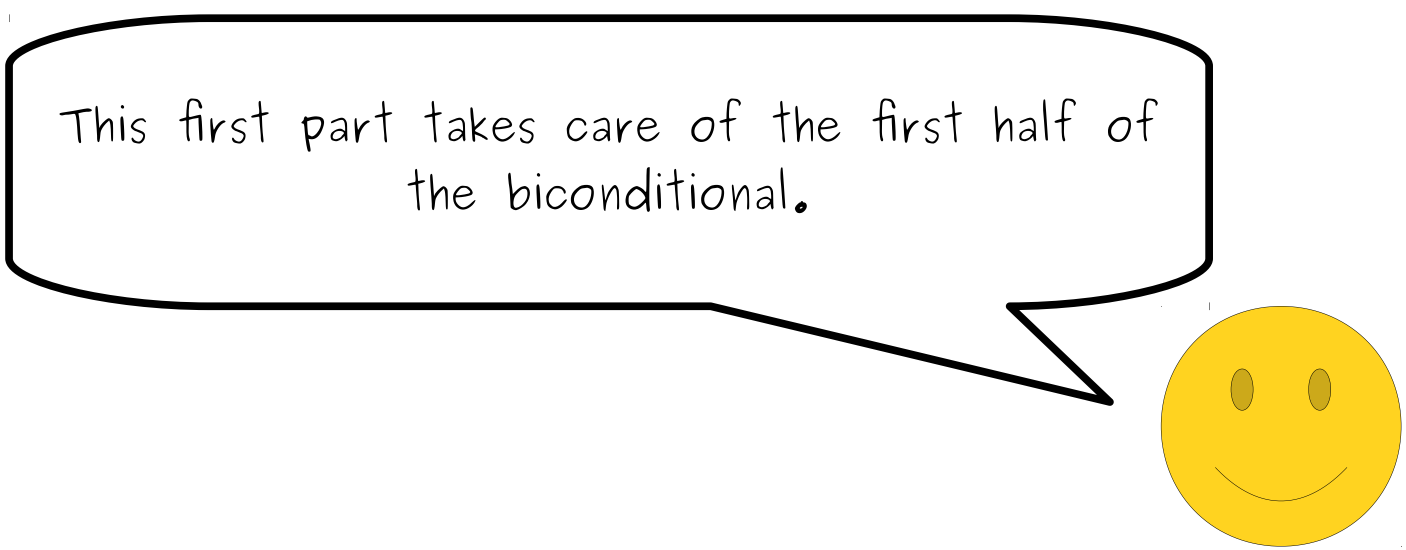
No, *M* is not a secure voting machine.

```
bool isSecure(string program)
```

Program *P* design specification:

If *P* is a secure voting machine, then

P is not a secure voting machine.



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Program *P* is secure if and only if program *P* is not secure.

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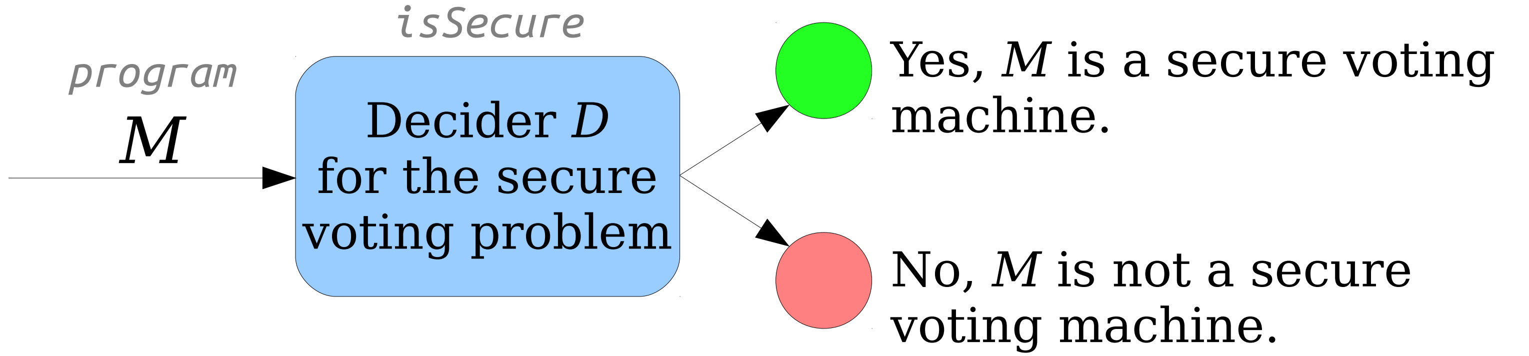


```
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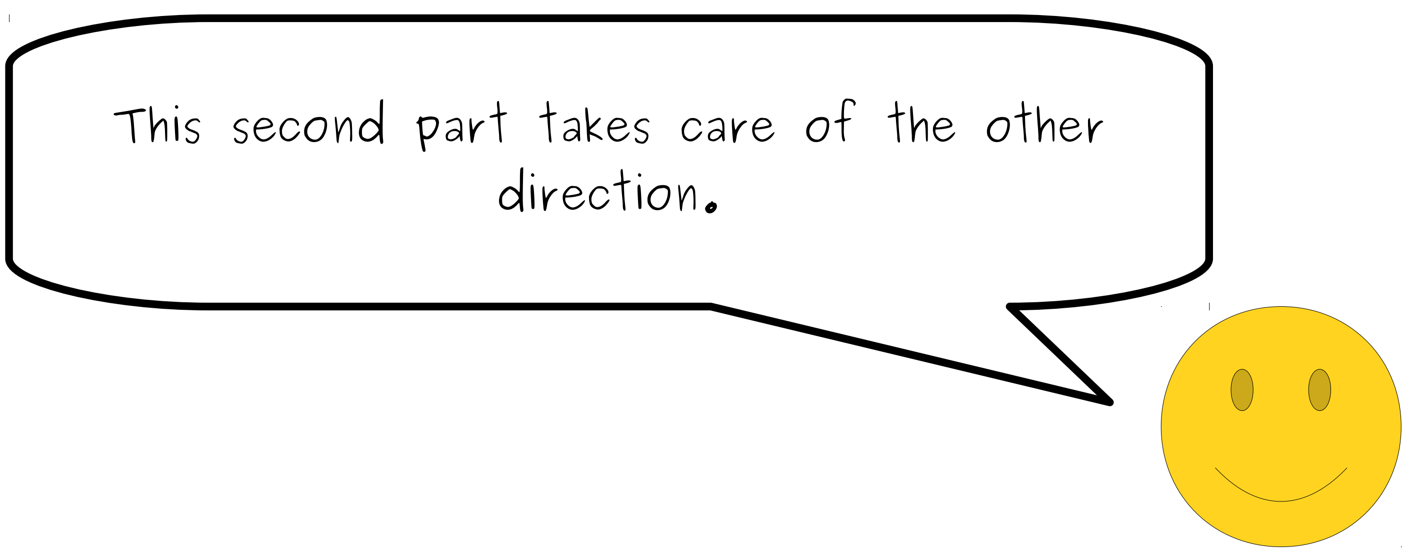

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There is a decider *D* for the secure voting problem

We can write programs that use *D* as a helper method

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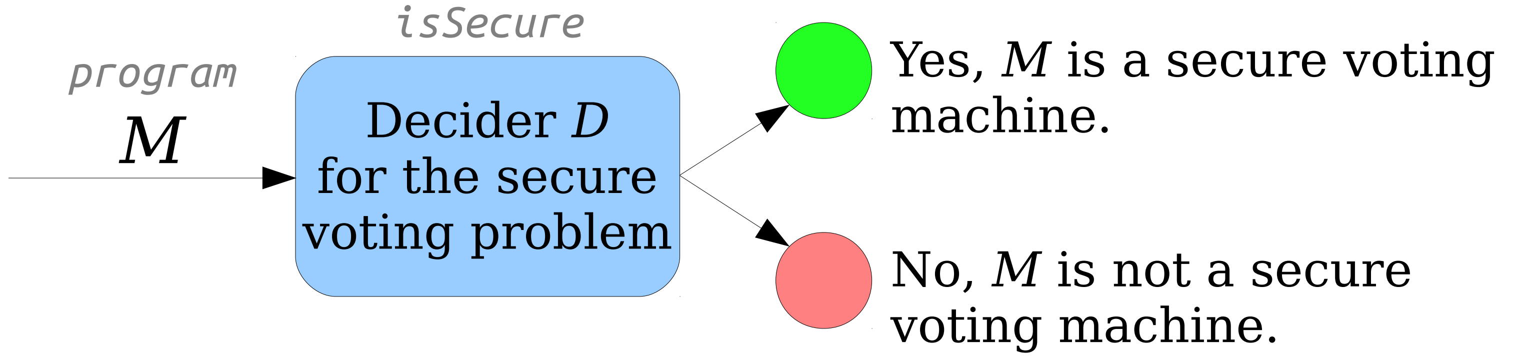


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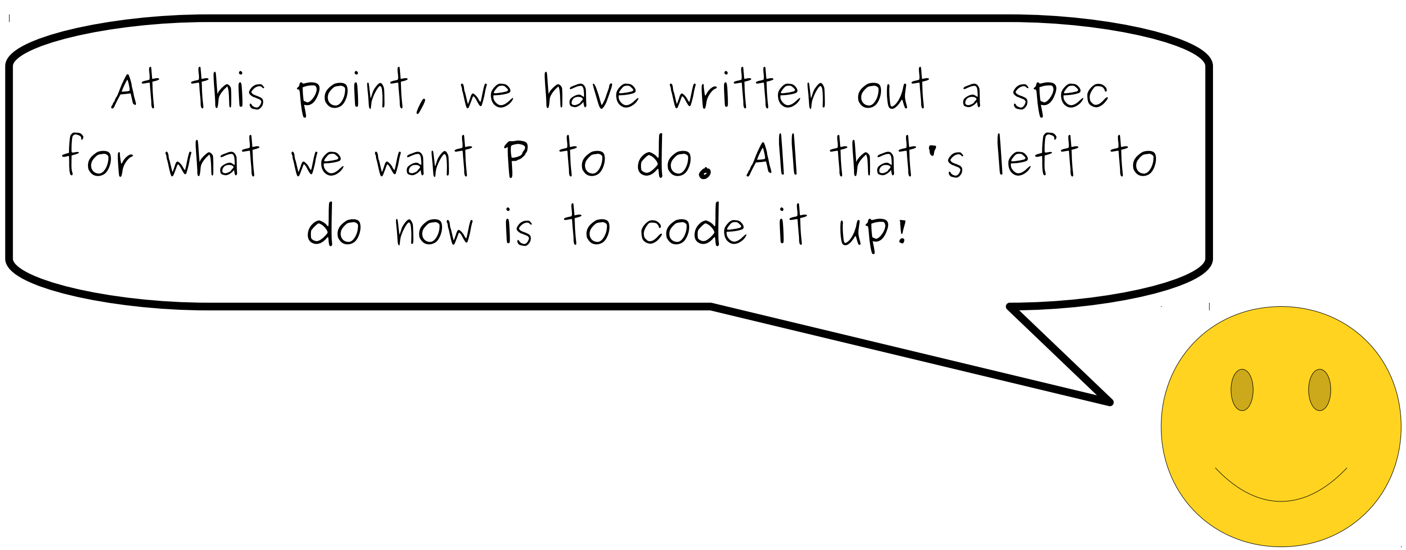

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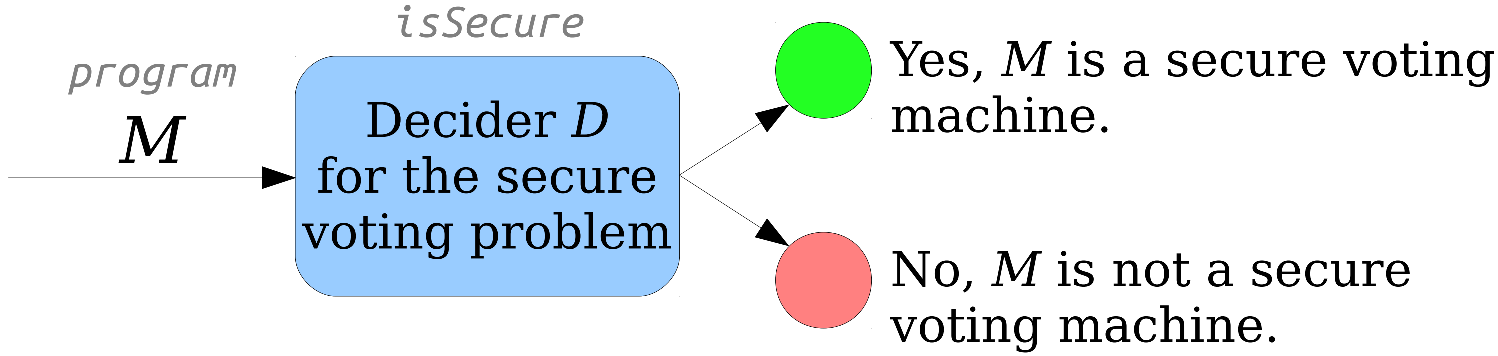


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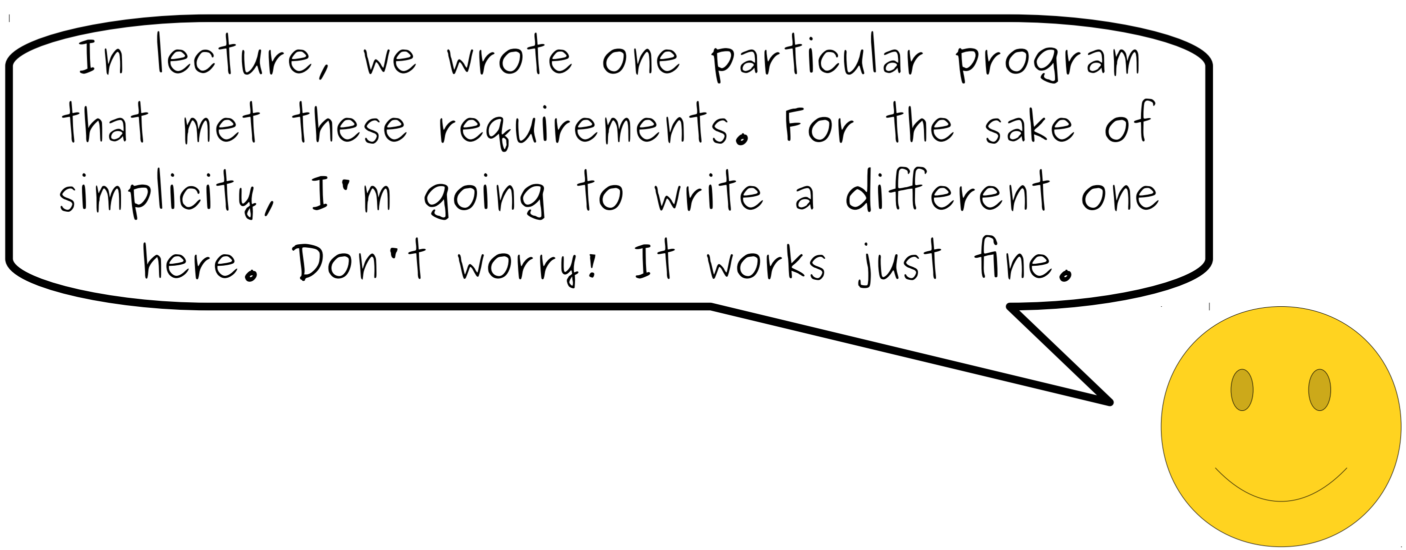

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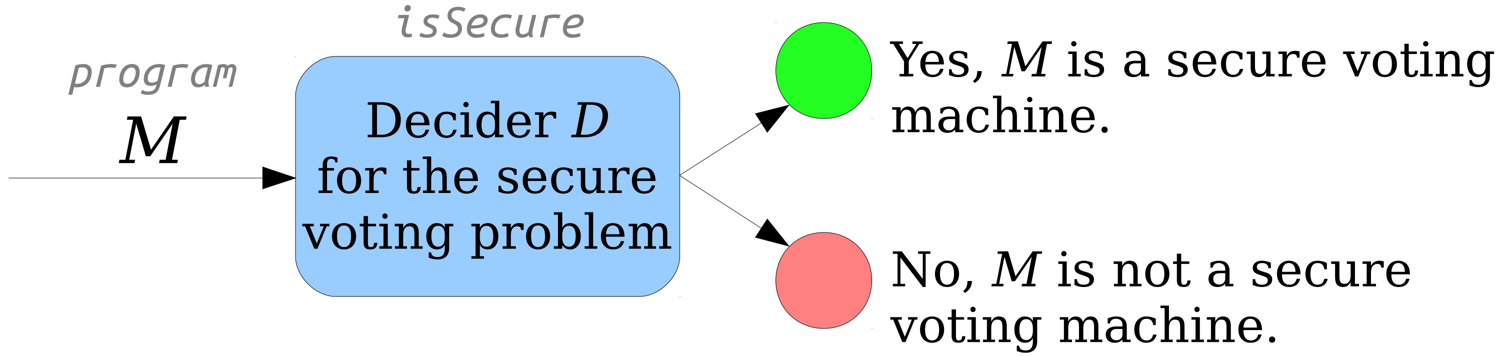


```
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Program *P* design specification:

If *P* is a secure voting machine, then
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If *P* is not a secure voting machine, then
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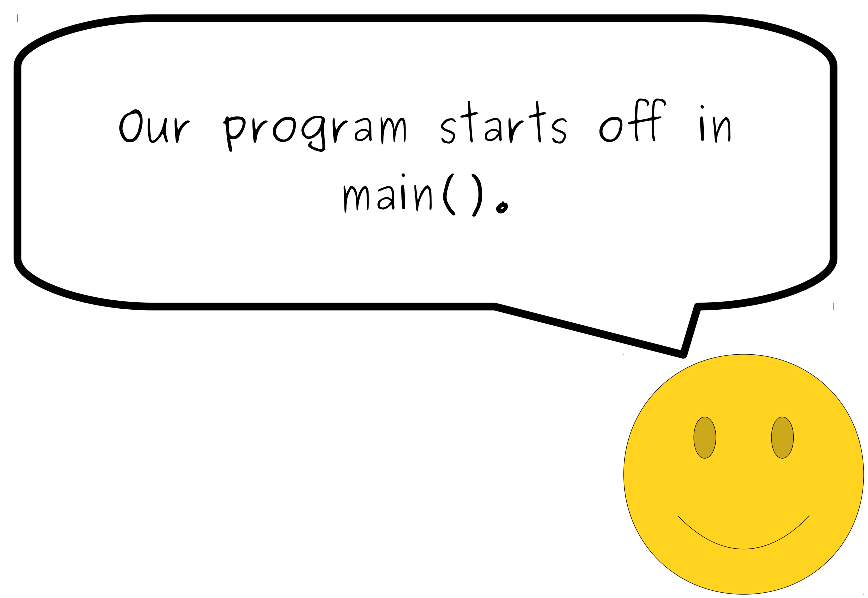
```
// Program P

int main() {



}
```



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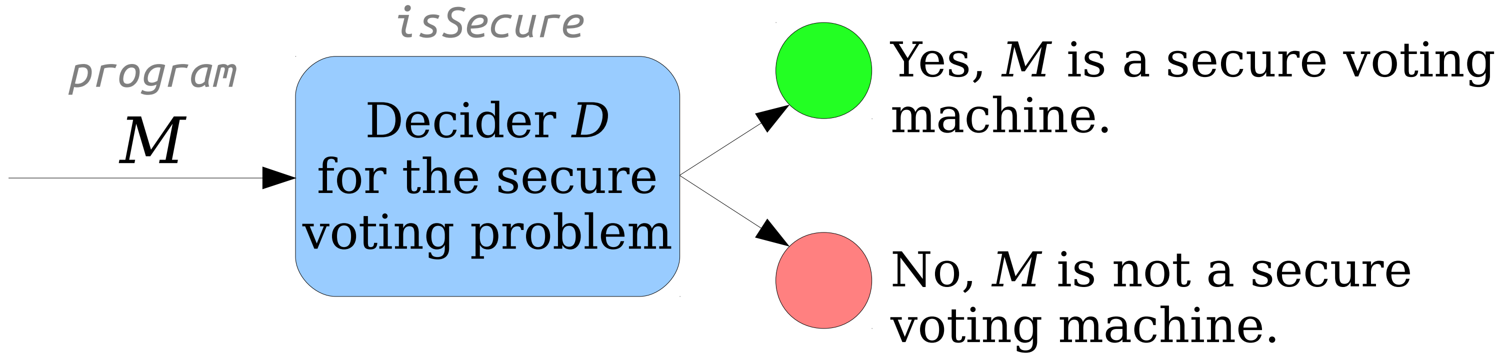


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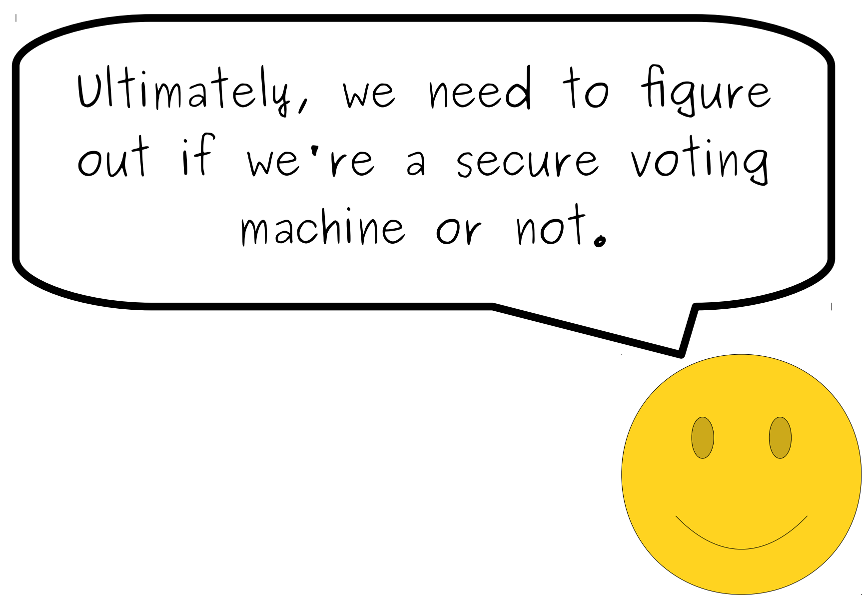

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Program *P* is secure if and only if program *P* is not secure.

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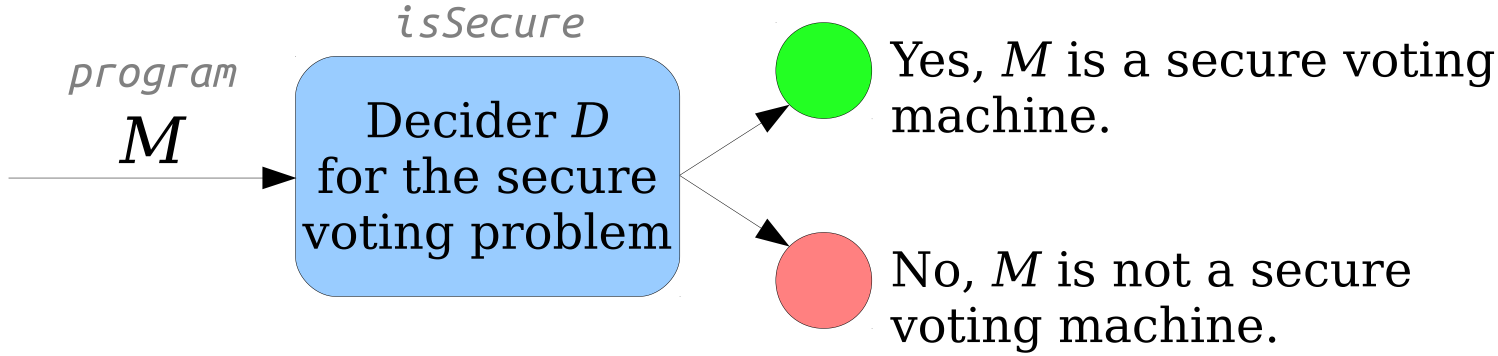


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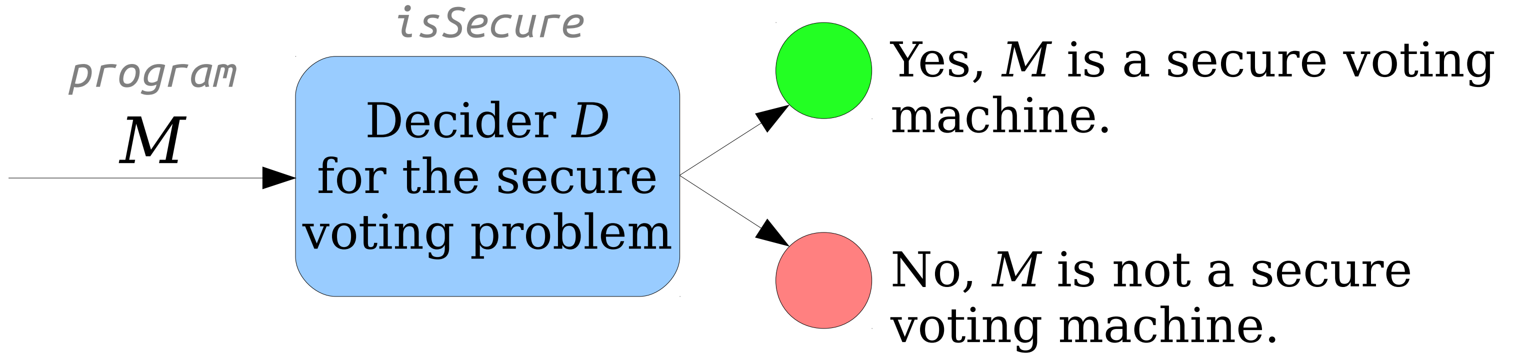


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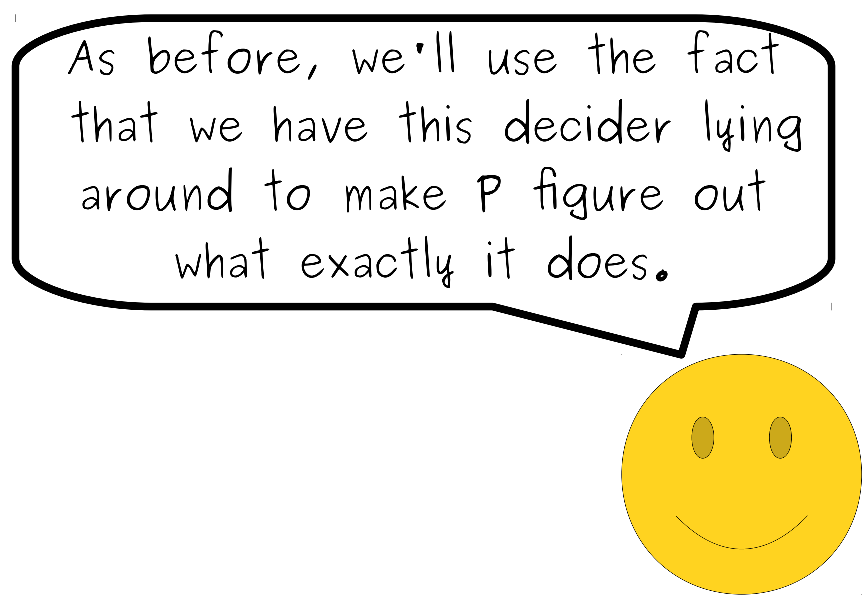

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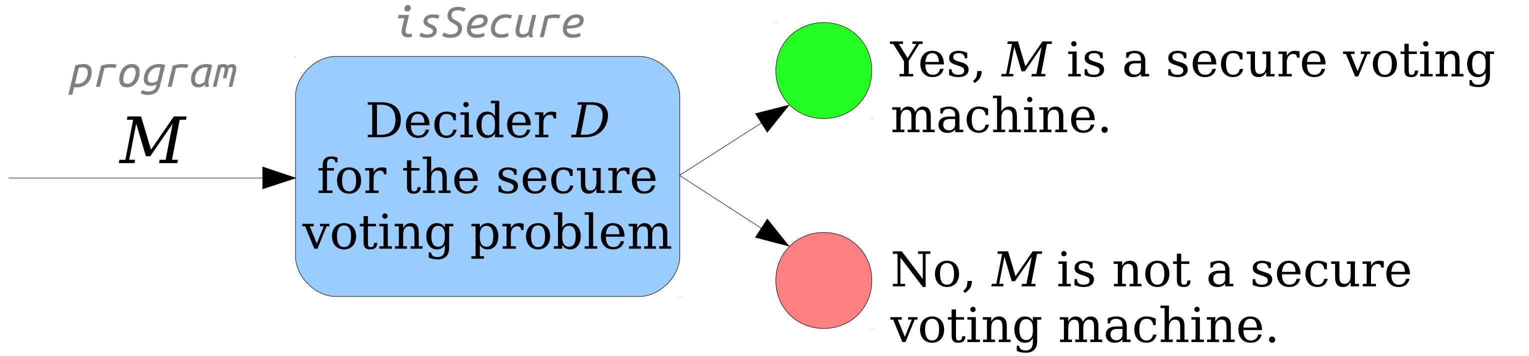


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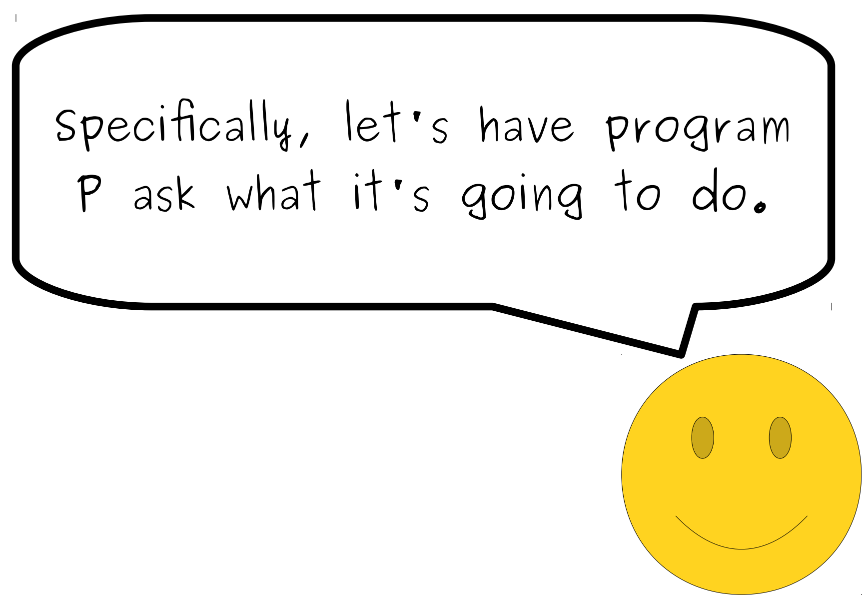

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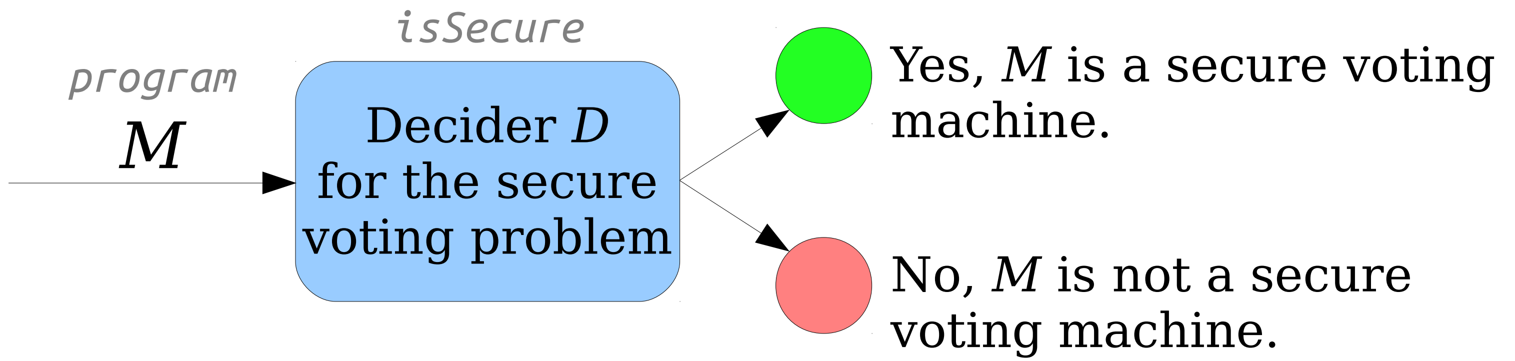


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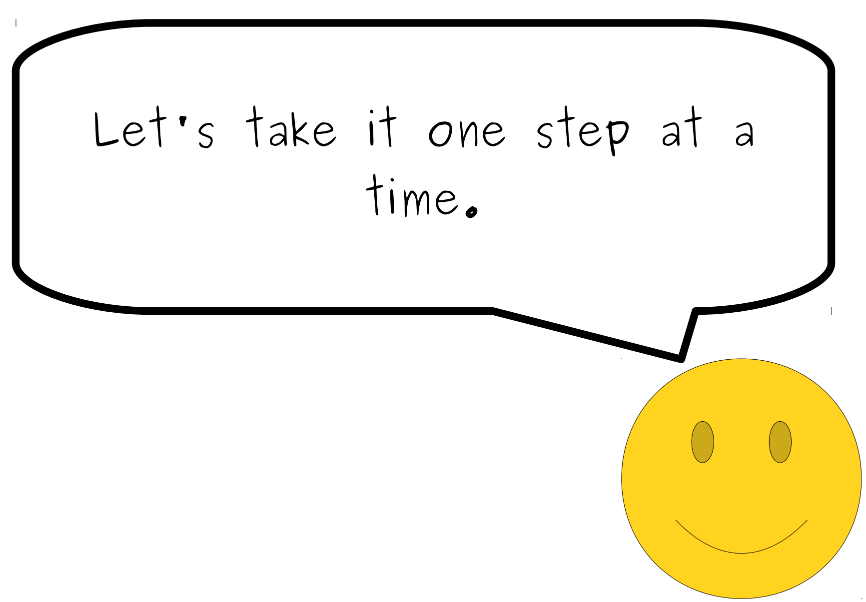

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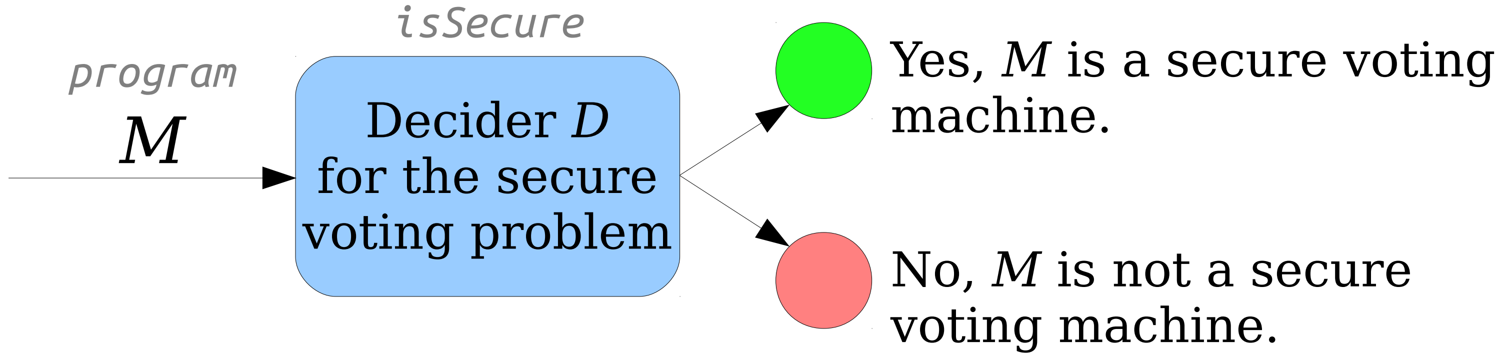


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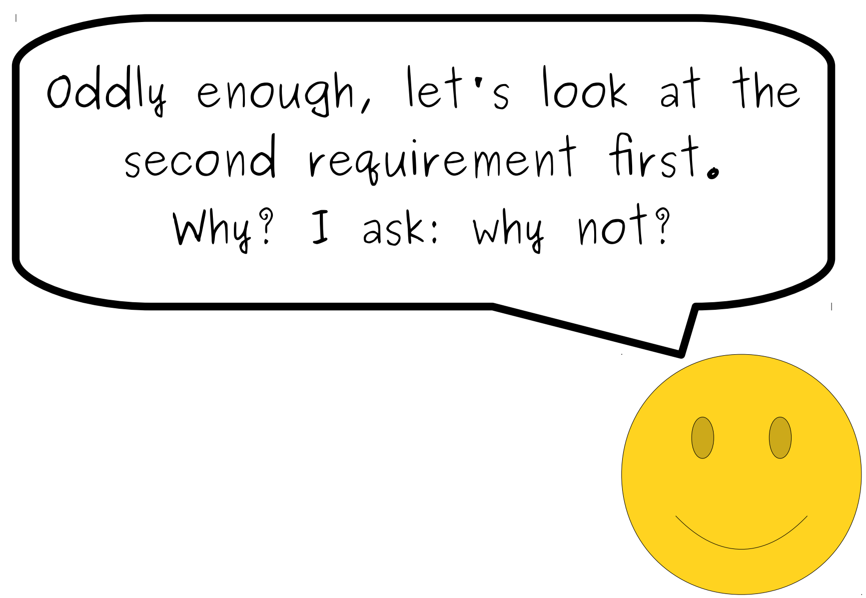

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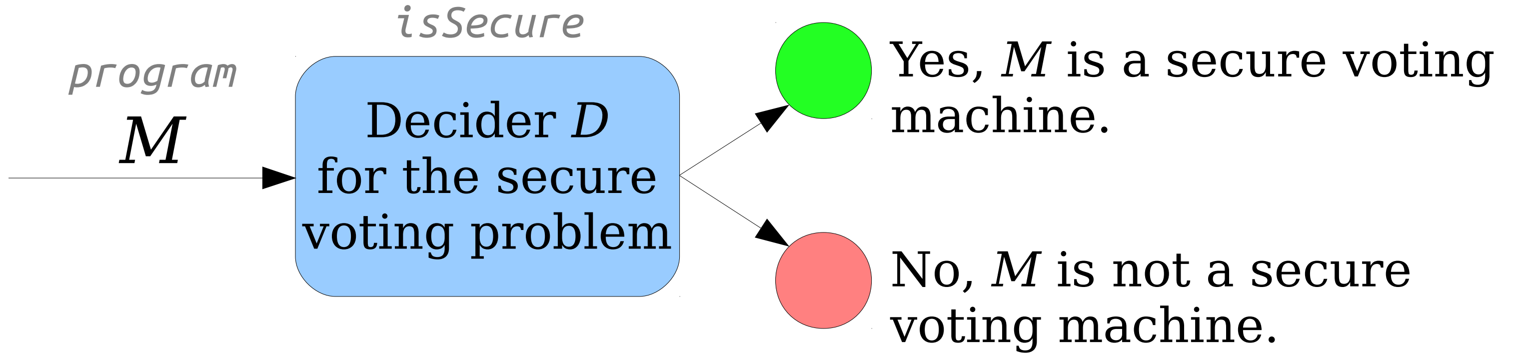


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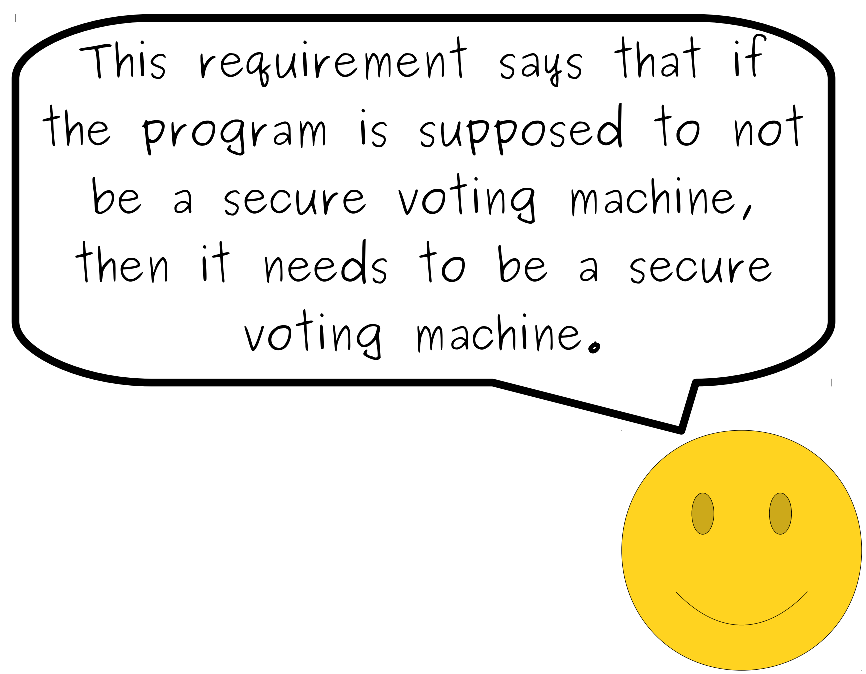

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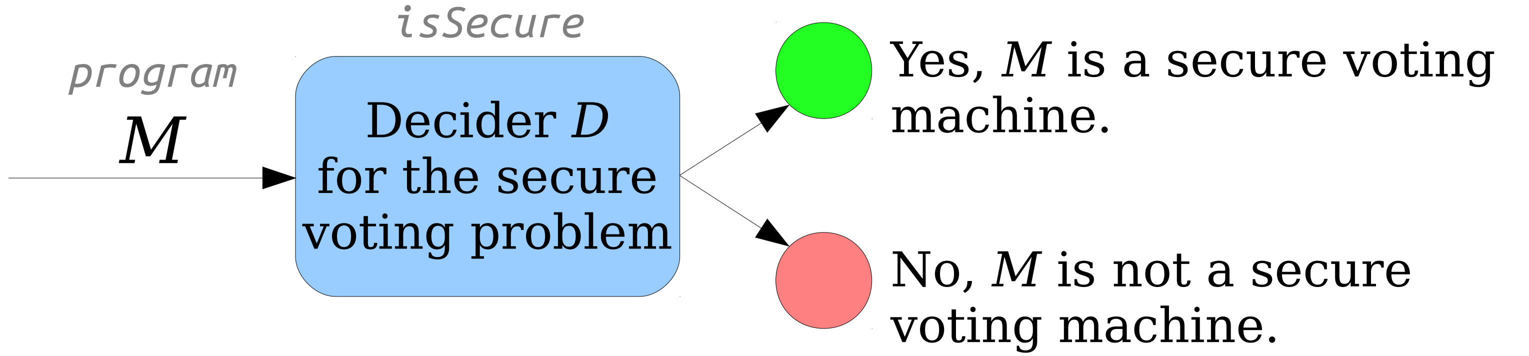


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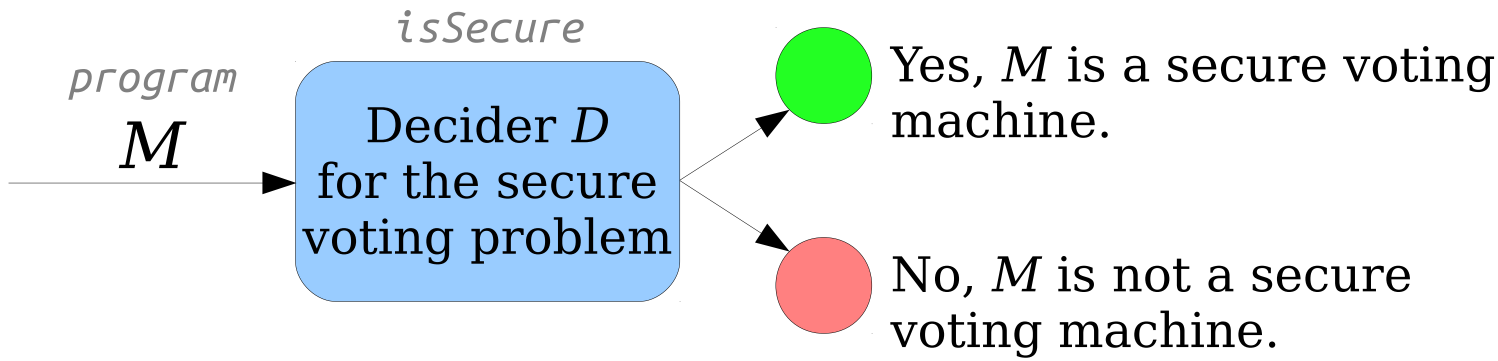


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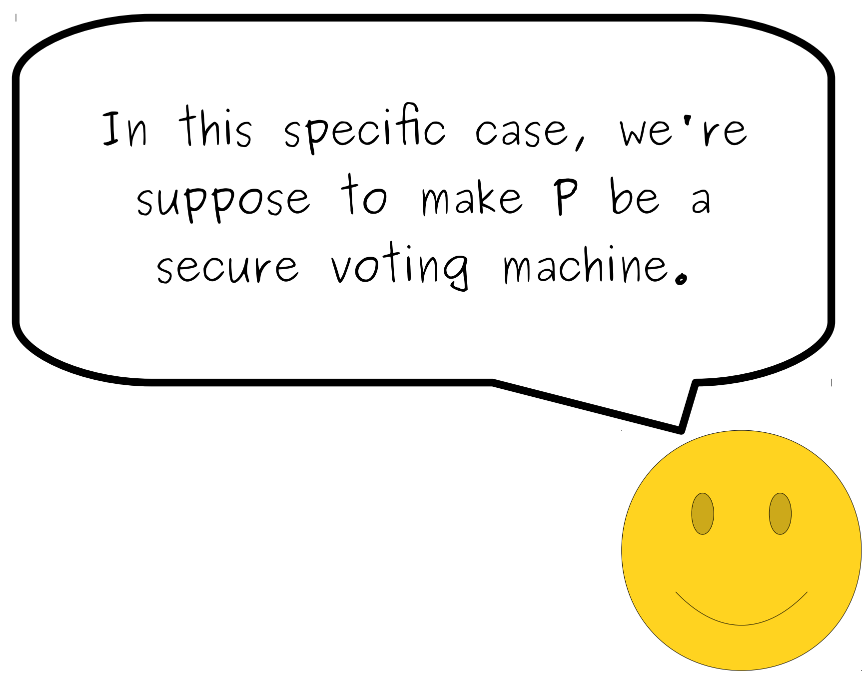

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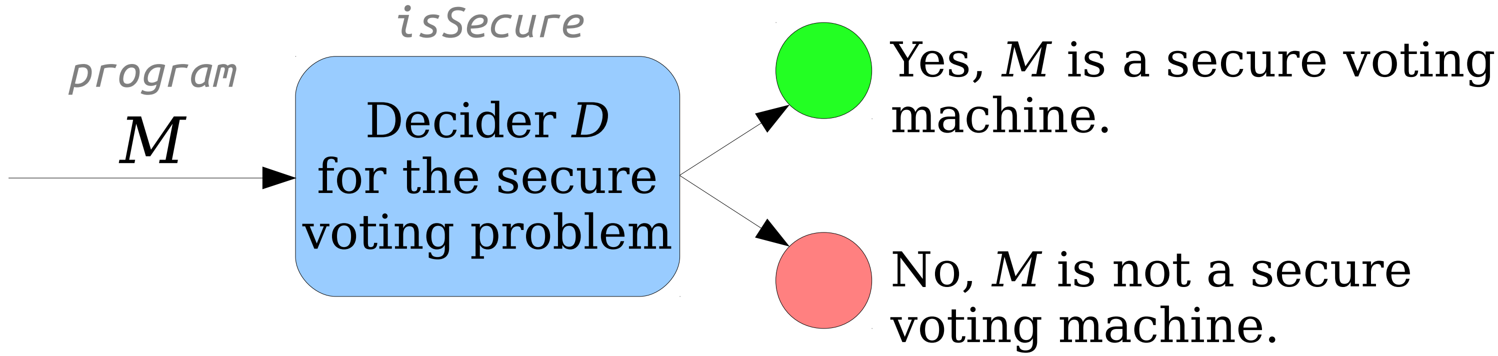


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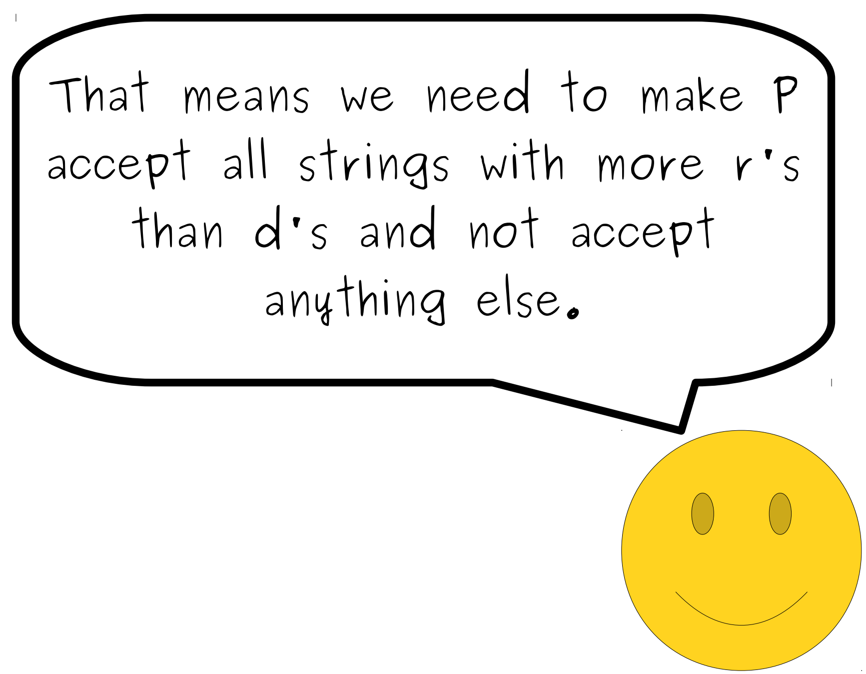

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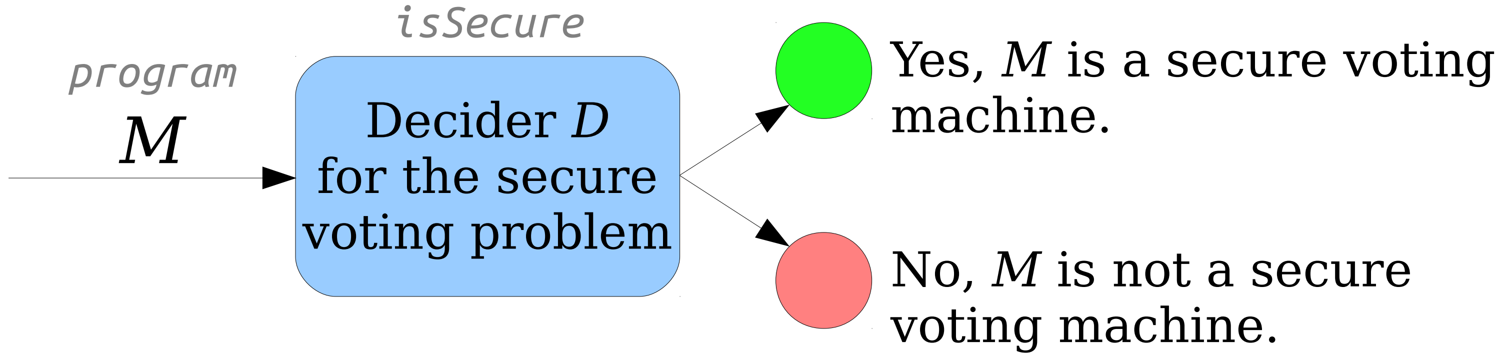


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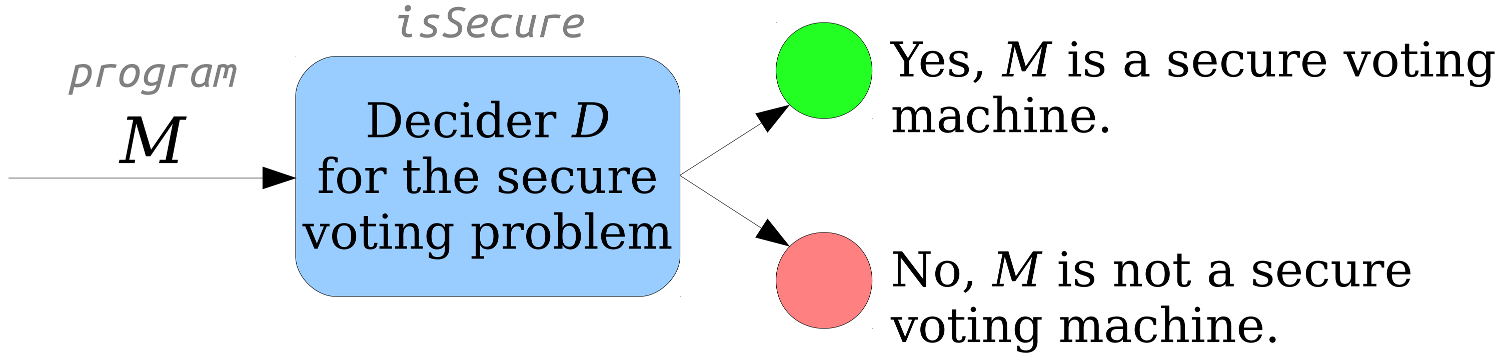


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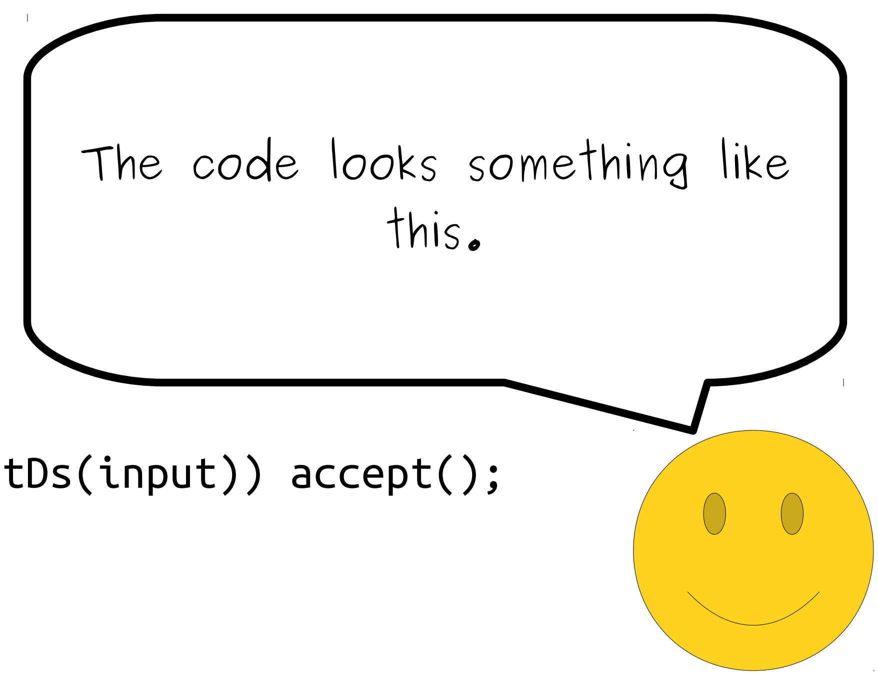

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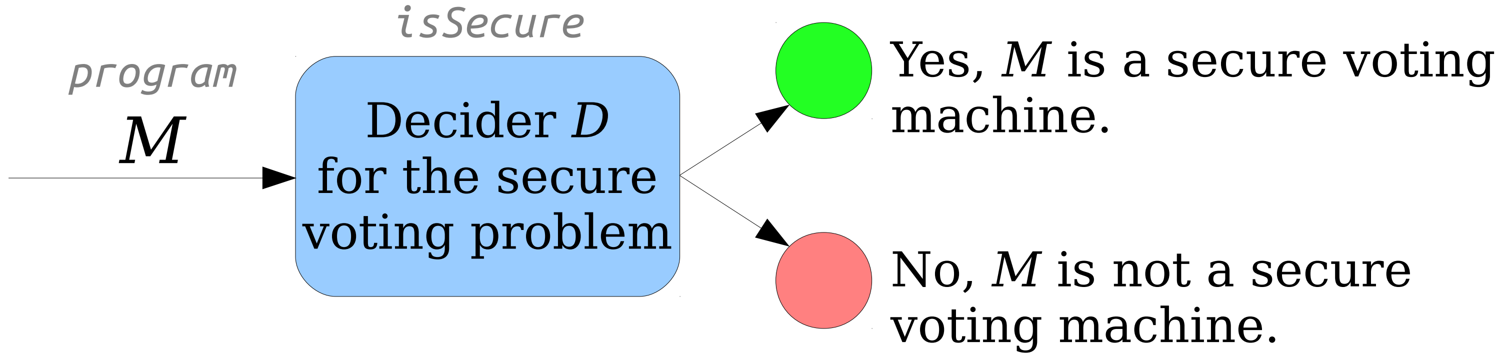


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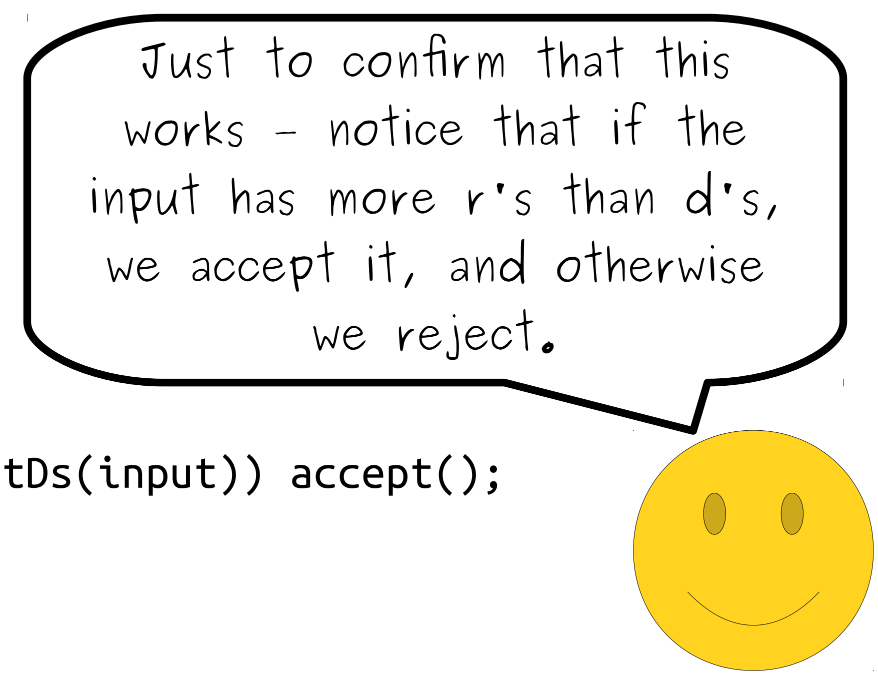

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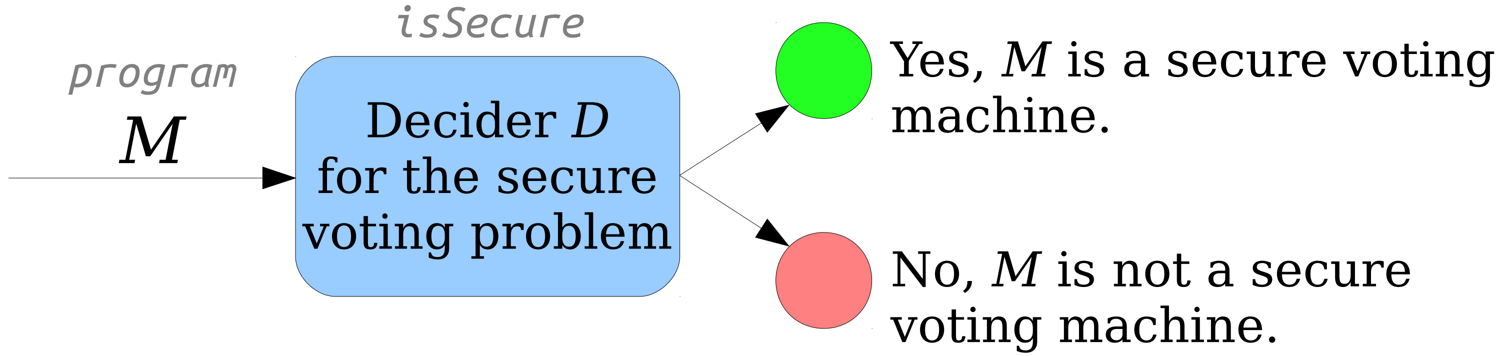


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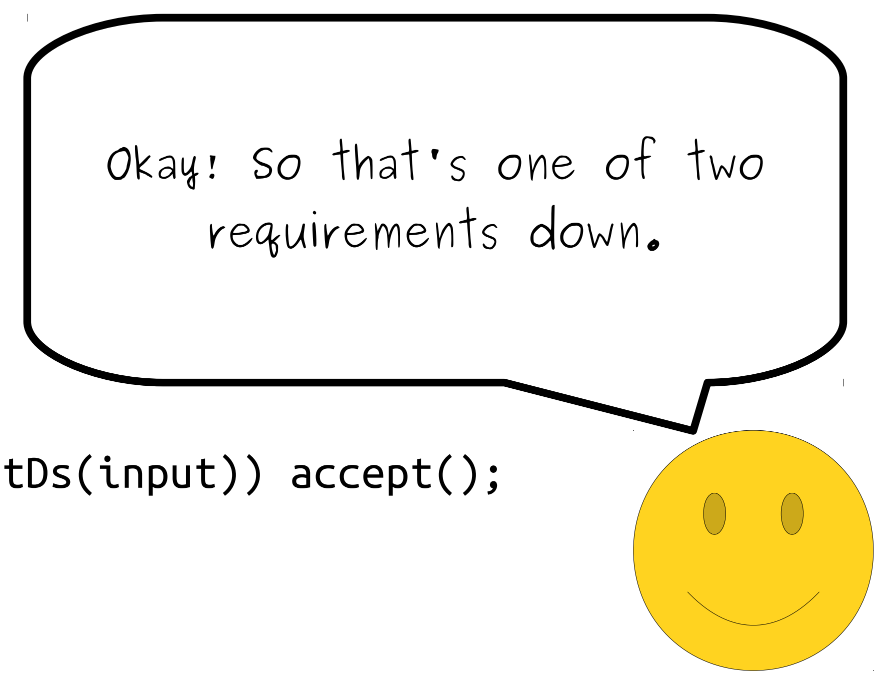

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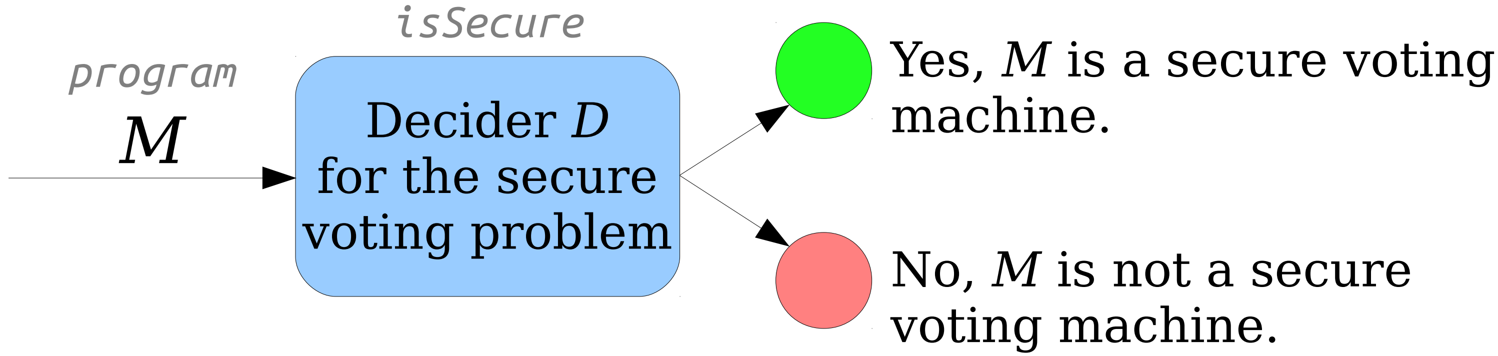


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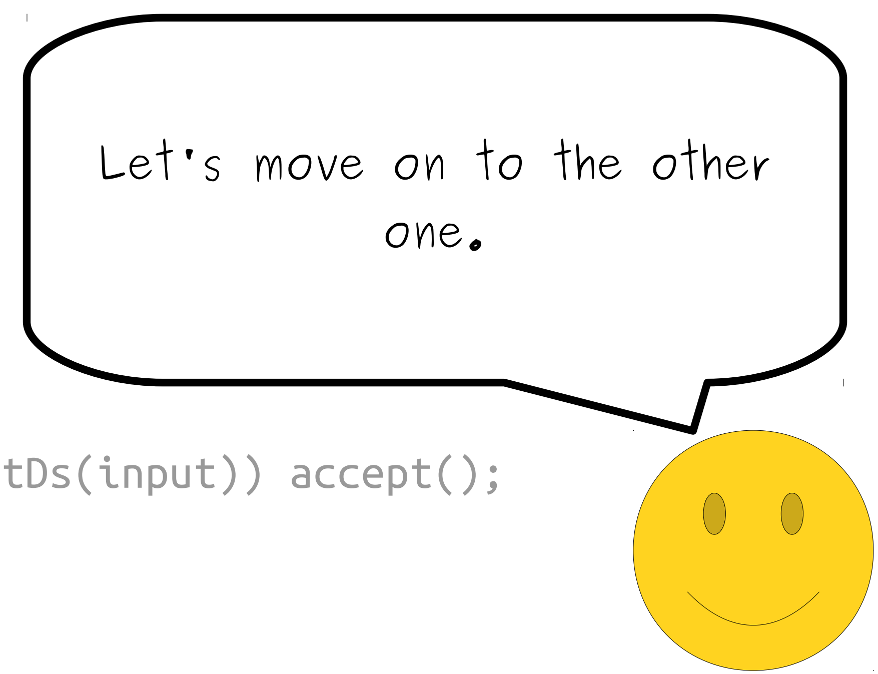

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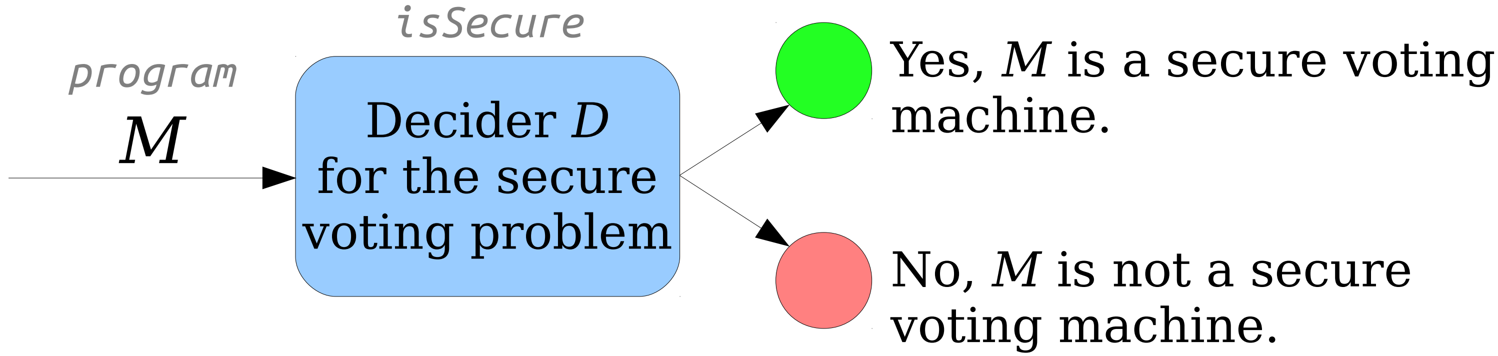


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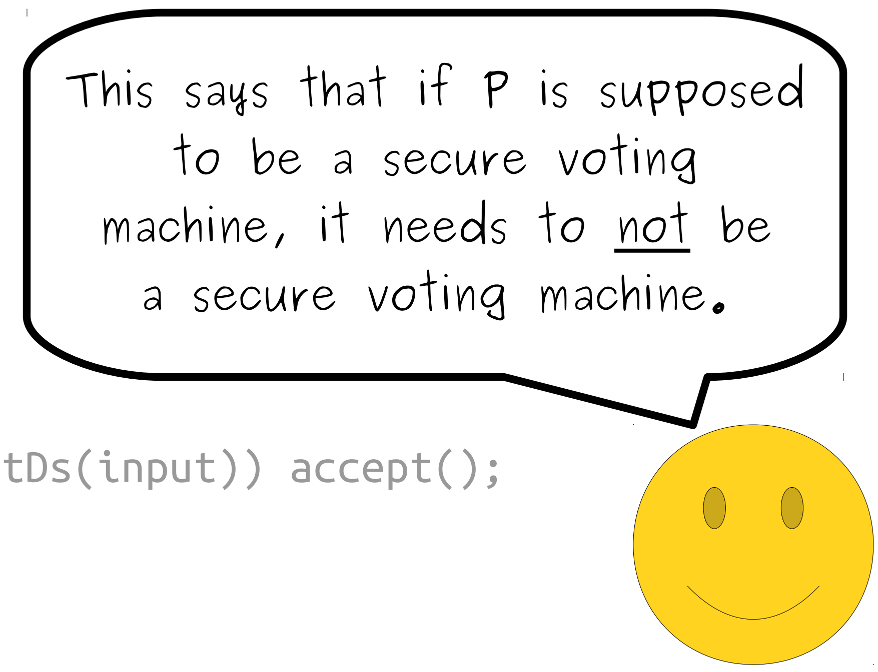

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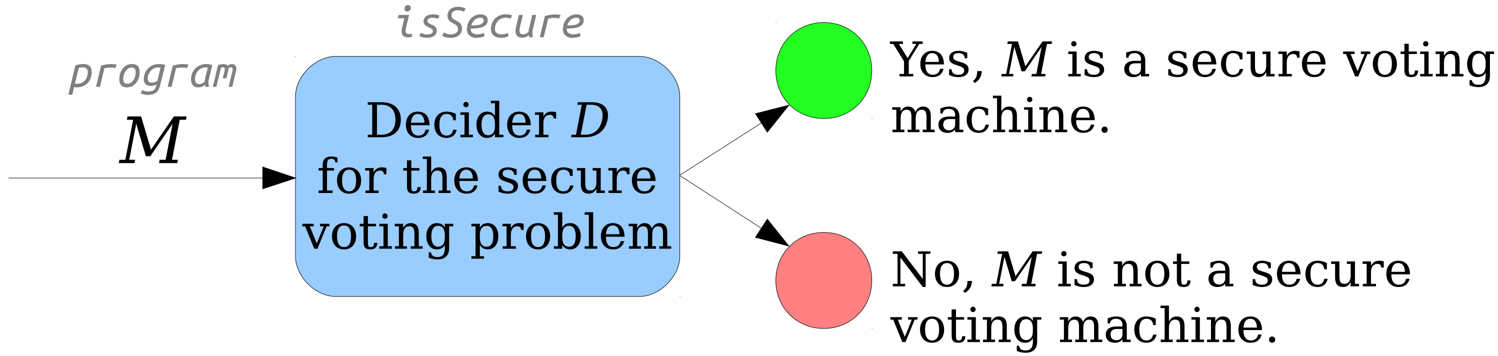


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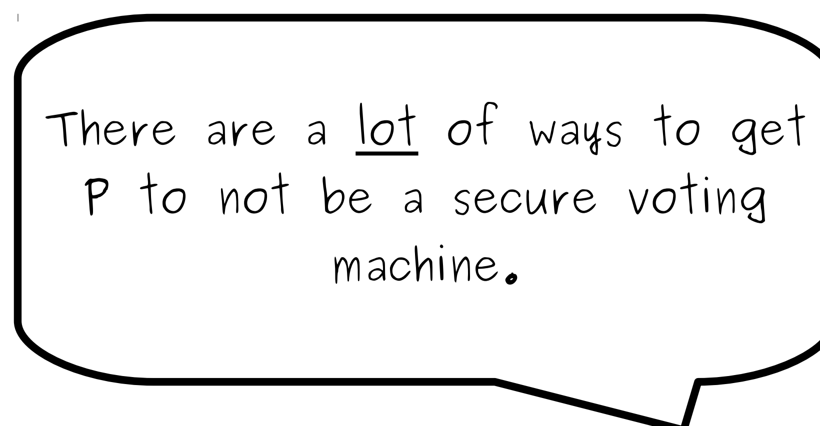

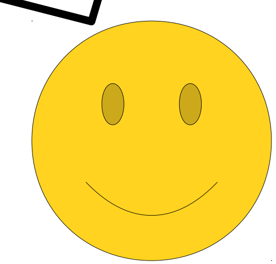

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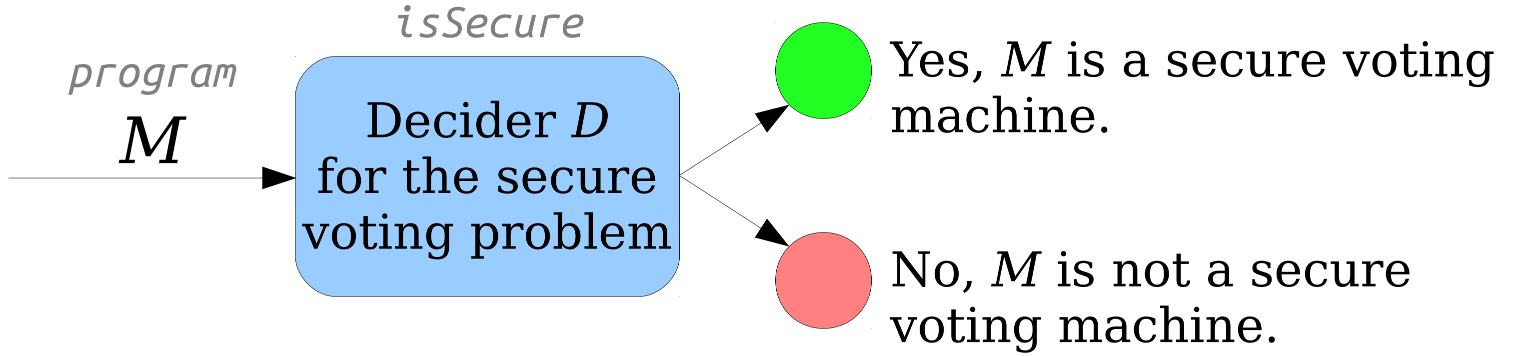


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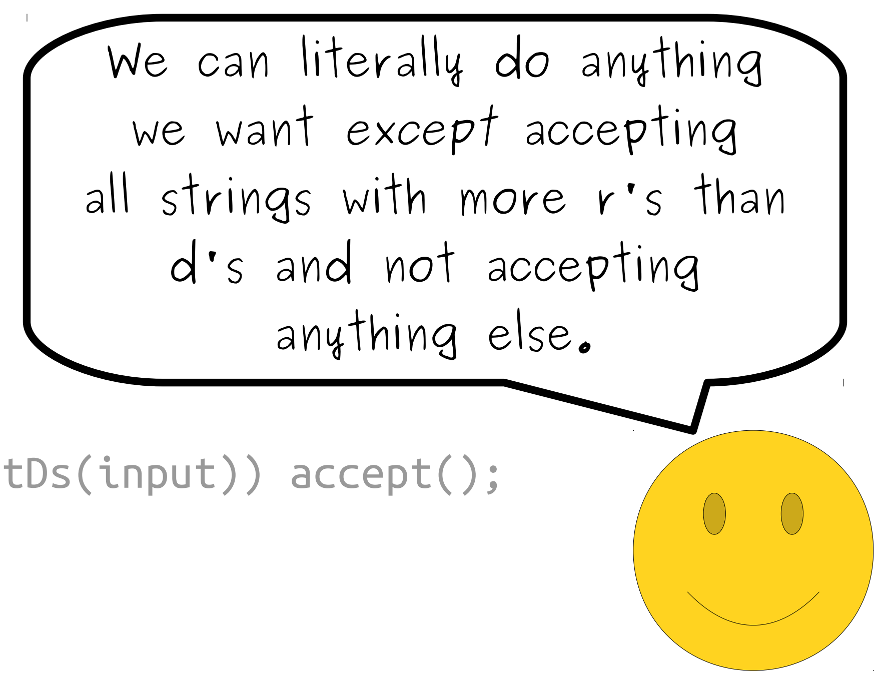

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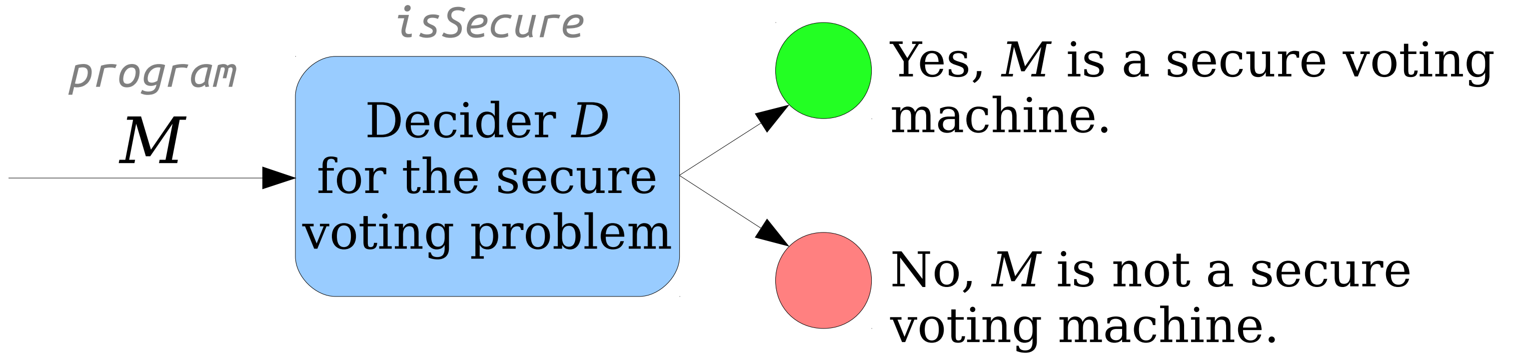

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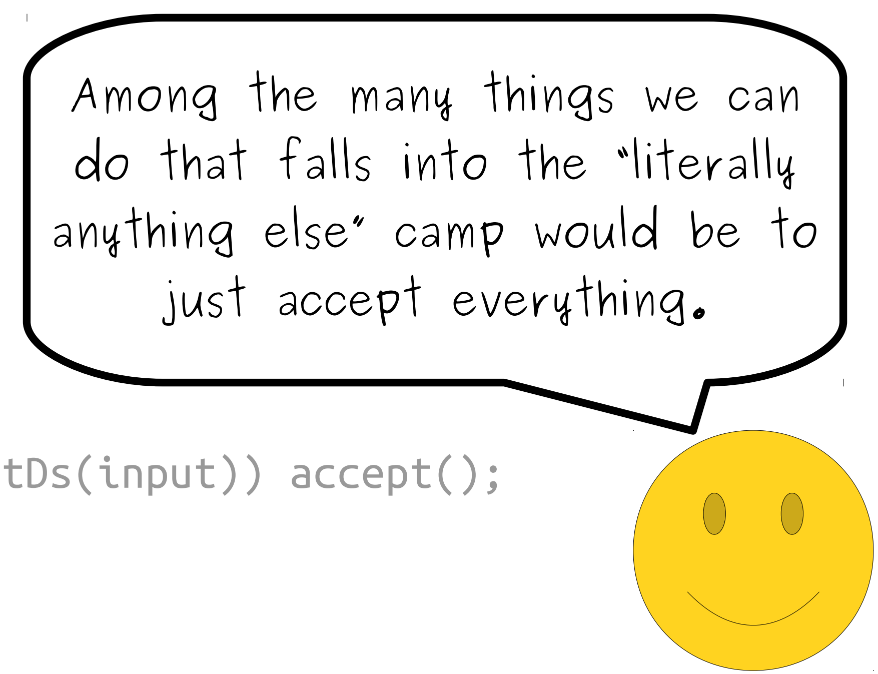

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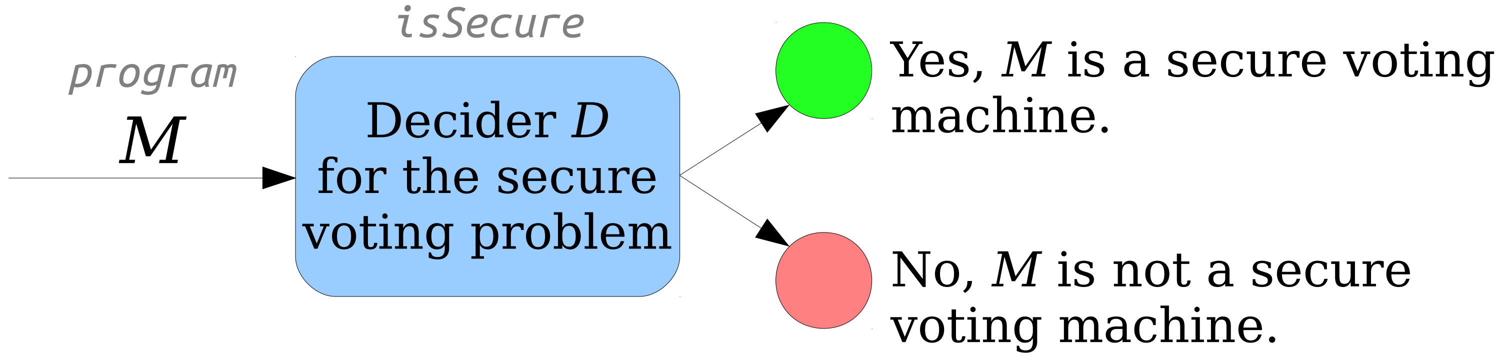


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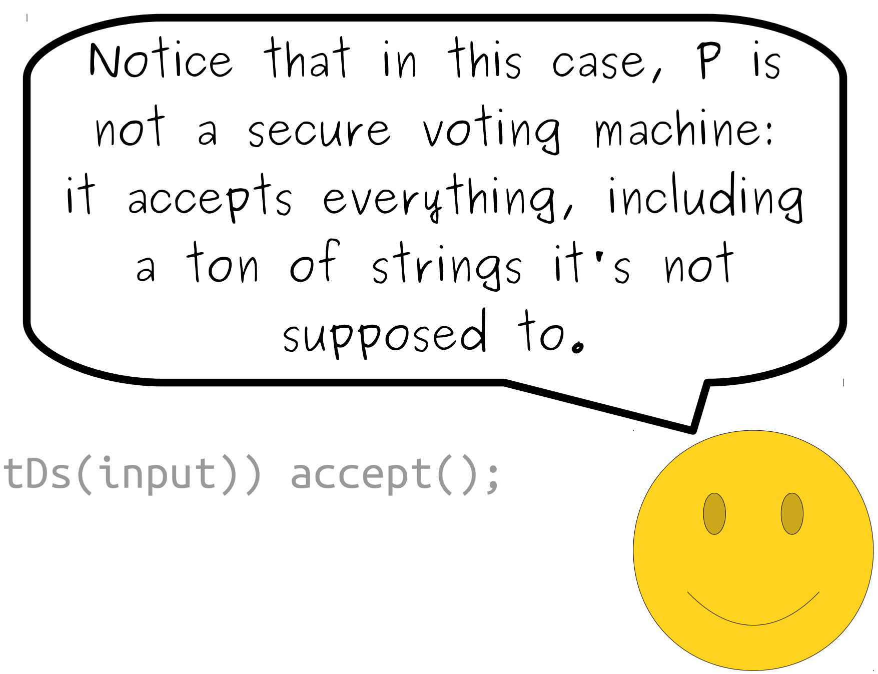

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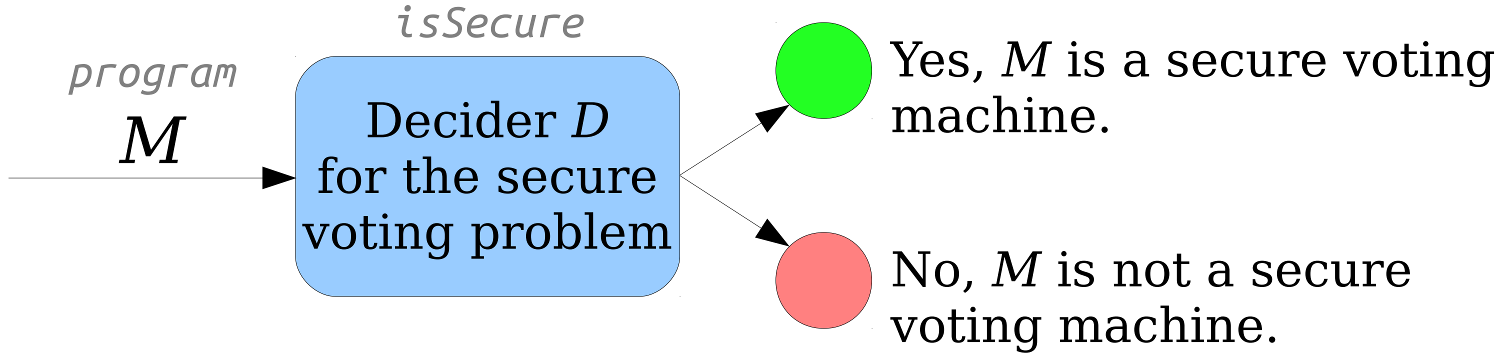


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- ✓ If *P* is a secure voting machine, then *P* is not a secure voting machine.
- ✓ If *P* is not a secure voting machine, then *P* is a secure voting machine.

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// Program P

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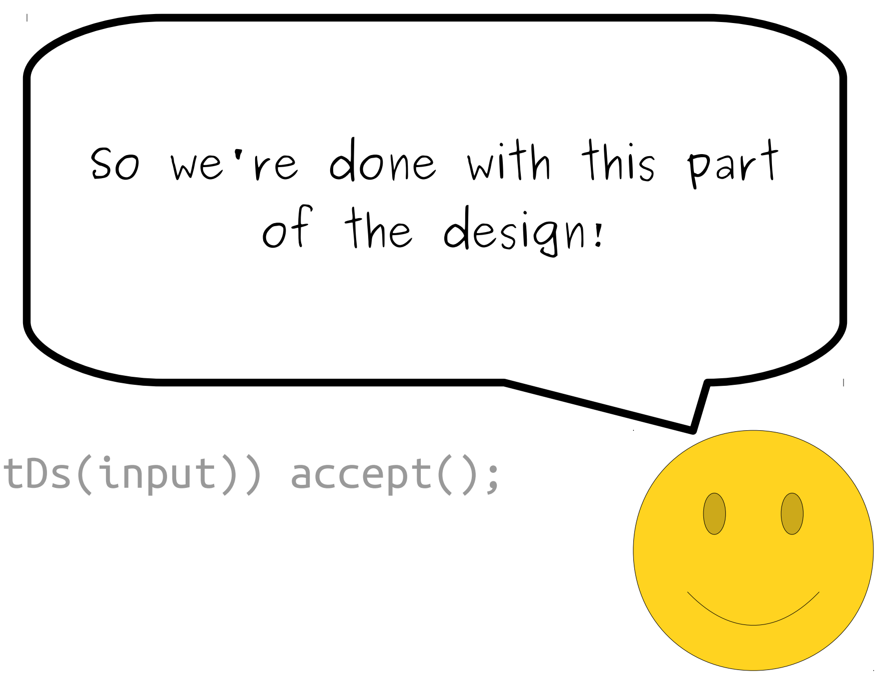


The secure voting problem is decidable.

↓

There is a decider *D* for the secure voting problem

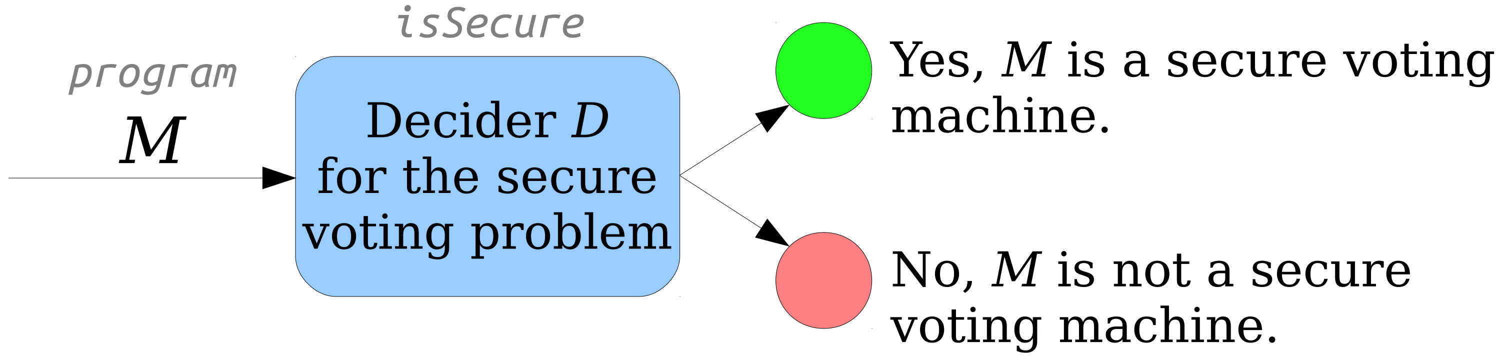
↓

We can write programs that use *D* as a helper method

↓

Program *P* is secure if and only if program *P* is not secure.

Contradiction!

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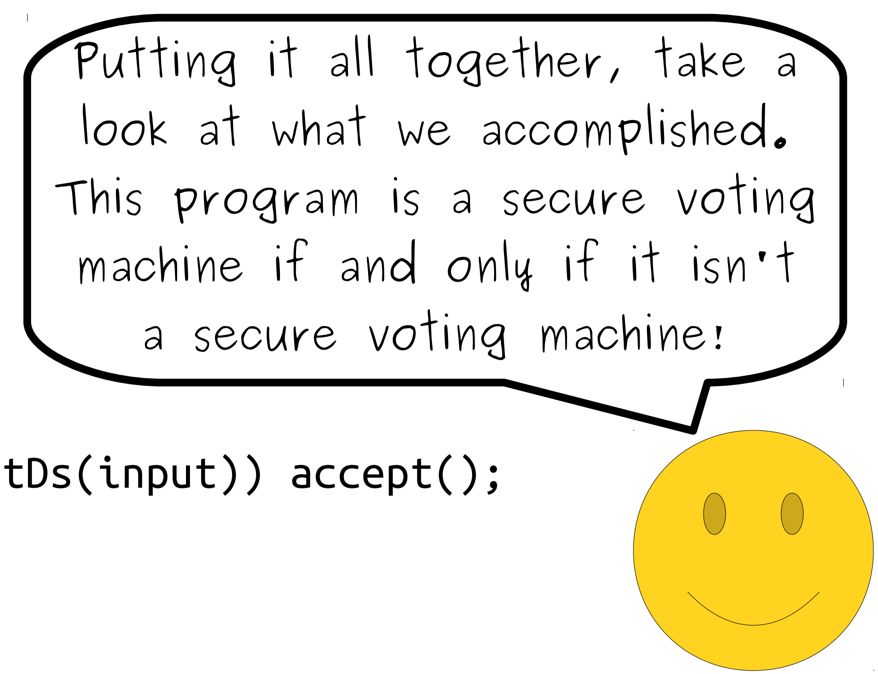

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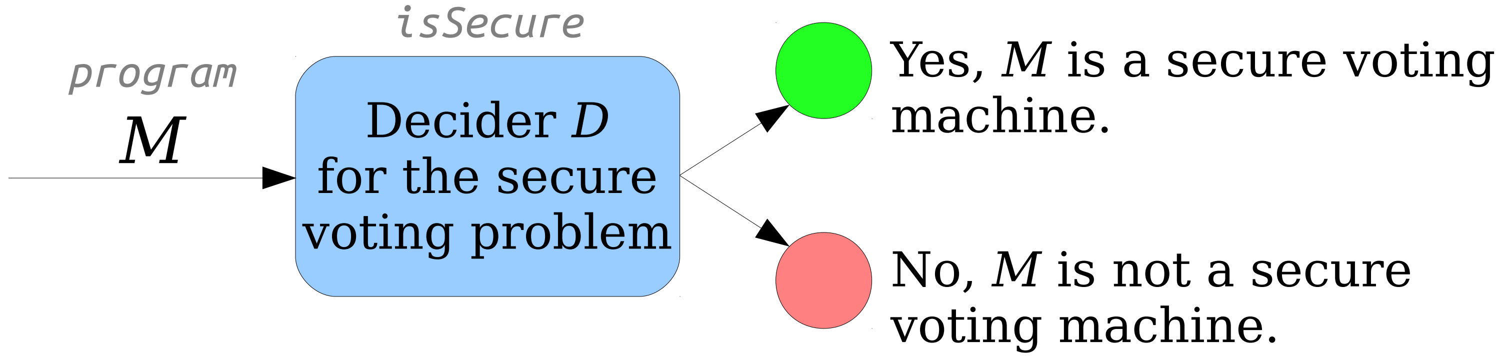


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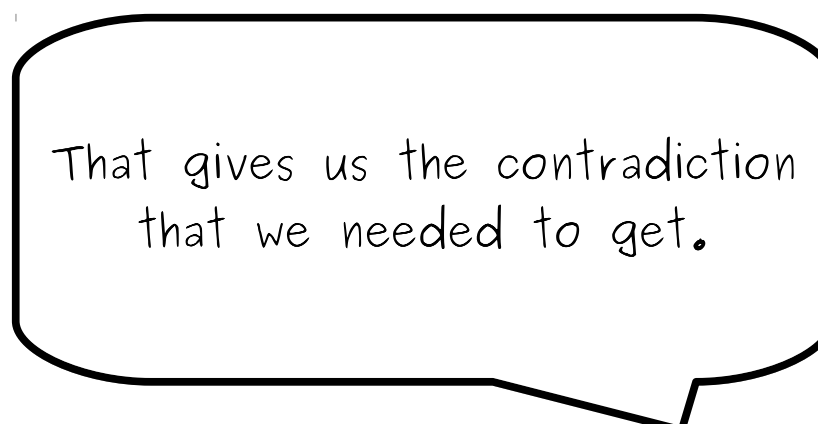

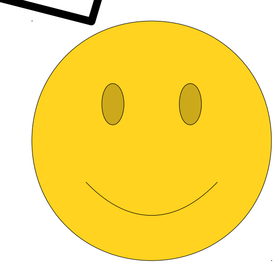

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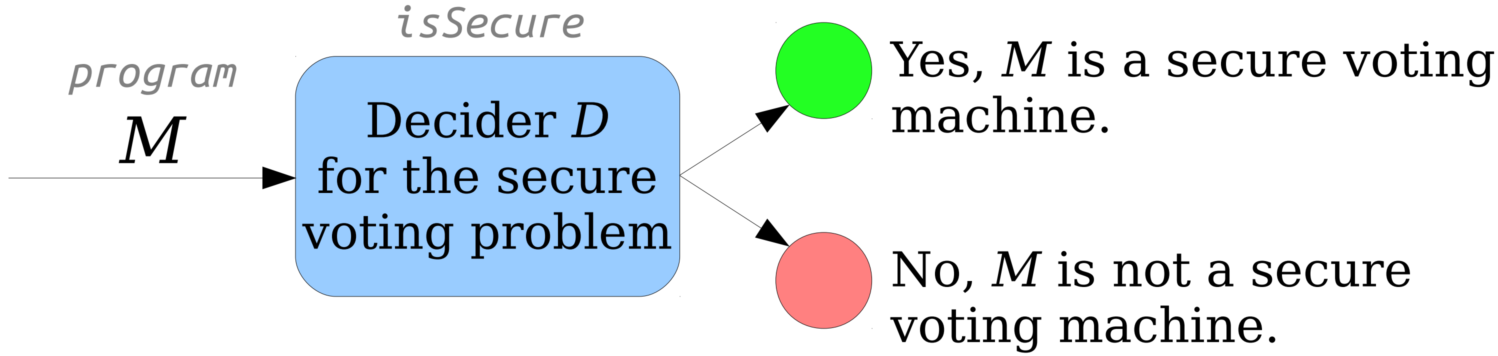


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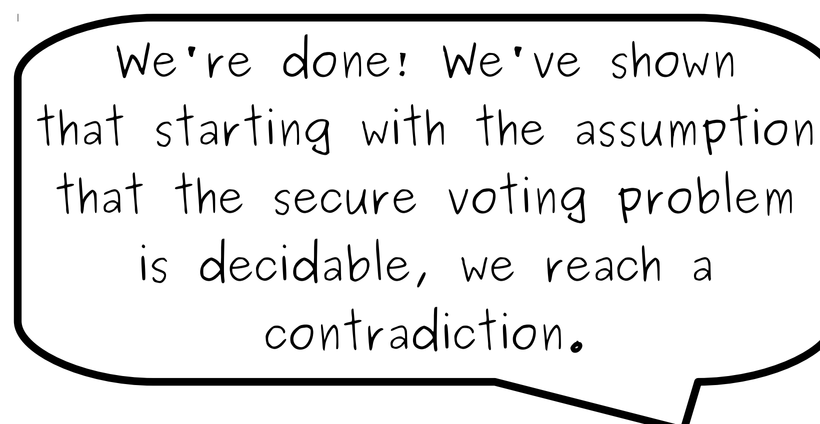

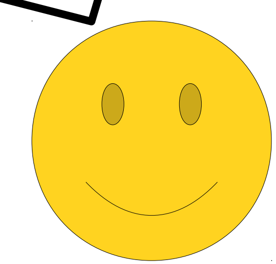

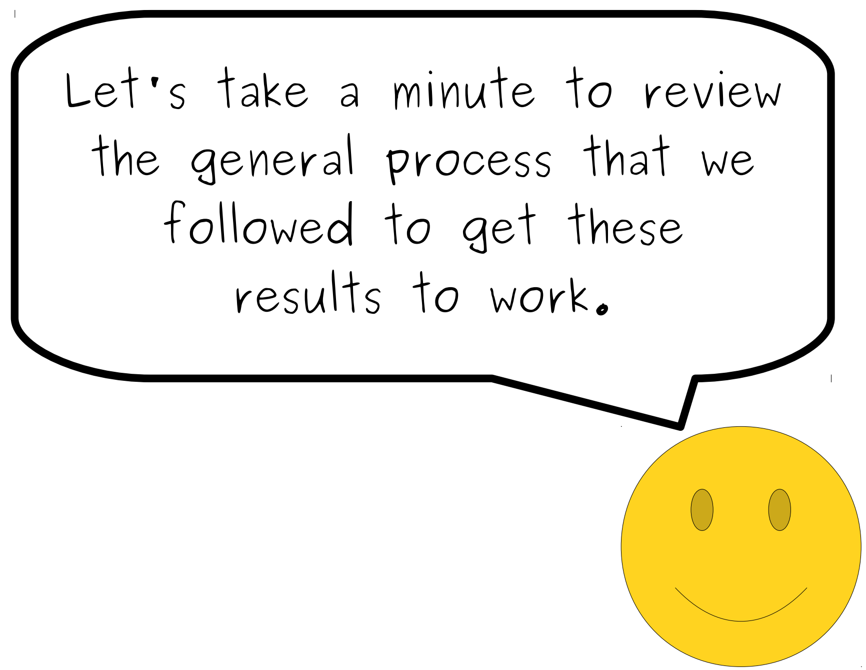
Let's take a minute to review the general process that we followed to get these results to work.


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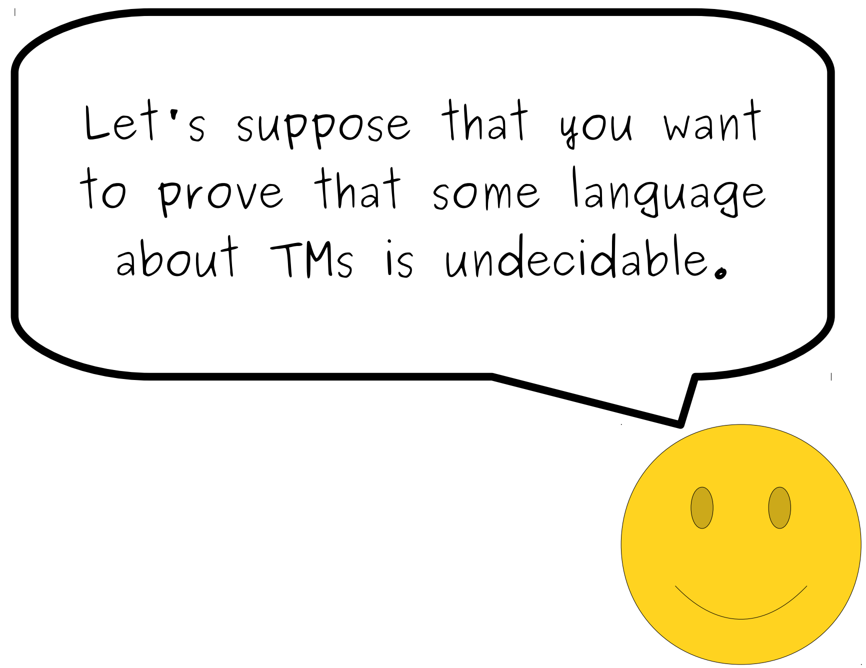
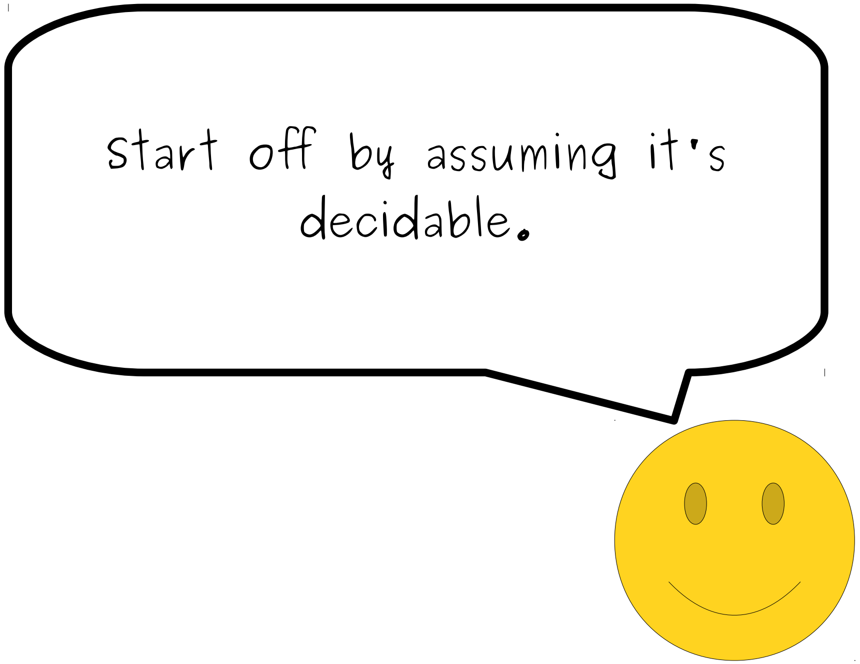
That other guy is going to tell you a general pattern to follow. You might want to take notes.


Let's suppose that you want to prove that some language about TMs is undecidable.

The problem in question is decidable



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Contradiction!

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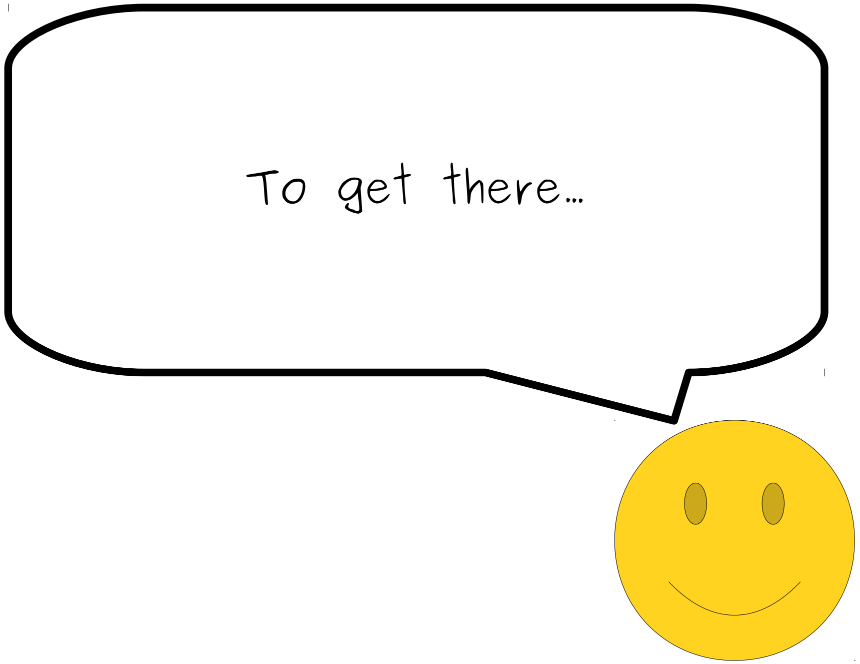

Contradiction!

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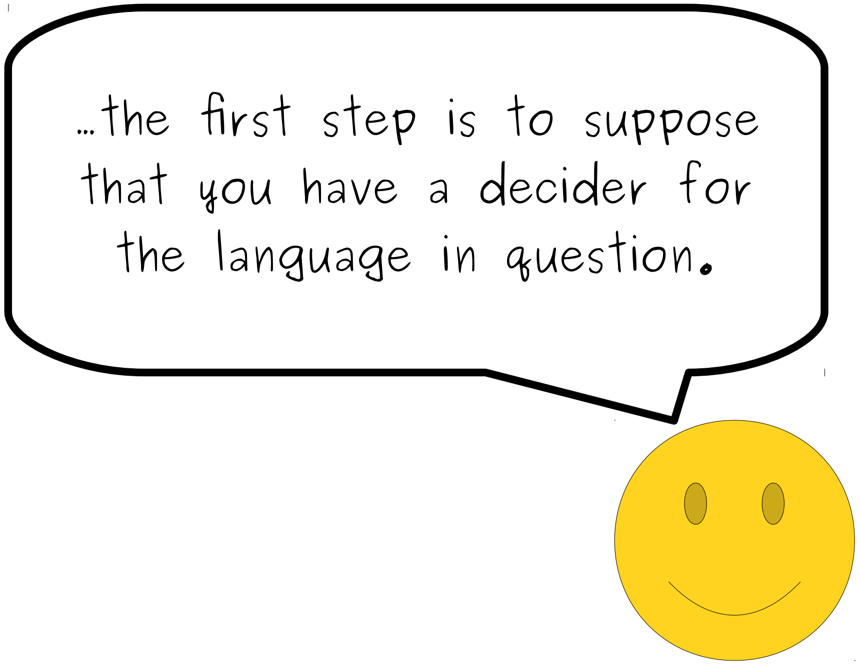

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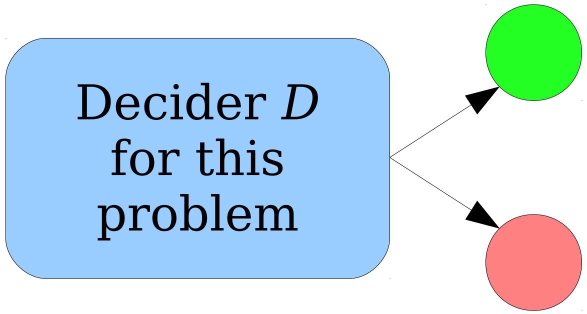

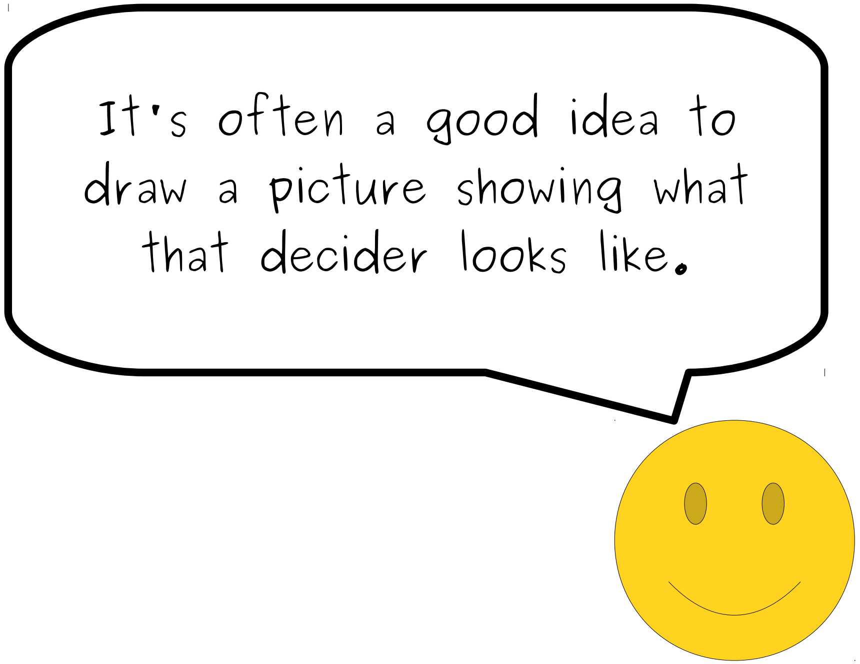

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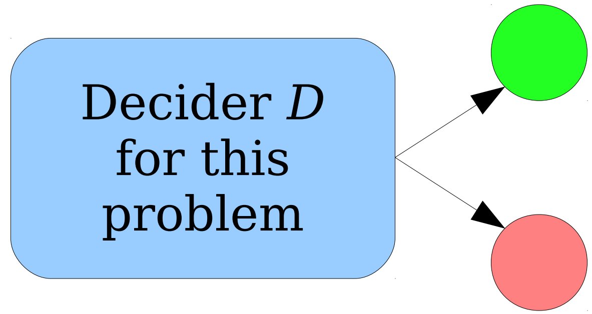

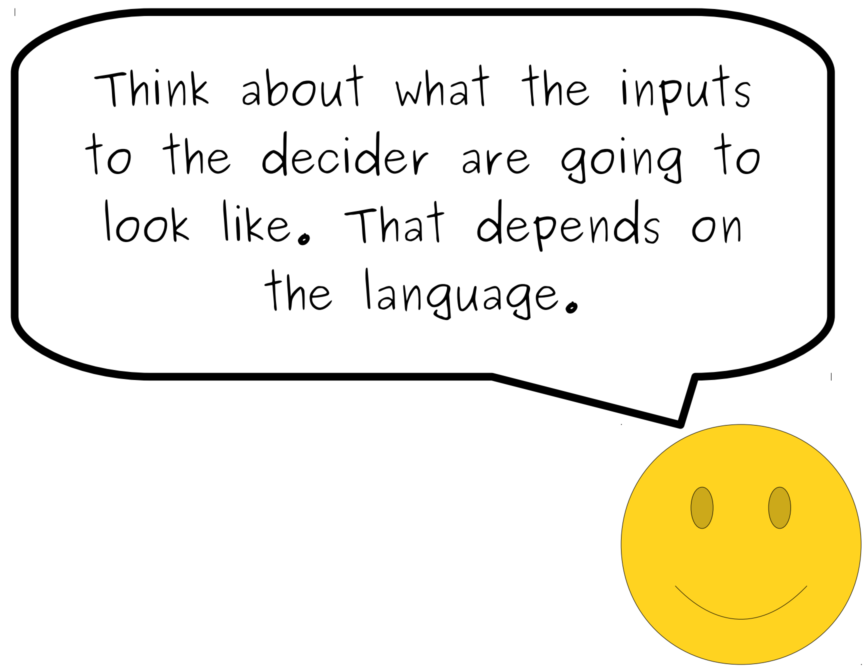

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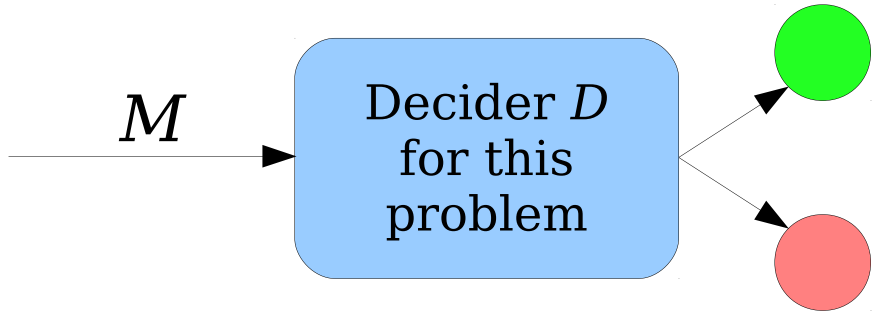

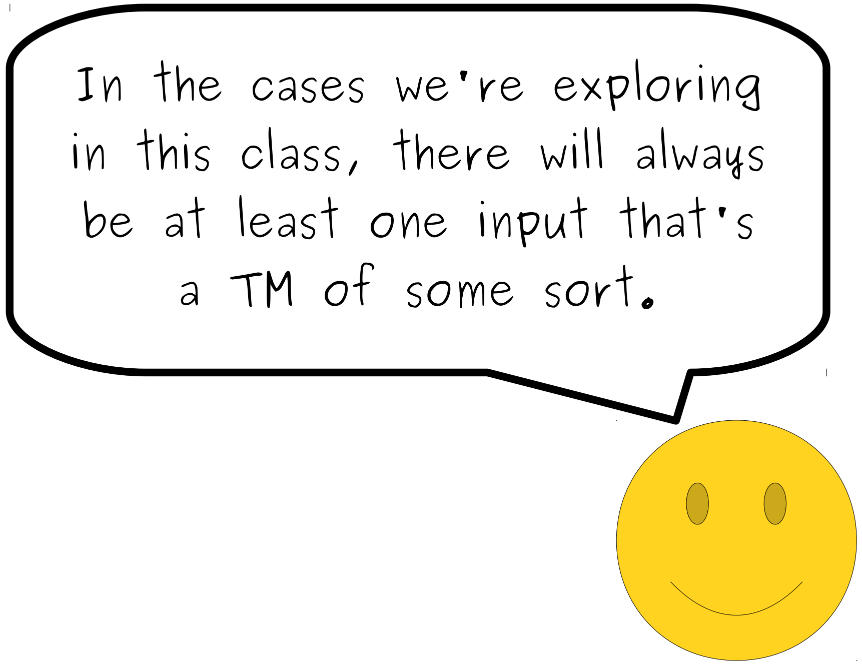

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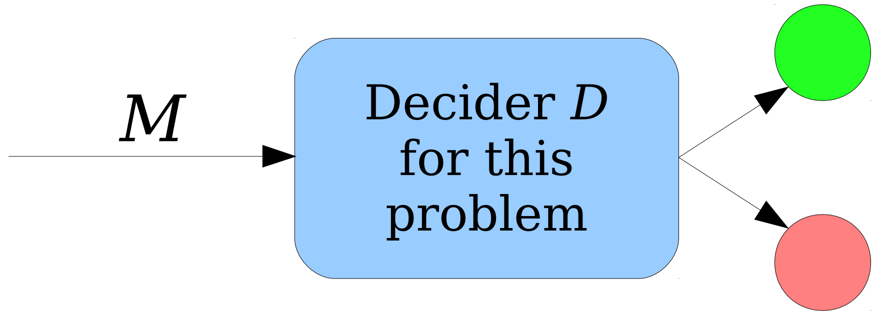

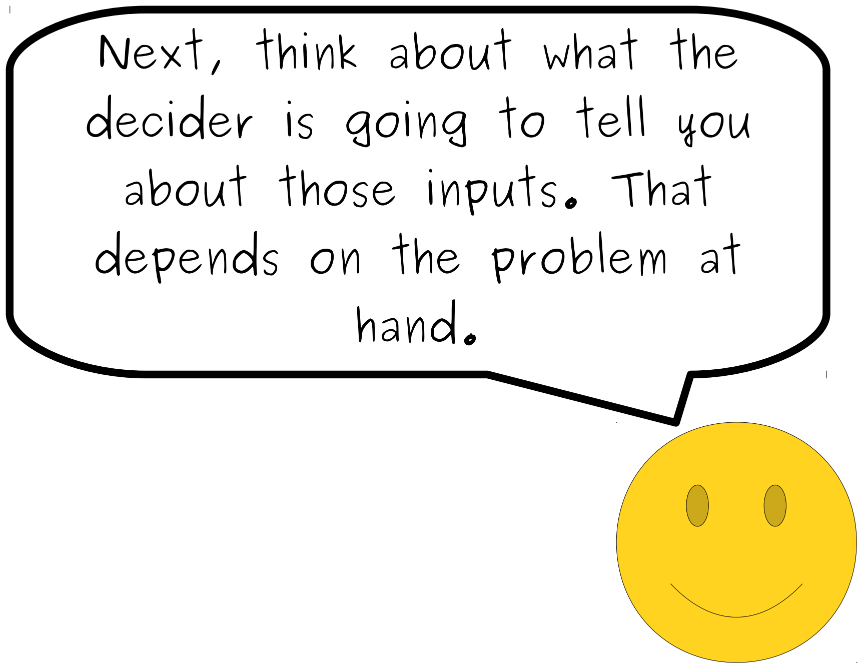

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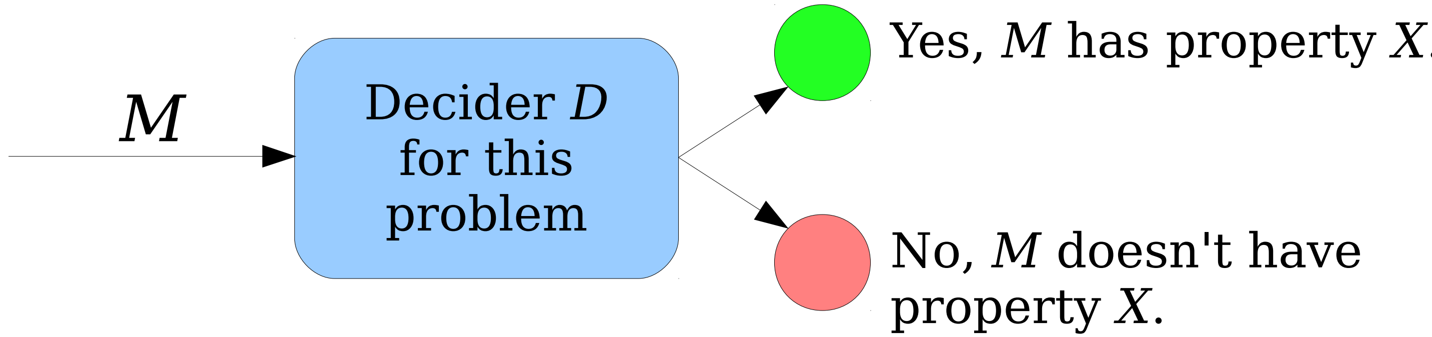

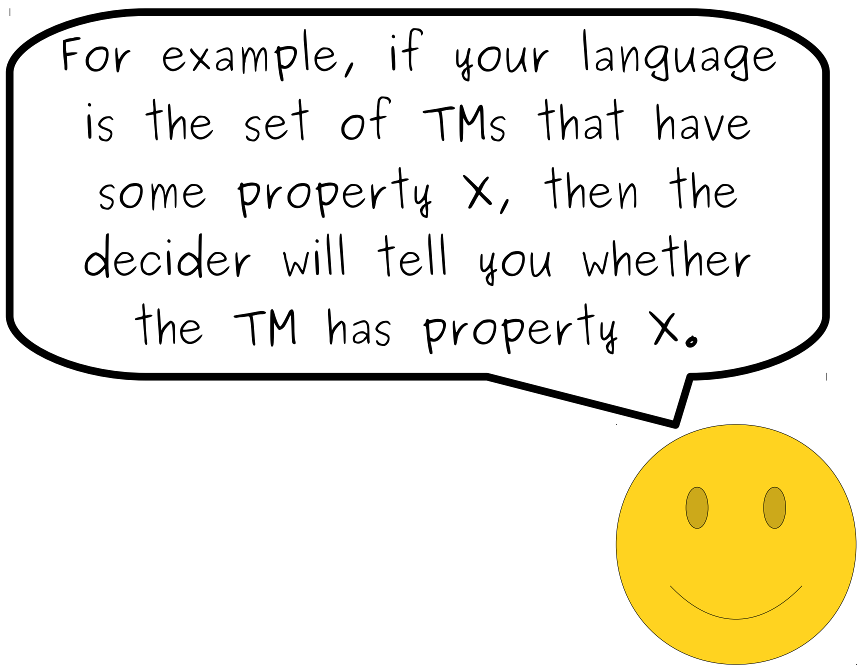

Contradiction!

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We can write programs that use D as a helper method

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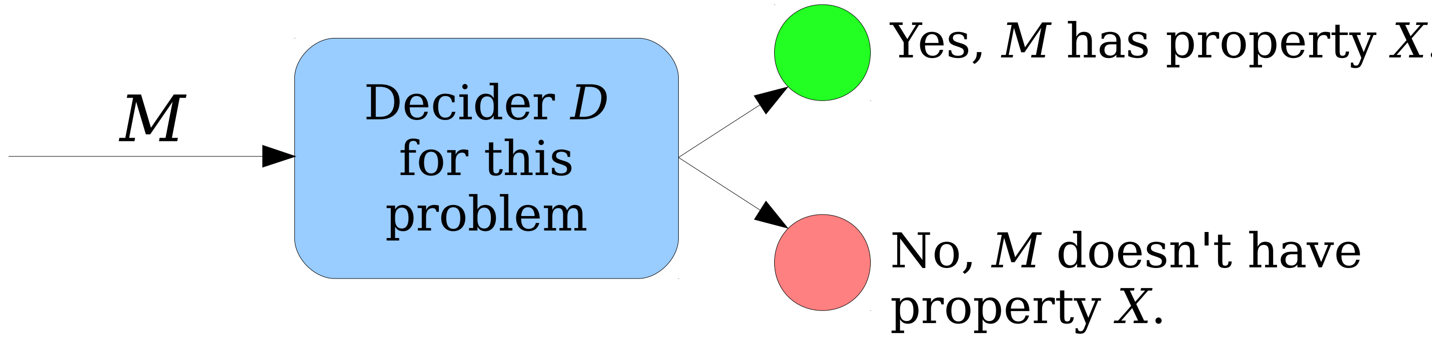

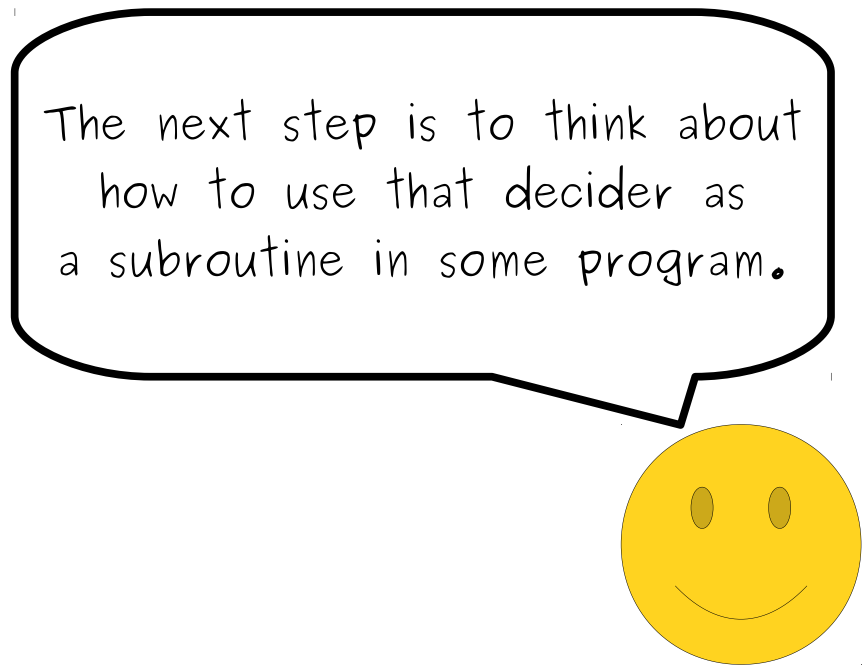

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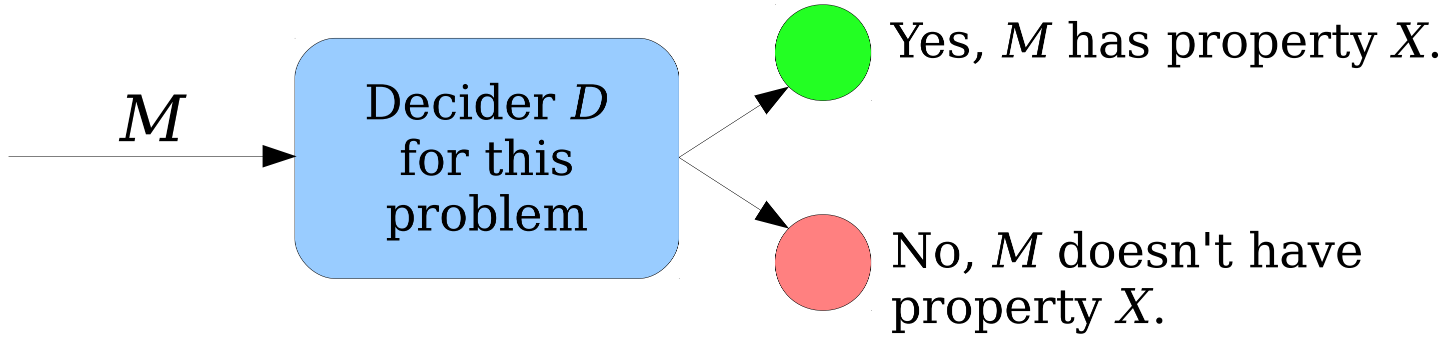

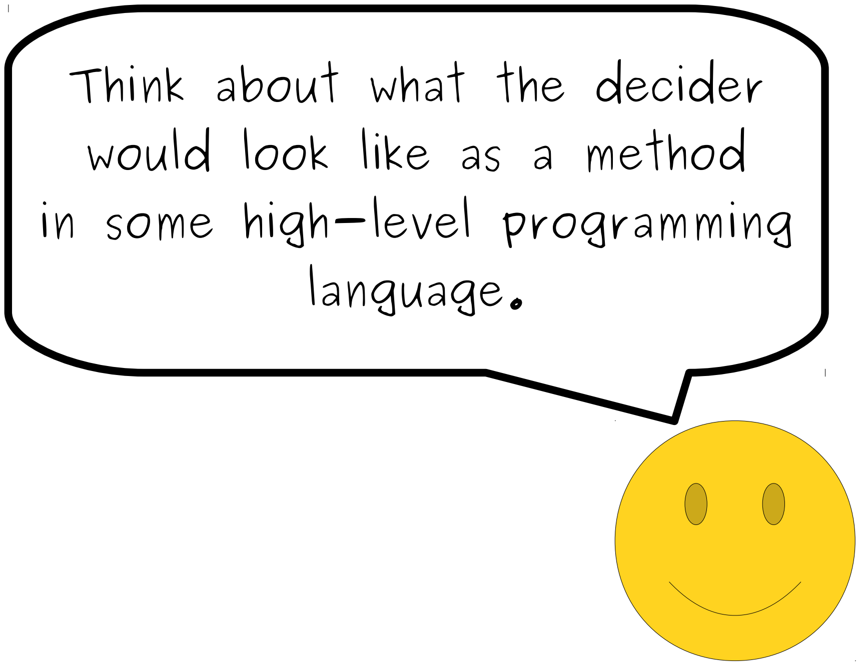

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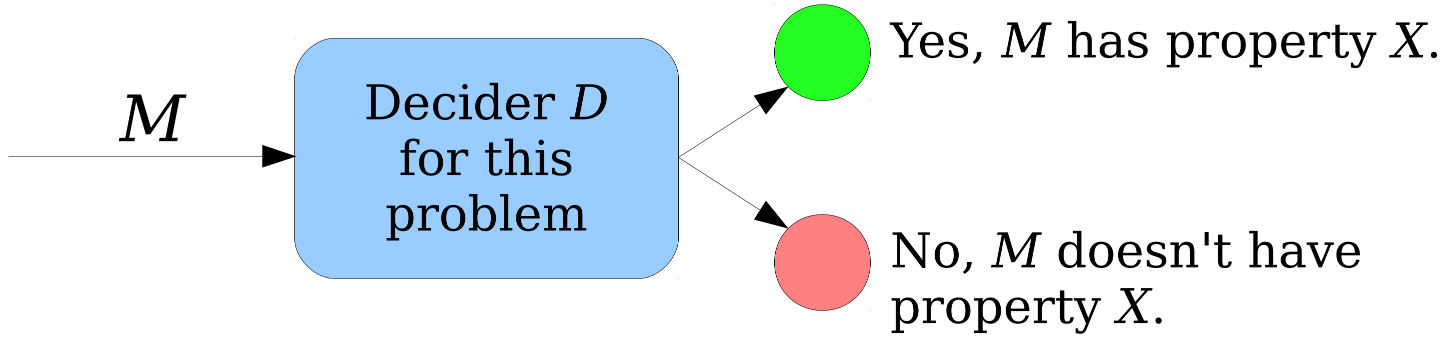

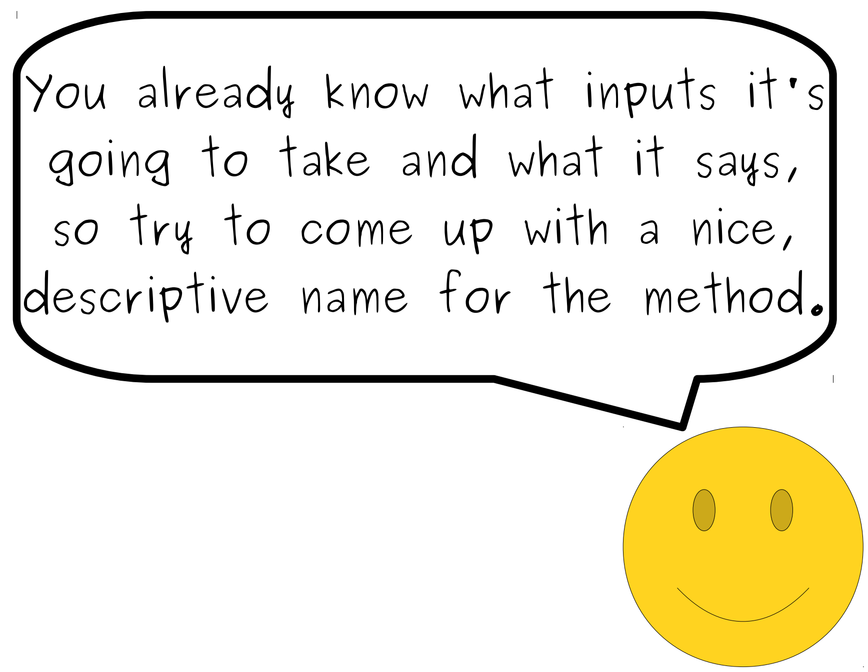

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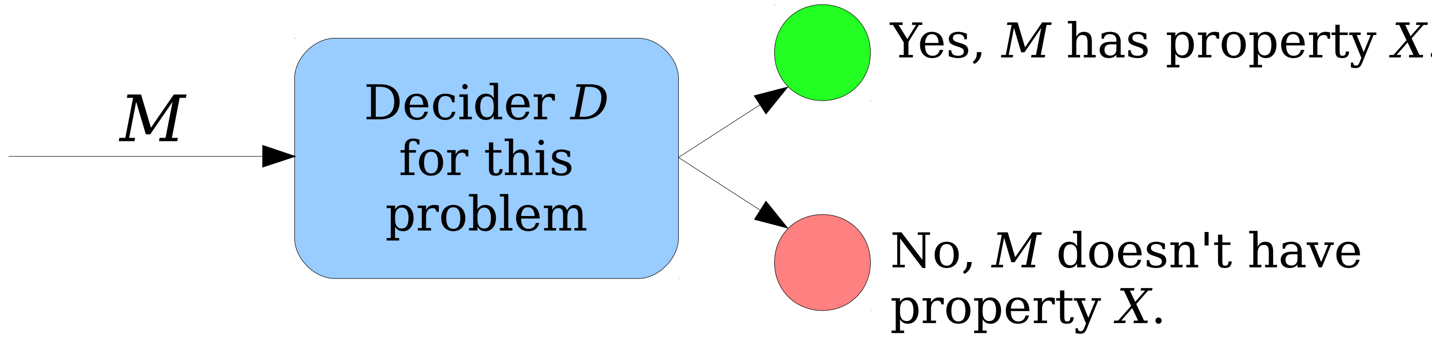


```
bool hasPropertyX(string program)
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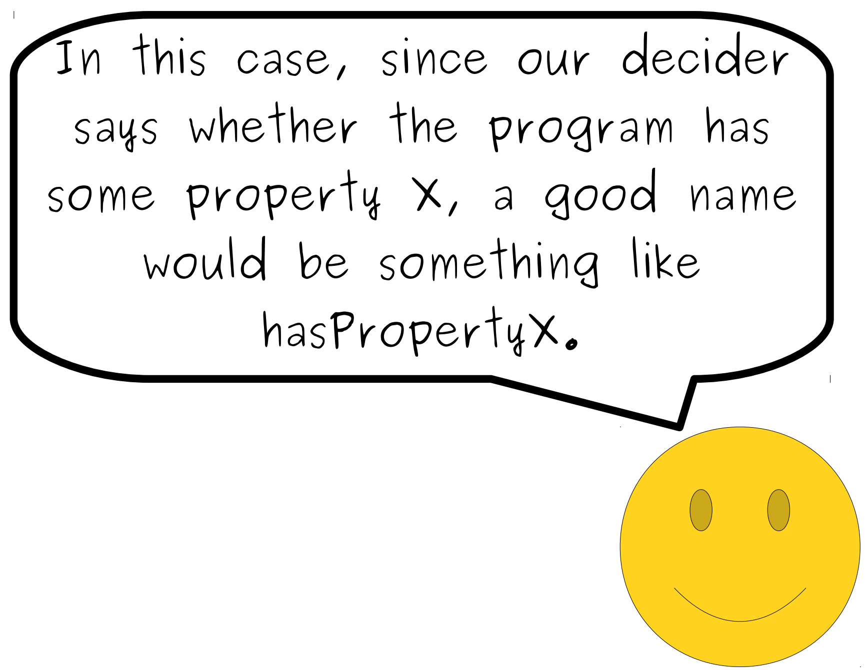

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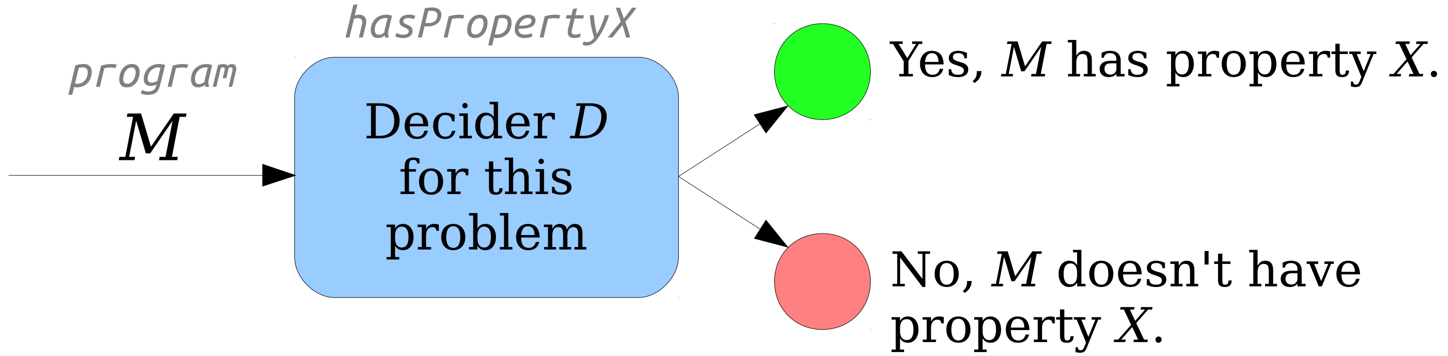


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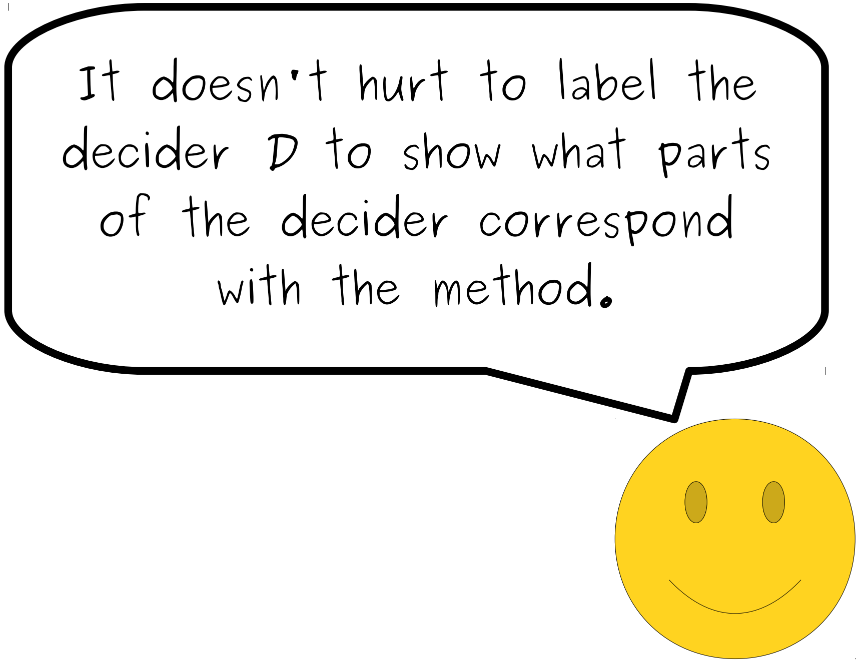

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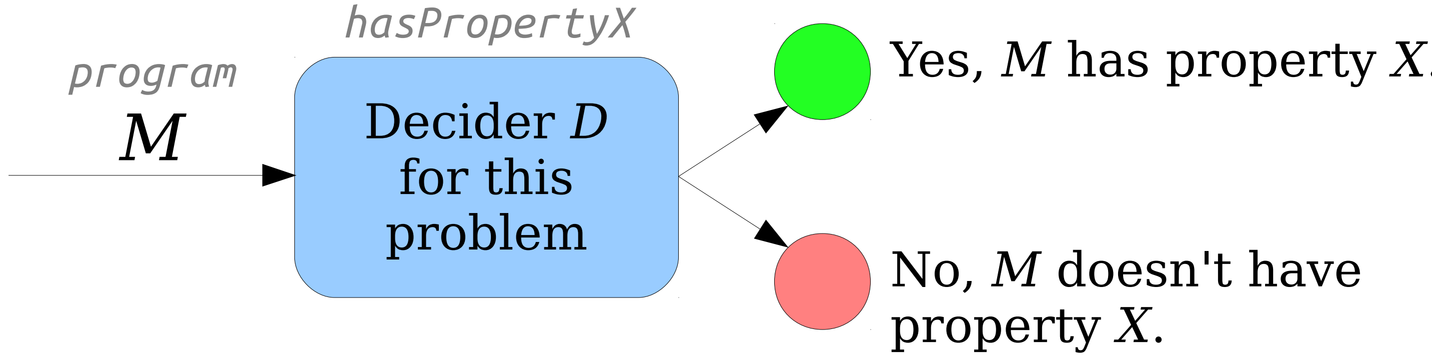


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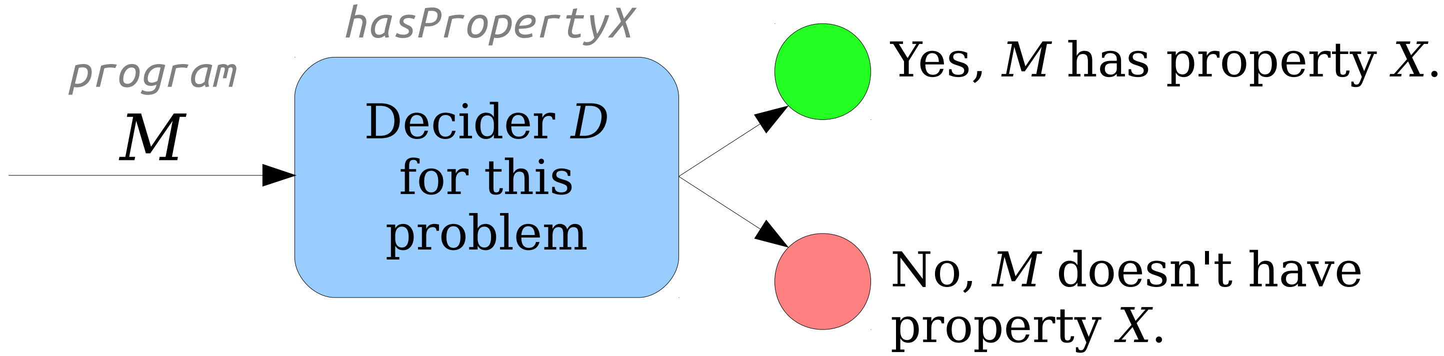


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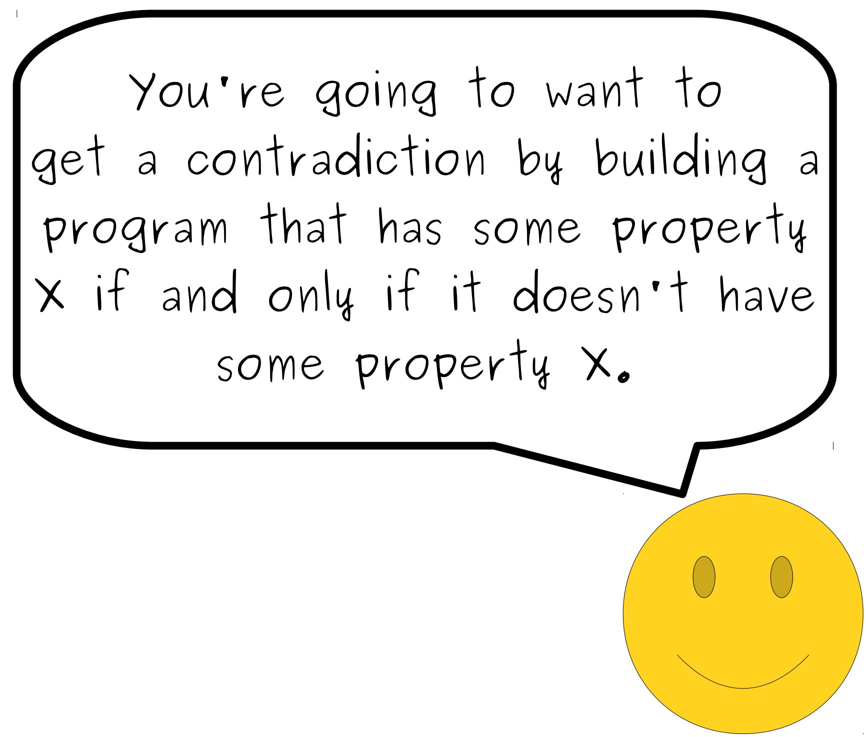

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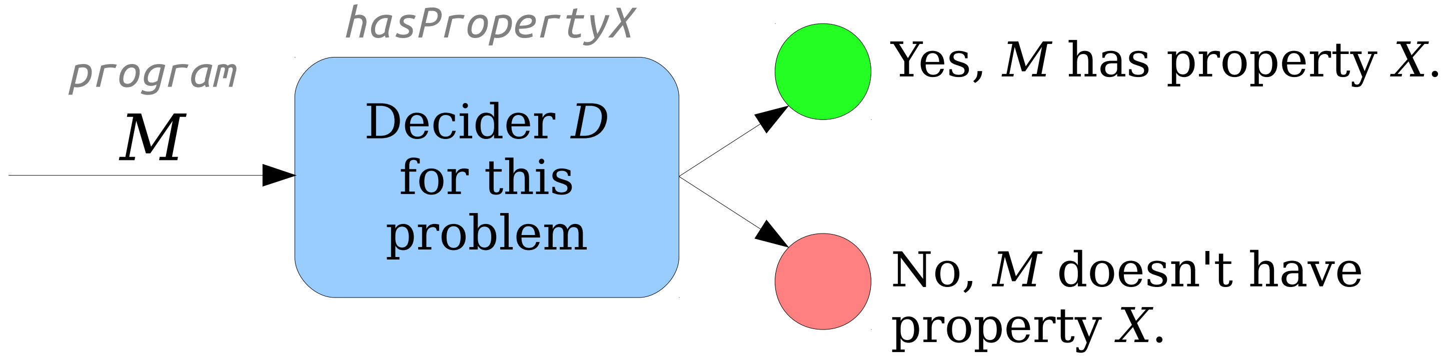

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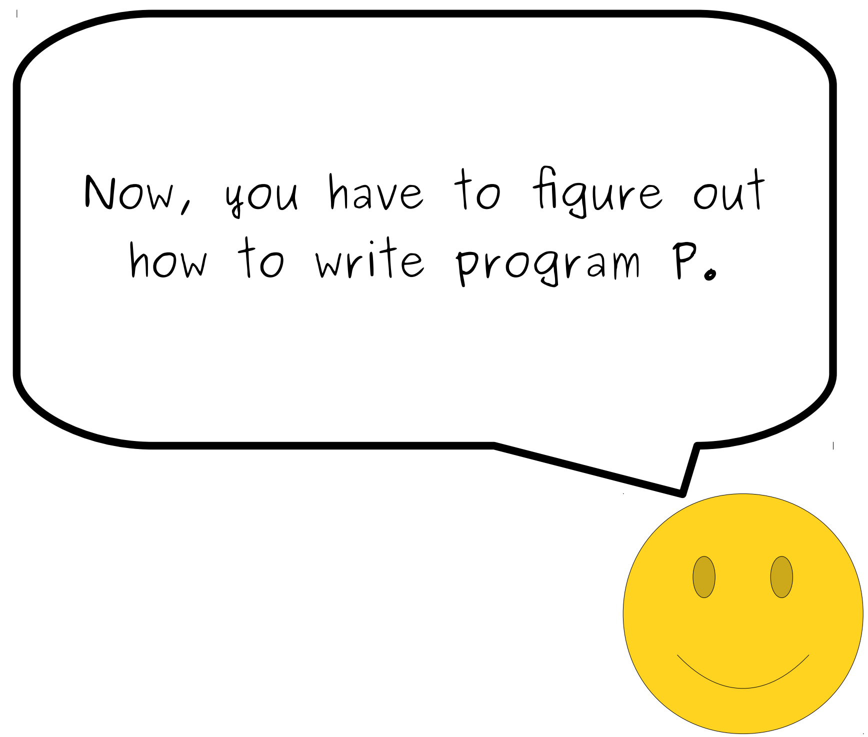

Program P has property X if and only if P doesn't have property X

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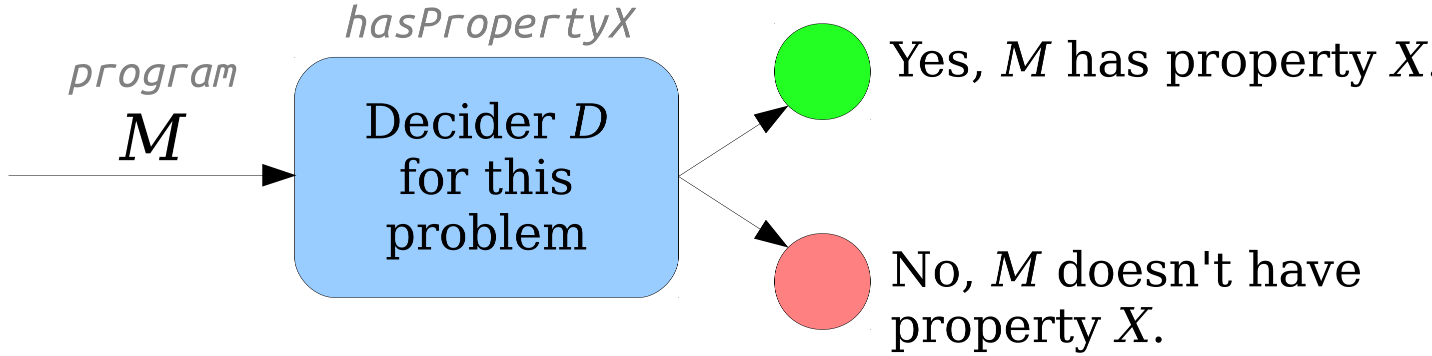


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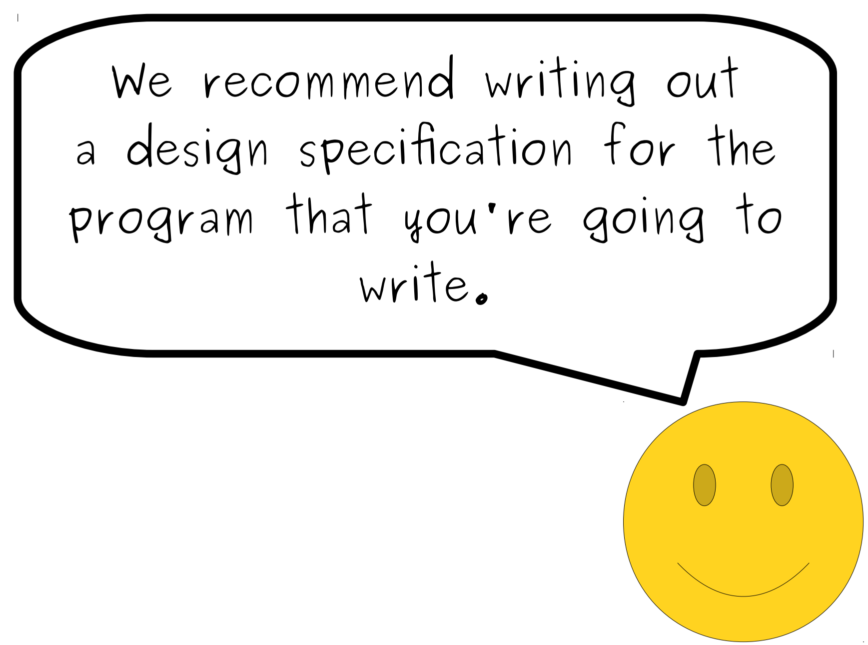

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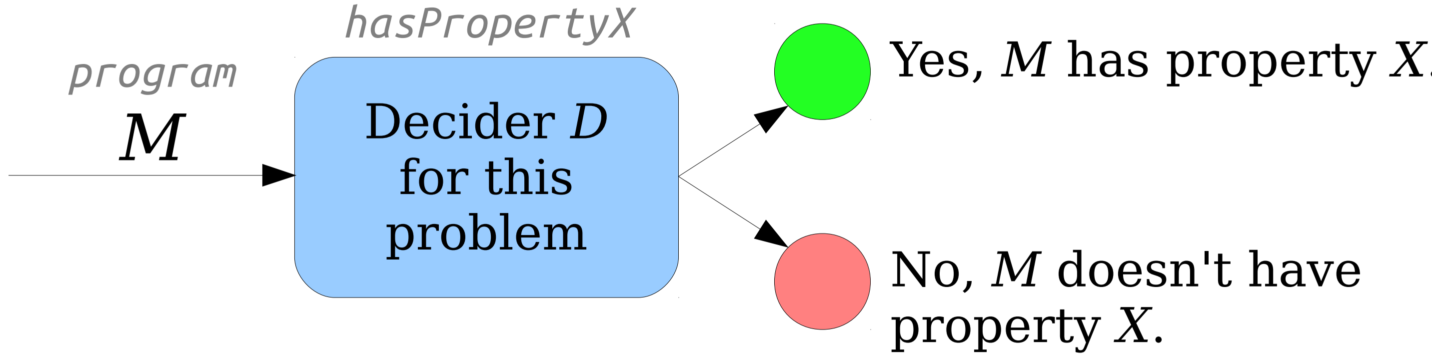


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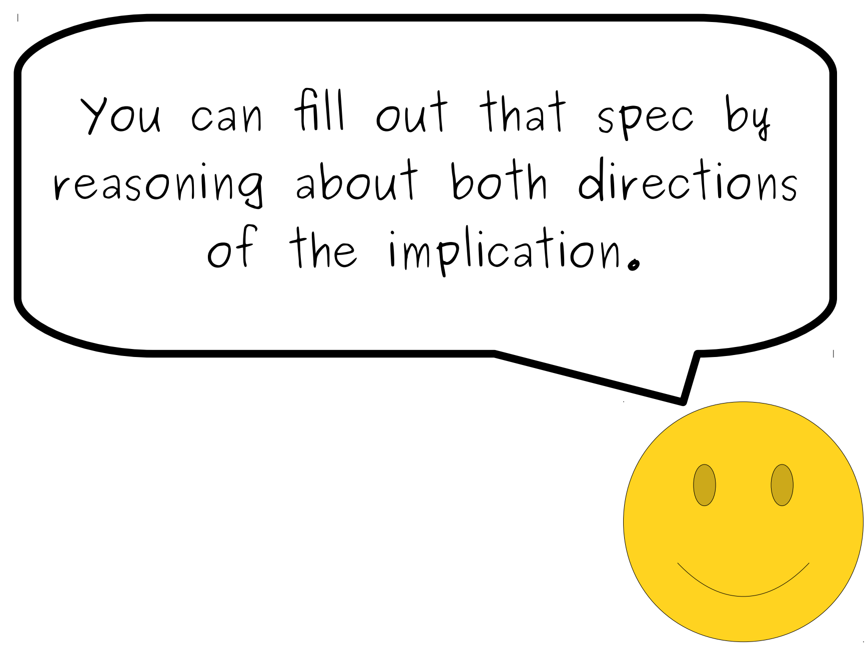

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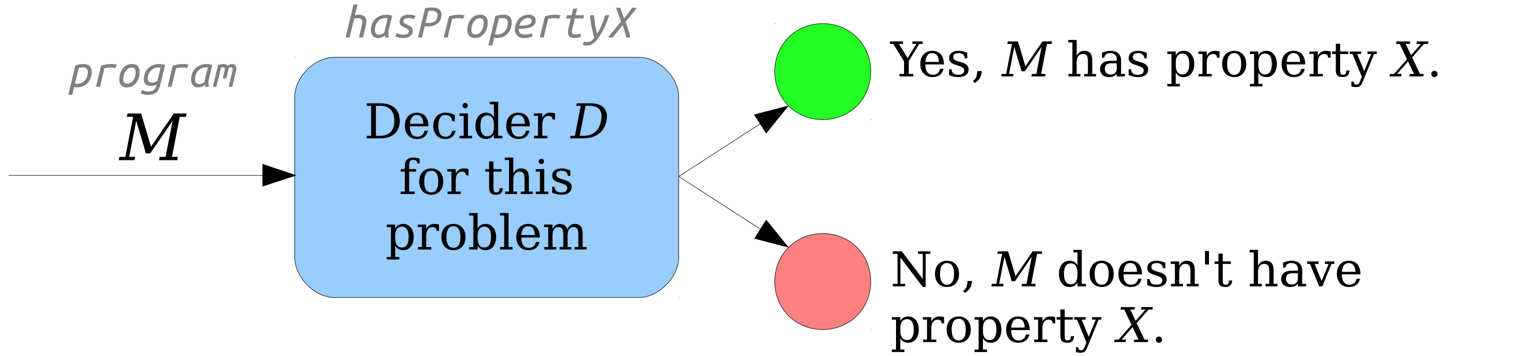


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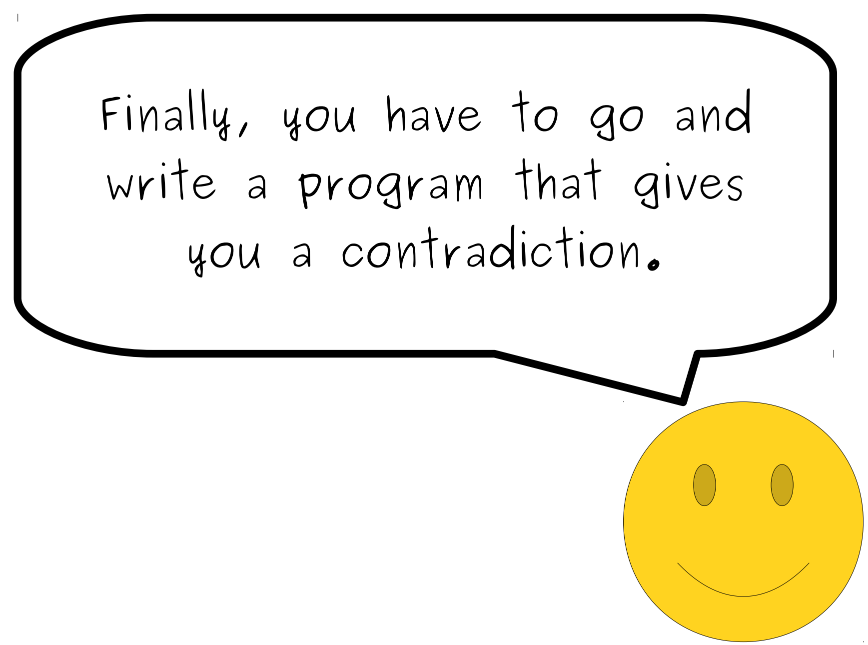

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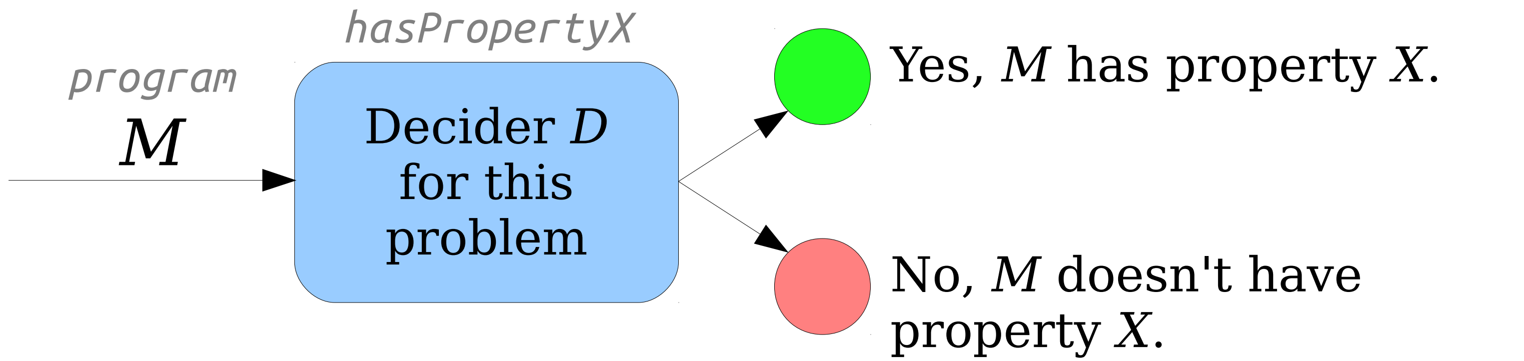


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// Program P

int main() {
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  } else {
    // Do something so you do
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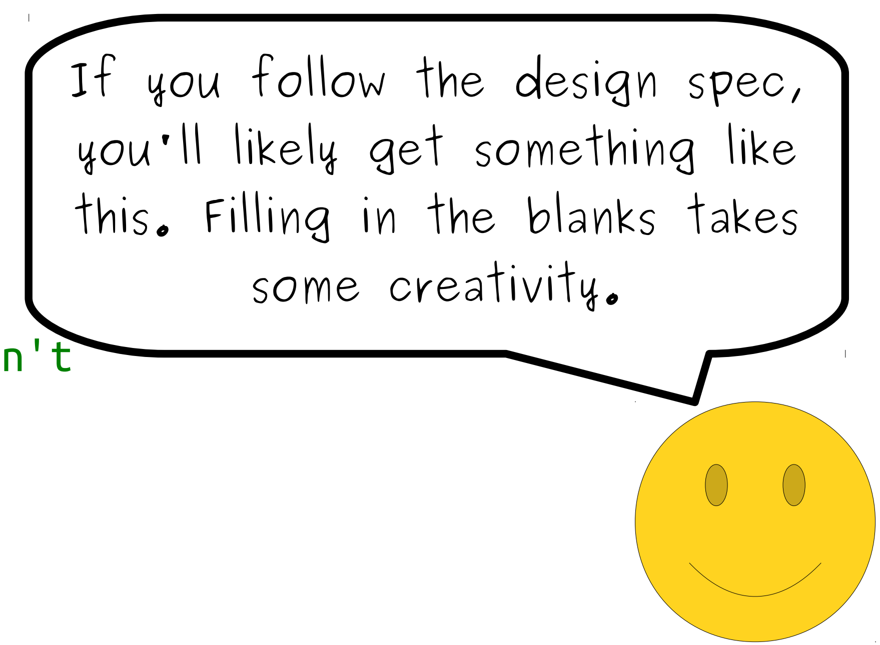

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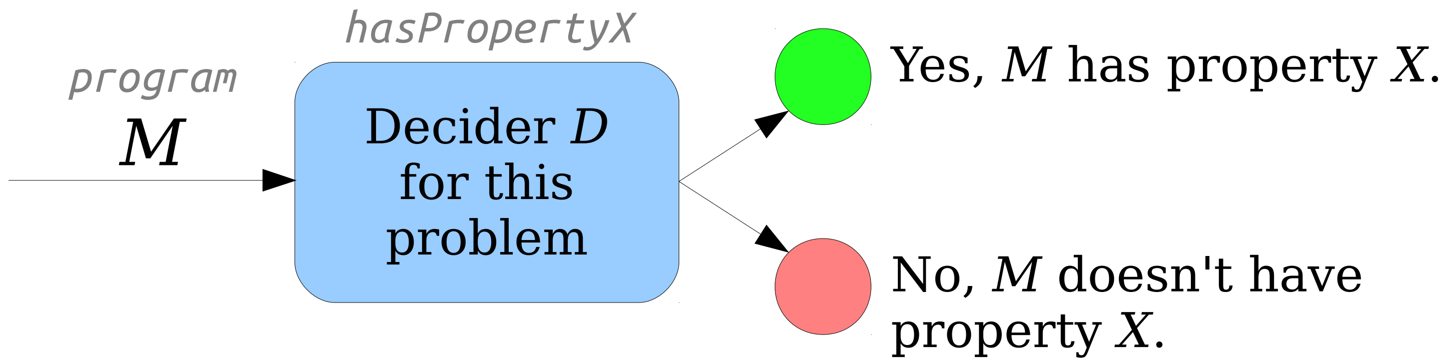


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  }
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Hope this helps!

Please feel free to ask questions if you have them.

Did you find this useful? If so, let us know! We can go and make more guides like these.